TEST - 1 (Paper-I)

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(1)	31.	(2)	61.	(1)
2.	(4)	32.	(2)	62.	(2)
3.	(3)	33.	(2)	63.	(3)
4.	(3)	34.	(3)	64.	(4)
5.	(2)	35.	(3)	65.	(2)
6.	(3)	36.	(2)	66.	(4)
7.	(4)	37.	(4)	67.	(4)
8.	(1)	38.	(3)	68.	(2)
9.	(3)	39.	(4)	69.	(1)
10.	(4)	40.	(2)	70.	(1)
11.	(4)	41.	(3)	71.	(3)
12.	(2)	42.	(3)	**72.**	**(deleted)**
13.	(4)	**43.**	**(2)**	73.	(3)
14.	(2)	44.	(4)	74.	(1)
15.	(4)	45.	(2)	75.	(1)
16.	(3)	46.	(4)	76.	(2)
17.	(3)	47.	(3)	77.	(3)
18.	(2)	48.	(3)	78.	(2)
19.	(3)	49.	(4)	79.	(4)
20.	(2)	50.	(3)	80.	(2)
21.	(3)	51.	(3)	81.	(1)
22.	(2)	52.	(2)	82.	(2)
23.	(3)	53.	(1)	83.	(2)
24.	(1)	54.	(3)	84.	(1)
25.	(2)	55.	(3)	85.	(2)
26.	(4)	**56.**	**(4)**	86.	(4)
27.	(4)	57.	(4)	87.	(1)
28.	(1)	58.	(4)	88.	(3)
29.	(1)	59.	(4)	89.	(1)
30.	(2)	60.	(4)	90.	(4)

PART - A (PHYSICS)

1. Answer (1)

Number of significant figures = 3

$\Rightarrow$ Answer must have 3 significant digits = 164

$\Rightarrow$ Uncertainty must not have decimals because the result 164 has no decimal

$= 164 \pm 3\ cm^2$

2. Answer (4)

A and L are not independent quantities ($\because A = L^2$)

3. Answer (3)

Given,

$P^2 + Q^2 + 2PQ\cos\theta = R^2$ ...(i)

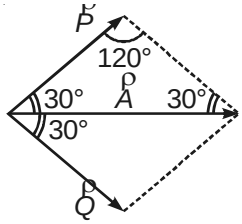
$P^2 + (2Q)^2 + 2P(2Q)\cos\theta = (2R)^2$ ...(ii)

$P^2 + Q^2 - 2PQ\cos\theta = (2R)^2$ ...(iii)

Solving these,

$P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$

4. Answer (3)



$\vec{P}$ and $\vec{Q}$ are components of $\vec{R}$, then

$\vec{P} + \vec{Q} = \vec{A}$

$\Rightarrow \quad \frac{A}{\sin 120^\circ} = \frac{P}{\sin 30^\circ}$

$\Rightarrow \quad P = \frac{A\sin 30^\circ}{\sin 120^\circ} = \frac{4 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}}$

Similarly, $Q = \frac{4}{\sqrt{3}}$

5. Answer (2)

$\frac{dx}{dt} = t^2 - 5t + 6 = 0 \quad \Rightarrow \quad t = 2, 3$

$\frac{d^2x}{dt^2} = 2t - 5$

At $t = 2$ s, $\frac{d^2x}{dt^2}$ = –ve $\Rightarrow$ x is maximum

At $t = 3$ s, $\frac{d^2x}{dt^2}$ = +ve $\Rightarrow$ x is minimum

$\Rightarrow$ At $t = 2$ s, velocity changes from +ve to –ve

At $t = 3$ s, velocity changes from –ve to +ve

After $t = 3$ s, x goes on increasing without limit

$\Rightarrow$ Maximum value of x is not $\frac{20}{3}$ m

Body starts from $x = 2$ m at $t = 0$

$\Rightarrow$ Minimum value of $x = 2$ m

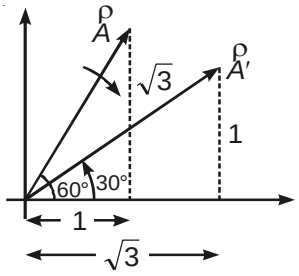
6. Answer (3)

