The length of mercury column at 30°C is $l_\theta$. Suppose the length of the mercury column, if it were at 0°C, is $l_0$. Then,

$$l_\theta = l_0 \left[1 + \frac{1}{3}\gamma\,(30°C)\right]. \quad \dots \text{(ii)}$$

By (i) and (ii),

$$l_0\left[1 + \frac{1}{3}\gamma\,(30°C)\right] = 75\text{ cm}[1 + \alpha(30°C)]$$

or,
$$l_0 = 75\text{ cm}\frac{[1 + \alpha(30°C)]}{\left[1 + \frac{1}{3}\gamma\,(30°C)\right]}$$

$$= 75\text{ cm}\left[1 + \left(\alpha - \frac{\gamma}{3}\right)(30°C)\right]$$

$$= 74\cdot89\text{ cm}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. If two bodies are in thermal equilibrium in one frame, will they be in thermal equilibrium in all frames ?

2. Does the temperature of a body depend on the frame from which it is observed ?

3. It is heard sometimes that mercury is used in defining the temperature scale because it expands uniformly with the temperature. If the temperature scale is not yet defined, is it logical to say that a substance expands uniformly with the temperature ?

4. In defining the ideal gas temperature scale, it is assumed that the pressure of the gas at constant volume is proportional to the temperature $T$. How can we verify whether this is true or not ? Are we using the kinetic theory of gases ? Are we using the experimental result that the pressure is proportional to temperature ?

5. Can the bulb of a thermometer be made of an adiabatic wall ?

6. Why do marine animals live deep inside a lake when the surface of the lake freezes ?

7. The length of a brass rod is found to be smaller on a hot summer day than on a cold winter day as measured by the same aluminium scale. Do we conclude that brass shrinks on heating ?

8. If mercury and glass had equal coefficient of volume expansion, could we make a mercury thermometer in a glass tube ?

9. The density of water at 4°C is supposed to be 1000 kg/m$^3$. Is it same at the sea level and at a high altitude ?

10. A tightly closed metal lid of a glass bottle can be opened more easily if it is put in hot water for some time. Explain.

11. If an automobile engine is overheated, it is cooled by putting water on it. It is advised that the water should be put slowly with engine running. Explain the reason.

12. Is it possible for two bodies to be in thermal equilibrium if they are not in contact ?

13. A spherical shell is heated. The volume changes according to the equation $V_\theta = V_0\,(1 + \gamma\theta)$. Does the volume refer to the volume enclosed by the shell or the volume of the material making up the shell ?

## OBJECTIVE I

1. A system $X$ is neither in thermal equilibrium with $Y$ nor with $Z$. The systems $Y$ and $Z$
   (a) must be in thermal equilibrium
   (b) cannot be in thermal equilibrium
   (c) may be in thermal equilibrium.

2. Which of the curves in figure (23-Q1) represents the relation between Celsius and Fahrenheit temperatures ?



Figure 23-Q1

3. Which of the following pairs may give equal numerical values of the temperature of a body ?
   (a) Fahrenheit and kelvin    (b) Celsius and kelvin
   (c) kelvin and platinum.

4. For a constant volume gas thermometer, one should fill the gas at
   (a) low temperature and low pressure
   (b) low temperature and high pressure
   (c) high temperature and low pressure
   (d) high temperature and high pressure.

5. Consider the following statements.
   (A) The coefficient of linear expansion has dimension $K^{-1}$.
   (B) The coefficient of volume expansion has dimension $K^{-1}$.
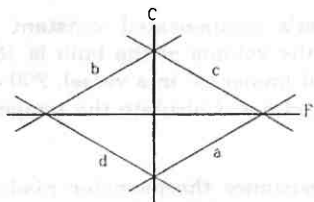   (a) $A$ and $B$ are both correct.
   (b) $A$ is correct but $B$ is wrong.

(c) $B$ is correct but $A$ is wrong.

(d) $A$ and $B$ are both wrong.

6. A metal sheet with a circular hole is heated. The hole
   (a) gets larger          (b) gets smaller
   (c) remains of the same size    (d) gets deformed.

7. Two identical rectangular strips, one of copper and the other of steel, are rivetted together to form a bimetallic strip ($\alpha_{copper} > \alpha_{steel}$). On heating, this strip will
   (a) remain straight
   (b) bend with copper on convex side
   (c) bend with steel on convex side
   (d) get twisted.

8. If the temperature of a uniform rod is slightly increased by $\Delta t$, its moment of inertia $I$ about a perpendicular

bisector increases by
   (a) zero     (b) $\alpha I \Delta t$     (c) $2\alpha I \Delta t$     (d) $3\alpha I \Delta t$.

9. If the temperature of a uniform rod is slightly increased by $\Delta t$, its moment of inertia $I$ about a line parallel to itself will increase by
   (a) zero     (b) $\alpha I \Delta t$     (c) $2\alpha I \Delta t$     (d) $3\alpha I \Delta t$.

10. The temperature of water at the surface of a deep lake is 2°C. The temperature expected at the bottom is
    (a) 0°C     (b) 2°C     (c) 4°C     (d) 6°C.

11. An aluminium sphere is dipped into water at 10°C. If the temperature is increased, the force of buoyancy
    (a) will increase          (b) will decrease
    (c) will remain constant
    (d) may increase or decrease depending on the radius of the sphere.

## OBJECTIVE II

1. A spinning wheel is brought in contact with an identical wheel spinning at identical speed. The wheels slow down under the action of friction. Which of the following energies of the first wheel decrease ?
   (a) kinetic   (b) total   (c) mechanical   (d) internal.

2. A spinning wheel $A$ is brought in contact with another wheel $B$ initially at rest. Because of the friction at contact, the second wheel also starts spinning. Which of the following energies of the wheel $B$ increase ?
   (a) kinetic   (b) total   (c) mechanical   (d) internal.

3. A body $A$ is placed on a railway platform and an identical body $B$ in a moving train. Which of the following energies of $B$ are greater than those of $A$ as seen from the ground ?
   (a) kinetic   (b) total   (c) mechanical   (d) internal.

4. In which of the following pairs of temperature scales, the size of a degree is identical ?

   (a) mercury scale and ideal gas scale
   (b) Celsius scale and mercury scale
   (c) Celsius scale and ideal gas scale
   (d) ideal gas scale and absolute scale.

5. A solid object is placed in water contained in an adiabatic container for some time. The temperature of water falls during the period and there is no appreciable change in the shape of the object. The temperature of the solid object
   (a) must have increased          (b) must have decreased
   (c) may have increased
   (d) may have remained constant.

6. As the temperature is increased, the time period of a pendulum
   (a) increases proportionately with temperature
   (b) increases              (c) decreases
   (d) remains constant.

## EXERCISES

1. The steam point and the ice point of a mercury thermometer are marked as 80° and 20°. What will be the temperature in centigrade mercury scale when this thermometer reads 32° ?

2. A constant volume thermometer registers a pressure of $1.500 \times 10^4$ Pa at the triple point of water and a pressure of $2.050 \times 10^4$ Pa at the normal boiling point. What is the temperature at the normal boiling point ?

3. A gas thermometer measures the temperature from the variation of pressure of a sample of gas. If the pressure measured at the melting point of lead is 2·20 times the pressure measured at the triple point of water, find the melting point of lead.

4. The pressure measured by a constant volume gas thermometer is 40 kPa at the triple point of water. What will be the pressure measured at the boiling point of water (100°C) ?

5. The pressure of the gas in a constant volume gas thermometer is 70 kPa at the ice point. Find the pressure at the steam point.

6. The pressures of the gas in a constant volume gas thermometer are 80 cm, 90 cm and 100 cm of mercury at the ice point, the steam point and in a heated wax bath respectively. Find the temperature of the wax bath.