$[B] = [x] = L$

$[dx] = L$

$\Rightarrow \quad \int \frac{dx}{B - x} = M^0L^0T^0 \quad \Rightarrow \quad \frac{A}{B} = L^{-1}$

7. Answer (4)



$\vec{A'} = \sqrt{3}\hat{i} + \hat{j}$

$\Rightarrow \quad \hat{A'} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

8. Answer (1)

$[E] = ML^2T^{-2} = (M^{-1}L^3T^{-2})^p(ML^2T^{-1})^q(LT^{-1})^r$

Equating powers

$\Rightarrow$ $-p + q = 1$ ...(i)

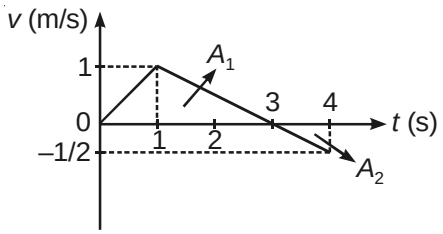
$3p + 2q + r = 2$ ...(ii)

$-2p - q - r = -2$ ...(iii)

Solve and get,

$p = \frac{-1}{2},\ q = \frac{1}{2},\ r = \frac{5}{2}$

9. Answer (3)



Displacement = $A_1 - A_2$

$= \frac{5}{4}$ m

Distance = $A_1 + A_2$

$= \frac{7}{4}$ m

Average velocity = $\frac{\frac{5}{4}}{4}$

$= \frac{5}{16}$ m/s

10. Answer (4)

$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = \vec{A}\times\vec{A} - \vec{A}\times\vec{B} + \vec{B}\times\vec{A} - \vec{B}\times\vec{B}$

$= 2(\vec{B}\times\vec{A})$

11. Answer (4)

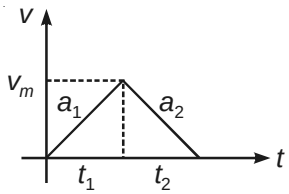
$t = \frac{T}{2} = \frac{u\sin 30°}{g\cos 45°}$

At this point, $v_y = 0$

$v_x = 10\cos 30° - g\sin 45° \times t$

$= 5(\sqrt{3}-1)$ m/s

12. Answer (2)



$a_1t_1 = a_2t_2 = v_m$

$\Rightarrow v_1 = \frac{\left\{\frac{1}{2}\times(t_1+t_2)\times v_m\right\}}{t_1+t_2}$

$\Rightarrow \frac{v_m}{2} = \frac{1}{2}a_1t_1 = \frac{1}{2}a_2t_2$

13. Answer (4)

$y = 9x^2$

$\Rightarrow \frac{dy}{dt} = 18x\cdot\frac{dx}{dt} = 6x$

$\frac{dx}{dt} = \frac{1}{3}$ m/s = constant

$\Rightarrow \frac{d^2x}{dt^2} = 0$

$\Rightarrow \frac{d^2y}{dt^2} = 6\cdot\frac{dx}{dt} = 2$ m/s^2

$\Rightarrow$ Total acceleration = 2 m/s^2

14. Answer (2)

When velocity = 0

$\Rightarrow \frac{dx}{dt} = 0$

$\Rightarrow -\left[\frac{1}{a} - \frac{2t}{b}\right] = 0$

$\Rightarrow t = \frac{b}{2a}$

15. Answer (4)

$\frac{dv}{dt} = -v$

$\Rightarrow \int_u^v \frac{dv}{v} = \int_0^t -dt$

$\Rightarrow v = ue^{-t}$

16. Answer (3)

$\frac{\Delta P}{P} = 3\frac{\Delta A}{A} + \frac{1}{2}\frac{\Delta B}{B} + 4\frac{\Delta C}{C} + \frac{3}{2}\frac{\Delta D}{D}$

$\Rightarrow$ Contribution of error in C is maximum

17. Answer (3)

50.14 cm has 4 significant digits, whereas 0.00025 A has only 2 significant digits.