7. In a Callender's compensated constant pressure air thermometer, the volume of the bulb is 1800 cc. When the bulb is kept immersed in a vessel, 200 cc of mercury has to be poured out. Calculate the temperature of the vessel.

8. A platinum resistance thermometer reads 0° when its resistance is 80 Ω and 100° when its resistance is 90 Ω. Find the temperature at the platinum scale at which the resistance is 86 Ω.

9. A resistance thermometer reads $R = 20.0\ \Omega$, $27.5\ \Omega$, and $50.0\ \Omega$ at the ice point (0°C), the steam point (100°C) and the zinc point (420°C) respectively. Assuming that the resistance varies with temperature as $R_\theta = R_0(1 + \alpha\theta + \beta\theta^2)$, find the values of $R_0$, $\alpha$ and $\beta$. Here $\theta$ represents the temperature on Celsius scale.

10. A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of concrete is $1.0 \times 10^{-5}/°C$.

11. A metre scale made of steel is calibrated at 20°C to give correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at 10°C. Coefficient of linear expansion of steel is $1.1 \times 10^{-5}/°C$.

12. A railway track (made of iron) is laid in winter when the average temperature is 18°C. The track consists of sections of 12.0 m placed one after the other. How much gap should be left between two such sections so that there is no compression during summer when the maximum temperature goes to 48°C? Coefficient of linear expansion of iron = $11 \times 10^{-6}/°C$.

13. A circular hole of diameter 2.00 cm is made in an aluminium plate at 0°C. What will be the diameter at 100°C? $\alpha$ for aluminium = $2.3 \times 10^{-5}/°C$.

14. Two metre scales, one of steel and the other of aluminium, agree at 20°C. Calculate the ratio aluminium-centimetre/steel-centimetre at (a) 0°C, (b) 40°C and (c) 100°C. $\alpha$ for steel = $1.1 \times 10^{-5}/°C$ and for aluminium = $2.3 \times 10^{-5}/°C$.

15. A metre scale is made up of steel and measures correct length at 16°C. What will be the percentage error if this scale is used (a) on a summer day when the temperature is 46°C and (b) on a winter day when the temperature is 6°C? Coefficient of linear expansion of steel = $11 \times 10^{-6}/°C$.

16. A metre scale made of steel reads accurately at 20°C. In a sensitive experiment, distances accurate upto 0.055 mm in 1 m are required. Find the range of temperature in which the experiment can be performed with this metre scale. Coefficient of linear expansion of steel = $11 \times 10^{-6}/°C$.

17. The density of water at 0°C is 0.998 g/cm³ and at 4°C is 1.000 g/cm³. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

18. Find the ratio of the lengths of an iron rod and an aluminium rod for which the difference in the lengths is independent of temperature. Coefficients of linear expansion of iron and aluminium are $12 \times 10^{-6}/°C$ and $23 \times 10^{-6}/°C$ respectively.

19. A pendulum clock gives correct time at 20°C at a place where $g = 9.800$ m/s². The pendulum consists of a light steel rod connected to a heavy ball. It is taken to a different place where $g = 9.788$ m/s². At what temperature will it give correct time? Coefficient of linear expansion of steel = $12 \times 10^{-6}/°C$.

20. An aluminium plate fixed in a horizontal position has a hole of diameter 2.000 cm. A steel sphere of diameter 2.005 cm rests on this hole. All the lengths refer to a temperature of 10°C. The temperature of the entire system is slowly increased. At what temperature will the ball fall down? Coefficient of linear expansion of aluminium is $23 \times 10^{-6}/°C$ and that of steel is $11 \times 10^{-6}/°C$.

21. A glass window is to be fit in an aluminium frame. The temperature on the working day is 40°C and the glass window measures exactly 20 cm × 30 cm. What should be the size of the aluminium frame so that there is no stress on the glass in winter even if the temperature drops to 0°C? Coefficients of linear expansion for glass and aluminium are $9.0 \times 10^{-6}/°C$ and $24 \times 10^{-6}/°C$ respectively.