$\Rightarrow$ 50.14 cm is more accurate

$\Rightarrow$ 470 km + 380 m = 470.380 km

Round off to no decimal places = 470 km

18. Answer (2)

$K_i = \frac{1}{2}mu^2 = 100$ mJ

At half KE,

$K = 50$ mJ

$\Rightarrow mgy = 50$ m

$\Rightarrow y = 5$ m

$$y = u\sin\theta \cdot t - \frac{1}{2}gt^2$$

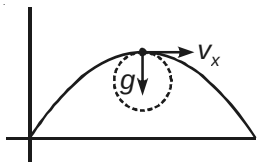
$$\Rightarrow \quad 5 = 5\sqrt{6}\cdot t - 5t^2$$

$$\Rightarrow \quad t^2 - \sqrt{6}t + 1 = 0$$

$$\Rightarrow \quad t = \frac{\sqrt{6} \pm \sqrt{6-4}}{2} = \frac{\sqrt{6} \pm \sqrt{2}}{2}$$

$$\Rightarrow \quad t_1 - t_2 = \sqrt{2}\ \text{s}$$

19. Answer (3)



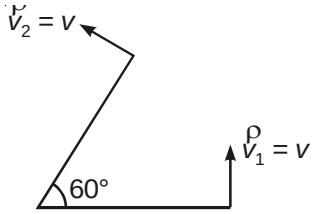
$$r = \frac{v_x^2}{g} = \frac{a^2}{g}$$

20. Answer (2)

$\vec{a}, \vec{b}$ and $\vec{c}$ lie in the same plane

$\Rightarrow$ Scalar triple product is zero

21. Answer (3)



$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1$$

$$\Rightarrow \quad v_{21} = \sqrt{v^2 + v^2 - 2vv\cos 60°}$$

$$= v$$

22. Answer (2)

$$\frac{dv}{dt} = 4 - 2v$$

$$\Rightarrow \quad \int_0^v \frac{dv}{4-2v} = \int_0^t dt$$

$$\Rightarrow \quad v = 2(1 - e^{-2t})$$

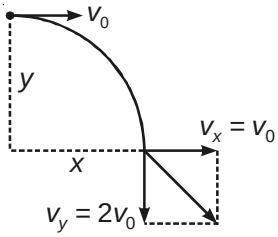
$$\Rightarrow \quad a = \frac{dv}{dt} = 4e^{-2t} = \text{always} + \text{ve}$$

Finally, $v_f = 2$ m/s

23. Answer (3)

$$t = \frac{x}{v_0}$$

$$y = \frac{1}{2}gt^2 = \frac{gx^2}{2v_0^2} = x$$



$$\Rightarrow \quad x = \frac{2v_0^2}{g} = y$$

$$\Rightarrow \quad v_y = \sqrt{2gy} = 2v_0$$

$$\Rightarrow \quad v = \sqrt{5}v_0$$

24. Answer (1)

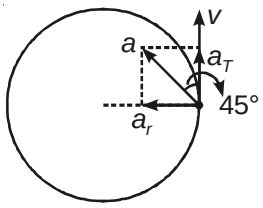
100 Pa = 100 N/m^2

$$= 100 \times \frac{10^5 \text{ dyne}}{(10^2)^2 \text{ cm}^2}$$

= 1000 dyne/cm^2

25. Answer (2)

$$\tan 45° = \frac{a_r}{a_T}$$



$$\Rightarrow \quad a_r = a_T$$

$$\Rightarrow \quad \alpha \cdot r = \omega^2 r$$

$$\Rightarrow \quad \alpha = (\alpha t)^2$$

$$\Rightarrow \quad t = \sqrt{\frac{1}{\alpha}} = \sqrt{\frac{8}{\pi}}\ \text{s}$$

26. Answer (4)

Direction of acceleration keeps on changing

$\Rightarrow$ Non-uniform acceleration.

27. Answer (4)

A straight line can also be $P = mQ + c$

$$\Rightarrow \quad \frac{P}{Q} = m + \frac{c}{Q} = \text{not constant}$$

28. Answer (1)

| Displacement | $\leq$ Distance

29. Answer (1)

The least count error is distributed over each oscillation.

$\Rightarrow$ Larger number of oscillation means less percentage error.

30. Answer (2)

Both statements are outcome of air resistance force.

PART - B (CHEMISTRY)

31. Answer (2)

Let the atomic weights of A and B be 'a' and 'b' respectively.

Molecular weight of A_2B_3 = 2a + 3b

Molecular weight of AB_2 = a + 2b

$$\text{Molecular weight} = \frac{\text{Weight}}{\text{Number of mole}}$$

Molecular weight of A_2B_3 = 2a + 3b

$$= \frac{31.8}{0.30}$$

Molecular weight of AB_2 = a + 2b

$$= \frac{18.6}{0.30}$$

Solving these two,

a = 26 and b = 18

32. Answer (2)

$x \times 1 + 2 \times 2 = 1.5(2 + x)$

$\Rightarrow$ $x + 4 = 3 + 1.5x$

$\Rightarrow$ $0.5x = 1$

$\Rightarrow$ $x = 2$

$\Rightarrow$ $y = 4$

33. Answer (2)

meq of H_2SO_4 (initial) = 30 × 1

meq of H_2SO_4 after passing NH_3 = 30 × 0.2 = 6

meq of H_2SO_4 reacted = meq of NH_3

= 30 – 6

= 24

$$\therefore \quad \frac{W_{NH_3}}{17} \times 1000 = 24; \; W_{NH_3} = 0.408 \text{ g}$$

$$V_{NH_3} \text{ at STP} = \frac{0.408}{17} \times 22.4$$

= 0.5376 L

= 537.6 mL

34. Answer (3)

Moles of Mg^{2+} = 0.3 + 0.2 = 0.5

Number of gram equivalent = 0.5 × 2 = 1

Moles of SO_4^{2-} = 0.2

Equivalent of SO_4^{2-} = 0.2 × 2 = 0.4

Equivalent of Cl^- = 0.6 × 1 = 0.6

Total equivalent = 1 + 0.4 + 0.6 = 2

$\therefore$ Equivalent fraction of $Mg^{2+} = \frac{1}{2} = 0.5$

35. Answer (3)

36. Answer (2)

$C^{12}\ O^{16}\ O^{16}$

$C^{12}\ O^{17}\ O^{17}$

$C^{12}\ O^{18}\ O^{18}$

$C^{12}\ O^{16}\ O^{17}$

$C^{12}\ O^{16}\ O^{18}$

$C^{12}\ O^{17}\ O^{18}$

Similarly, six molecule with C^{12}.

37. Answer (4)

Energy of 1 mole photon in 1 mole.

$$E = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^{-7}} \times 6.022 \times 10^{23}$$

$\approx$ 297 kJ/mole

Percentage of energy converted to KE

$$= \frac{297 - 246.5}{297} \times 100$$

= 17%

38. Answer (3)

Radial node occurs where probability of finding electron is zero.

$\therefore$ $\psi^2 = 0$ or $\psi = 0$

$\therefore$ $6 - 6\sigma + \sigma^2 = 0$

$\therefore$ $\sigma = 3 \pm \sqrt{3}$

For maximum distance, $r = \frac{3(3+\sqrt{3})a_0}{2Z}$

39. Answer (4)

40. Answer (2)

Initial energy of photon = Energy of photon after scattering + KE of scattered electron,

$$h\nu_1 = h\nu_2 + \frac{1}{2}mv^2$$

$\Rightarrow \frac{1}{2}mv^2 = h(\nu_1 - \nu_2)$

$\Rightarrow v^2 = \frac{2h(\nu_1 - \nu_2)}{m}$

$= \frac{2 \times 6.63 \times 10^{-34}(10^{11} - 0.9 \times 10^{11})}{9.1 \times 10^{-31}}$

$\Rightarrow v = \sqrt{14571428} = 3.8 \times 10^3 \text{ ms}^{-1}$

41. Answer (3)

42. Answer (3)

For first excited state,

n = 2

$\Rightarrow \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \frac{n_2^4}{n_1^4} = \frac{16}{1} \quad \left(r = \frac{n^2}{Z} \times 0.529 \text{ Å}\right)$