22. The volume of a glass vessel is 1000 cc at 20°C. What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficients of cubical expansion of mercury and glass are $1.8 \times 10^{-4}/°C$ and $9.0 \times 10^{-6}/°C$ respectively.

23. An aluminium can of cylindrical shape contains 500 cm³ of water. The area of the inner cross-section of the can is 125 cm². All measurements refer to 10°C. Find the rise in the water level if the temperature increases to 80°C. The coefficient of linear expansion of aluminium = $23 \times 10^{-6}/°C$ and the average coefficient of volume expansion of water = $3.2 \times 10^{-4}/°C$ respectively.

24. A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. It is filled completely with mercury at this temperature. When the temperature is raised to 10°C, 1.6 cm³ of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass = $6.5 \times 10^{-6}/°C$.

25. The densities of wood and benzene at 0°C are 880 kg/m³ and 900 kg/m³ respectively. The coefficients of volume expansion are $1.2 \times 10^{-3}/°C$ for wood and $1.5 \times 10^{-3}/°C$ for benzene. At what temperature will a piece of wood just sink in benzene?

26. A steel rod of length 1 m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

27. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = $1.2 \times 10^{-5}/°C$.

28. A steel wire of cross-sectional area 0.5 mm² is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is $1.2 \times 10^{-5}/°C$ and its Young's modulus is $2.0 \times 10^{11}$ N/m².

29. A steel rod is rigidly clamped at its two ends. The rod is under zero tension at 20°C. If the temperature rises to 100°C, what force will the rod exert on one of the

clamps. Area of cross-section of the rod = $2.00 \text{ mm}^2$. Coefficient of linear expansion of steel = $12.0 \times 10^{-6}/°C$ and Young's modulus of steel = $2.00 \times 10^{11} \text{ N/m}^2$.

30. Two steel rods and an aluminium rod of equal length $l_0$ and equal cross-section are joined rigidly at their ends as shown in the figure below. All the rods are in a state of zero tension at 0°C. Find the length of the system when the temperature is raised to θ. Coefficient of linear expansion of aluminium and steel are $α_a$ and $α_s$ respectively. Young's modulus of aluminium is $Y_a$ and of steel is $Y_s$.

| Steel |
| Aluminium |
| Steel |
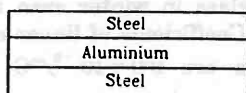
Figure 23-E1

31. A steel ball initially at a pressure of $1.0 \times 10^5$ Pa is heated from 20°C to 120°C keeping its volume constant.

Find the pressure inside the ball. Coefficient of linear expansion of steel = $12 \times 10^{-6}/°C$ and bulk modulus of steel = $1.6 \times 10^{11} \text{ N/m}^2$.

32. Show that moment of inertia of a solid body of any shape changes with temperature as $I = I_0 (1 + 2αθ)$, where $I_0$ is the moment of inertia at 0°C and α is the coefficient of linear expansion of the solid.

33. A torsional pendulum consists of a solid disc connected to a thin wire ($α = 2.4 \times 10^{-5}/°C$) at its centre. Find the percentage change in the time period between peak winter (5°C) and peak summer (45°C).

34. A circular disc made of iron is rotated about its axis at a constant velocity ω. Calculate the percentage change in the linear speed of a particle of the rim as the disc is slowly heated from 20°C to 50°C keeping the angular velocity constant. Coefficient of linear expansion of iron = $1.2 \times 10^{-5}/°C$.