43. Answer (2)

Change in PE $= -2 \times \frac{y}{4} - (-2y)$

$= \frac{-y}{2} + 2y$

$= \frac{3}{2}y$

44. Answer (4)

$x_1 = cR\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = cR$

$x_2 = cR\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3cR}{4}$

$x_3 = cR\left[\frac{1}{2^2} - \infty\right] = \frac{cR}{4}$

$\therefore \quad x_1 - x_2 = x_3$

45. Answer (2)

$_{105}Rf \longrightarrow 5f^{14}6d^3$

46. Answer (4)

n = 4

l = 0, m = 0

l = 1, m = –1, 0, +1

l = 2, m = –2, –1, 0, +1, +2

l = 3, m = –3, –2, –1, 0, +1, +2, +3

47. Answer (3)

$Fe^{2+} \rightarrow 3d^6 4s^0 \rightarrow$ 4 unpaired electrons

$Mn^{2+} \rightarrow 3d^5 4s^0 \rightarrow$ 5 unpaired electrons

$\therefore \quad \frac{\mu_{Fe^{2+}}}{\mu_{Mn^{2+}}} = \frac{\sqrt{24}}{\sqrt{35}}$

48. Answer (3)

49. Answer (4)

meq of NH_3 = 10

m moles of $(NH_4)_2SO_4$ = 5

Weight of $(NH_4)_2SO_4 = \frac{5}{1000} \times 132 = 0.66$ g

Percentage $= \frac{0.66}{0.70} \times 100 = 94.28\%$

50. Answer (3)

51. Answer (3)

52. Answer (2)

$\because$ 34 g of H_2O_2 give 16 g or 11200 mL of O_2

$\therefore \quad \frac{34}{1120} \text{ g } H_2O_2 = \frac{11200}{34} \times \frac{34}{1120} = 10 \text{ mL } O_2$

Volume strength = 10 volume

53. Answer (1)

$\frac{1-x}{49} + \frac{x}{40} = 53.4 \times 10^{-3} \times 0.4$

$\Rightarrow$ x = 0.206

Percentage of SO_3 = 20.6%

54. Answer (3)

55. Answer (3)

56. Answer (4)

57. Answer (4)

58. Answer (4)

59. Answer (4)

60. Answer (4)

PART - C (MATHEMATICS)

61. Answer (1)

$\cos^2\theta = \dfrac{x^2+y^2+1}{2x}$

$\Rightarrow \quad 0 \le \cos^2\theta \le 1$

$\Rightarrow \quad 0 \le \dfrac{x^2+y^2+1}{2x} \le 1$

If $\dfrac{x^2+y^2+1}{2x} \ge 0 \quad \Rightarrow \quad x > 0$

If $\dfrac{x^2+y^2+1}{2x} - 1 \le 0$

$\Rightarrow \quad \dfrac{(x-1)^2+y^2}{2x} \le 0$

$\because \quad x > 0 \quad \Rightarrow \quad (x-1)^2 + y^2 \le 0$

$\Rightarrow \quad x = 1, y = 0$

62. Answer (2)

$y = \dfrac{\sin^4 x + \cos^4 x}{\sin^6 x + \cos^6 x}$

$\Rightarrow \quad y = \dfrac{1 - \frac{1}{2}\sin^2 2x}{1 - \frac{3}{4}\sin^2 2x}$

Put $t^2 = \sin^2 2x$

$\Rightarrow \quad 0 \le \sin^2 2x \le 1$

$\Rightarrow \quad 0 \le t^2 \le 1$

$y = \dfrac{1 - \frac{1}{2}t^2}{1 - \frac{3}{4}t^2} \Rightarrow t^2 = \dfrac{4(1-y)}{2-3y} \Rightarrow 0 \le \dfrac{4(1-y)}{2-3y} \le 1$

On solving, we get $y \in [1, 2]$

63. Answer (3)

Let $x \sin\theta = y \cos\theta = z \tan 2\theta = k$

$\Rightarrow \quad x = k \operatorname{cosec}\theta, y = k \sec\theta, z = k \cot 2\theta$

$\Rightarrow \quad 4z^2(x^2 + y^2) = 4k^4 \cot^2 2\theta (\operatorname{cosec}^2\theta + \sec^2\theta)$

$= 4k^4 \dfrac{\cos^2 2\theta}{\sin^2 2\theta}\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cos^2\theta}\right)$

$= \dfrac{4k^4}{4}\left(\dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta\cos^2\theta}\right)^2$

$= k^4\left(\dfrac{x^2}{k^2} - \dfrac{y^2}{k^2}\right)^2$

64. Answer (4)

Case 1 : If $|\sin x| = 1 \Rightarrow x = \dfrac{(2n+1)\pi}{2}$, then $2 + \sin x - \sin^2 x$ may take any value.

Case 2 : If $|\sin x| \ne 1$, then

$2 + \sin x - \sin^2 x = 0$

$\Rightarrow \quad \sin x = -1, \sin x = 2$, But $|\sin x| \ne 1$

No values of x is possible.

65. Answer (2)

$3\cos 2\theta + 4\sin 2\theta = 5$

$\Rightarrow \quad 3\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 4\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) = 5$

Put $\tan\theta = t$

$\Rightarrow \quad 3\left(\dfrac{1-t^2}{1+t^2}\right) + 4\left(\dfrac{2t}{1+t^2}\right) = 5$

On solving, we get

$4t^2 - 4t + 1 = 0$

$\Rightarrow \quad 4\tan^2\theta - 4\tan\theta + 1 = 0$

Roots are $\tan\theta_1$ and $\tan\theta_2$

$\Rightarrow \quad \tan\theta_1 \tan\theta_2 = \dfrac{1}{4}$

66. Answer (4)

$\tan\dfrac{15^\circ}{2} \cdot \tan\dfrac{105^\circ}{2} \cdot \tan\dfrac{135^\circ}{2}$

$\Rightarrow \quad \tan\left(7\dfrac{1^\circ}{2}\right) \cdot \tan\left(52\dfrac{1^\circ}{2}\right) \cdot \tan\left(67\dfrac{1^\circ}{2}\right)$

$= \tan\left(3 \cdot 7\dfrac{1^\circ}{2}\right)$

$= \tan\left(22\dfrac{1^\circ}{2}\right)$

$= \sqrt{2} - 1$

67. Answer (4)

$\cos(\sin x) = \cos\left(\dfrac{\pi}{2} - \cos x\right)$

$\sin = 2n\pi \pm \left(\dfrac{\pi}{2} - \cos x\right)$

On taking +ve sign,

$$\sin x = 2n\pi + \frac{\pi}{2} - \cos x$$

$$\Rightarrow \quad \sin x + \cos x = \frac{(4n+1)\pi}{2}$$

For solution,

$$-\sqrt{2} \le (4n+1)\frac{\pi}{2} \le \sqrt{2}$$

No value of n is possible.

Similarly, for –ve sign,

$$\sin x - \cos x = \frac{(4n-1)\pi}{2}$$

No value of n is possible.

68. Answer (2)

$$\sqrt{3}\sin x - \cos x \le 0$$

$$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x \le 0$$

$$\sin\left(x - \frac{\pi}{6}\right) \le 0$$

$$\pi \le x - \frac{\pi}{6} \le 2\pi$$

$$\frac{7\pi}{6} \le x \le \frac{13\pi}{6}$$

$$\Rightarrow \quad x \in \left[2n\pi + \frac{7\pi}{6}, 2n\pi + \frac{13\pi}{6}\right], n \in Z$$

69. Answer (1)

$f(x) = 4[\sin^3 x \cos 3x + \cos^3 x \sin 3x] - 3 \sin 4x$

$4 \sin^3 x = 3 \sin x - \sin 3x$

$4 \cos^3 x = \cos 3x + 3 \cos x$

$\Rightarrow \quad f(x) = [3 \sin x - \sin 3x] \cos 3x$

$+ [\cos 3x + 3 \cos x] \times \sin 3x - 3 \sin 4x$

$\Rightarrow \quad f(x) = 0$

70. Answer (1)

$\sin 6x = -1$	$\cos 4x = -1$
$\Rightarrow \quad x = (4n-1)\frac{\pi}{12}$	$\Rightarrow \quad x = (2m+1)\frac{\pi}{4}$