## ANSWERS

### OBJECTIVE I

1. (c)    2. (a)    3. (a)    4. (c)    5. (a)    6. (a)
7. (b)    8. (c)    9. (c)    10. (c)    11. (b)

### OBJECTIVE II

1. (a), (c)    2. all    3. (a), (b), (c)
4. (c), (d)    5. (a)    6. (b)

### EXERCISES

1. 20°C
2. 373.3 K
3. 601 K
4. 55 kPa
5. 96 kPa
6. 200°C
7. 307 K
8. 60°
9. 20.0 Ω, $3.8 \times 10^{-3}/°C$, $-5.6 \times 10^{-7}/°C^2$
10. 10.0035 m
11. 1.00011 cm
12. 0.4 cm
13. 2.0046 cm

14. (a) 0.99977 (b) 1.00025 (c) 1.00096
15. (a) 0.033% (b) −0.011%
16. 15°C to 25°C
17. $-5 \times 10^{-4}/°C$
18. 23 : 12
19. −82°C
20. 219°C
21. 20.012 cm × 30.018 cm
22. 50 cc
23. 0.089 cm
24. $1.8 \times 10^{-4}/°C$
25. 83°C
26. zero
27. $-3.6 \times 10^{-4}$
28. 24 N
29. 384 N
30. $l_0 \left[ 1 + \dfrac{α_a Y_a + 2α_s Y_s}{Y_a + 2Y_s} θ \right]$
31. $5.8 \times 10^8$ Pa
33. $9.6 \times 10^{-2}$
34. $3.6 \times 10^{-2}$

$$T_2 = 0.99 \, T_1$$

and hence,

$$p_2 = 0.99 \, p_1$$
$$= 0.99 \times 800 \text{ mm of mercury} = 792 \text{ mm of mercury}.$$

The vapour is still saturated and hence, its pressure is 25 mm of mercury. The total pressure at the reduced temperature is

$$p = (792 + 25) \text{ mm of mercury}$$
$$= 817 \text{ mm of mercury}.$$

22. *Calculate the mass of 1 litre of moist air at 27°C when the barometer reads 753.6 mm of mercury and the dew point is 16.1°C. Saturation vapour pressure of water at 16.1°C = 13.6 mm of mercury, density of air at STP = 0.001293 g/cc, density of saturated water vapour at STP = 0.000808 g/cc.*

**Solution** : We have $pV = \dfrac{m}{M} RT$

or,                $\rho = \dfrac{m}{V} = \dfrac{Mp}{RT}$.          ... (i)

The dew point is 16.1°C and the saturation vapour pressure is 13.6 mm of mercury at the dew point. This means that the present vapour pressure is 13.6 mm of mercury.

At this pressure and temperature, the density of vapour will be

$$\rho = \frac{Mp}{RT}$$

$$= \frac{(18 \text{ g/mol}) (13.6 \times 10^{-3} \text{ m}) (13600 \text{ kg/m}^3) (9.8 \text{ m/s}^2)}{(8.3 \text{ J/mol-K}) (300 \text{ K})}$$

$$= 13.1 \text{ g/m}^3.$$

Thus, 1 litre of moist air at 27°C contains 0.0131 g of vapour.

The pressure of dry air at 27°C is 753.6 mm − 13.6 mm = 740 mm of mercury. The density of air at STP is 0.001293 g/cc. The density at 27°C is given by equation (i),

$$\frac{\rho_1}{\rho_2} = \frac{p_1/T_1}{p_2/T_2}$$

or,                $\rho_2 = \dfrac{p_2 T_1}{T_2 p_1} \rho_1$

$$= \frac{740 \times 273}{300 \times 760} \times 0.001293 \text{ g/cc}.$$

$$= 0.001457 \text{ g/cc}.$$

Thus, 1 litre of moist air contains 1.145 g of dry air. The mass of 1 litre of moist air is 1.1457 g + 0.0131 g ≈ 1.159 g.

□

## QUESTIONS FOR SHORT ANSWER

1. When we place a gas cylinder on a van and the van moves, does the kinetic energy of the molecules increase ? Does the temperature increase ?

2. While gas from a cooking gas cylinder is used, the pressure does not fall appreciably till the last few minutes. Why ?