71. Answer (3)

$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 - \tan\theta\tan\phi}$$

Put $\tan\theta = 3\tan\phi$

$$\Rightarrow \quad \tan(\theta - \phi) = \frac{2}{\cot\phi + 3\tan\phi}$$

By AM – GM,

$$\frac{\cot\phi + 3\tan\phi}{2} \ge \sqrt{3}$$

72. Question deleted

73. Answer (3)

$$y = 5 + 2\sin^2 x + \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) \times \sqrt{2}$$

$$\Rightarrow \quad \sqrt{2}\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)_{\max} = 2$$

Maximum value at $x = \frac{\pi}{2}$

At $x = \frac{\pi}{2}$, $(5 + 2\sin^2 x)_{\max} = 7$

74. Answer (1)

$$\tan(2 + x) = \tan\left(\frac{\pi}{2} - x\right)$$

$$\Rightarrow \quad 2 + x = n\pi + \frac{\pi}{2} - x$$

$$\Rightarrow \quad x = \frac{n\pi}{2} + \frac{\pi}{4} - 1$$

$\cot x > 0$, $\tan(x + 2) > 0$

Therefore, $x = \frac{\pi}{4} - 1, \frac{5\pi}{4} - 1$

75. Answer (1)

$$\frac{\sin^4 x}{a} + \frac{\cos^4 x}{b} = \frac{1}{a+b}$$

Then, $\sin^2 x = \frac{a}{a+b}, \cos^2 x = \frac{b}{a+b}$

$$\sin^2 x = \frac{1}{2}$$

$$\cos^2 x = \frac{1}{2}$$

$(\sin^2 x)^3 + (\cos^2 x)^3 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

76. Answer (2)

$2\sin^2 x - 7\sin x + 3 = 0$

$\Rightarrow \sin x = 3,\ \sin x = \frac{1}{2}$

$\Rightarrow \cos 2x = 1 - 2\sin^2 x$

$= 1 - 2 \times \frac{1}{4}$

$\cos 2x = \frac{1}{2}$

$\Rightarrow \cos^2 2x + A\cos 2x + B = 0$

$\Rightarrow \frac{1}{4} + \frac{A}{2} + B = 0$

77. Answer (3)

Minimum value of $\cos(\cos(\cot x)) = \cos 1$

Maximum value of $\sin(\sin(\tan y)) = \sin 1$

78. Answer (2)

$\left|\sin\left(\pi + \frac{\pi x}{2\sqrt{3}}\right)\right| = \left|(x - \sqrt{3})^2 + 1\right|$

$\Rightarrow \left|\sin\left(\frac{\pi x}{2\sqrt{3}}\right)\right| = \left|(x - \sqrt{3})^2 + 1\right|$

Only solution is $x = \sqrt{3}$.

79. Answer (4)

$4\sin^2\alpha + 4\cos^2\alpha + \cos^2\alpha + x^2 - 4x = 0$

$\Rightarrow \cos^2\alpha + (x - 2)^2 = 0$

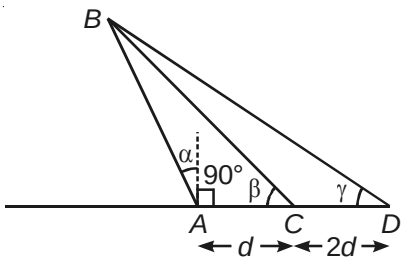
$\Rightarrow x = 2,\ \cos\alpha = 0$

80. Answer (2)

$\cos^6\frac{\pi}{24} - \sin^6\frac{\pi}{24}$

$\left(\cos^2\frac{\pi}{24} - \sin^2\frac{\pi}{24}\right)\left(1 - \sin^2\frac{\pi}{24}\cdot\cos^2\frac{\pi}{24}\right)$

$\cos\frac{\pi}{12}\cdot\left(1 - \frac{\sin^2\frac{\pi}{12}}{4}\right) \Rightarrow \frac{17 + 15\sqrt{3}}{32\sqrt{2}}$



By *m-n* theorem,

$(d + 2d)\cot\beta = d\cot\gamma - 2d\cot(90° + \alpha)$

$AC = d$

$CD = 2d$

AB = tower

81. Answer (1)

$\sqrt{4\sin^4\theta + 4\sin^2\theta\cos^2\theta} + 2 + 2\cos\left(\frac{\pi}{2} - \theta\right)$

$= 2|\sin\theta| + 2\sin\theta + 2$

82. Answer (2)

If $A + B + C = \pi$, then

$\cos 2A + \cos 2B - \cos 2C$

$= 1 - 4\sin A\sin B\cos C$

$\Rightarrow \cos C = 0$

83. Answer (2)

$\cos\theta\cos(2\theta)\cos(2^2\theta)\ldots\ldots\cos(2^{n-1}\theta) = \frac{\sin(2^n\theta)}{2^n\sin\theta}$

84. Answer (1)

$(\sin 47° + \sin 61°) - (\sin 11° + \sin 25°)$

$= 2\sin 54°\cos 7° - 2\sin 18°\cos 7°$

$= 2\cos 7°\left[\left(\frac{\sqrt{5}+1}{4}\right) - \left(\frac{\sqrt{5}-1}{4}\right)\right]$

$= \cos 7°$

85. Answer (2)

$y = 8\sin x\cos x + 5\cos^2 x - \sin^2 x + 3$

$\Rightarrow y = 4\sin 2x + 3\cos 2x + 5$

$\Rightarrow y_{max} = 10$

86. Answer (4)

$[2\sqrt{2}\cos x - \sin x] = -3\cdot 3^x$

Range of LHS is {–3, –2, –1, 0, 1, 2, 3}

Range of RHS is $(-3^{8\pi+1}, -3)$

87. Answer (1)

$\cot x - \tan x = 2\cot 2x$

$\Rightarrow 2\cot 2x - 2\tan 2x = 2^2\cot 2^2x$

$\Rightarrow 2^2\cot 2^2x - 2^2\tan 2^2x = 2^3\cot 2^3x$

$\Rightarrow 2^3\cot 2^3x - 2^3\tan 2^3x = 2^4\cot 2^4x$

On adding

88. Answer (3)

$-\sqrt{74} \le 7\cos x + 5\sin x \le \sqrt{74}$

$\Rightarrow \; -4.8 \le 7\cos x - 5\sin x \le 3.8$

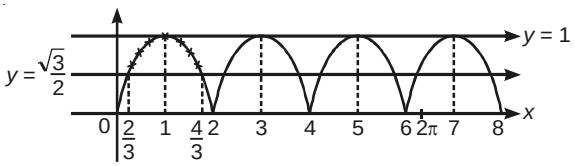
$\Rightarrow \; \lambda = -4, -3, -2, -1, 0, 1, 2, 3$

89. Answer (1)

The range of [sin x] is {0}.

The range of [cos x] is {0}.

90. Answer (4)

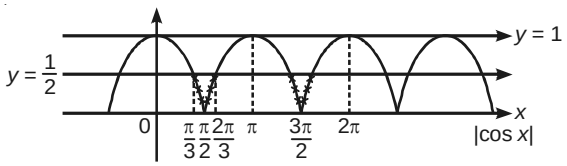


Graph of $\left|\sin\left(\frac{\pi x}{2}\right)\right|$

For $\left|\sin\left(\frac{\pi x}{2}\right)\right| \ge \frac{\sqrt{3}}{2}$

$x \in \left[2n + \frac{2}{3}, 2n + \frac{4}{3}\right], n \in Z$

x = 1, 3, 5, where $x \in (0, 2\pi)$



$x \in \left[n\pi + \frac{\pi}{3}, n\pi + \frac{2\pi}{3}\right]$

$x \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ and $\left[\pi + \frac{\pi}{3}, \pi + \frac{2\pi}{3}\right]$

x = 5 lies in $\left[\pi + \frac{\pi}{3}, \pi + \frac{2\pi}{3}\right]$

Therefore one solution.

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