3. Do you expect the gas in a cooking gas cylinder to obey the ideal gas equation ?

4. Can we define the temperature of vacuum ? The temperature of a single molecule ?

5. Comment on the following statement. The temperature of all the molecules in a sample of a gas is the same.

6. Consider a gas of neutrons. Do you expect it to behave much better as an ideal gas as compared to hydrogen gas at the same pressure and temperature ?

7. A gas is kept in a rigid cubical container. If a load of 10 kg is put on the top of the container, does the pressure increase ?

8. If it were possible for a gas in a container to reach the temperature 0 K, its pressure would be zero. Would the molecules not collide with the walls ? Would they not transfer momentum to the walls ?

9. It is said that the assumptions of kinetic theory are good for gases having low densities. Suppose a container is so evacuated that only one molecule is left in it. Which of the assumptions of kinetic theory will not be valid for such a situation ? Can we assign a temperature to this gas ?

10. A gas is kept in an enclosure. The pressure of the gas is reduced by pumping out some gas. Will the temperature of the gas decrease by Charles' law ?

11. Explain why cooking is faster in a pressure cooker.

12. If the molecules were not allowed to collide among themselves, would you expect more evaporation or less evaporation ?

13. Is it possible to boil water at room temperature, say 30°C ? If we touch a flask containing water boiling at this temperature, will it be hot ?

14. When you come out of a river after a dip, you feel cold. Explain.

## OBJECTIVE I

1. Which of the following parameters is the same for molecules of all gases at a given temperature ?
   (a) mass        (b) speed
   (c) momentum    (d) kinetic energy.

2. A gas behaves more closely as an ideal gas at
   (a) low pressure and low temperature
   (b) low pressure and high temperature
   (c) high pressure and low temperature
   (d) high pressure and high temperature.

3. The pressure of an ideal gas is written as $p = \dfrac{2E}{3V}$. Here $E$ refers to
   (a) translational kinetic energy
   (b) rotational kinetic energy
   (c) vibrational kinetic energy
   (d) total kinetic energy.

4. The energy of a given sample of an ideal gas depends only on its
   (a) volume   (b) pressure   (c) density   (d) temperature.

5. Which of the following gases has maximum rms speed at a given temperature ?
   (a) hydrogen        (b) nitrogen
   (c) oxygen          (d) carbon dioxide.

6. Figure 24-Q1 shows graphs of pressure vs. density for an ideal gas at two temperatures $T_1$ and $T_2$.
   (a) $T_1 > T_2$         (b) $T_1 = T_2$
   (c) $T_1 < T_2$         (d) any of the three is possible.



Figure 24-Q1

7. The mean square speed of the molecules of a gas at absolute temperature $T$ is proportional to
   (a) $\dfrac{1}{T}$      (b) $\sqrt{T}$      (c) $T$      (d) $T^2$.

8. Suppose a container is evacuated to leave just one molecule of a gas in it. Let $v_a$ and $v_{rms}$ represent the average speed and the rms speed of the gas.
   (a) $v_a > v_{rms}$.        (b) $v_a < v_{rms}$.
   (c) $v_a = v_{rms}$.        (d) $v_{rms}$ is undefined.

9. The rms speed of oxygen at room temperature is about 500 m/s. The rms speed of hydrogen at the same temperature is about
   (a) 125 m/s   (b) 2000 m/s   (c) 8000 m/s   (d) 31 m/s.

10. The pressure of a gas kept in an isothermal container is 200 kPa. If half the gas is removed from it, the pressure will be
    (a) 100 kPa   (b) 200 kPa   (c) 400 kPa   (d) 800 kPa.

11. The rms speed of oxygen molecules in a gas is $v$. If the temerature is doubled and the oxygen molecules dissociate into oxygen atoms, the rms speed will become
    (a) $v$        (b) $v\sqrt{2}$        (c) $2v$        (d) $4v$.

12. The quantity $\dfrac{pV}{kT}$ represents
    (a) mass of the gas
    (b) kinetic energy of the gas
    (c) number of moles of the gas
    (d) number of molecules in the gas.

13. The process on an ideal gas, shown in figure (24-Q2), is
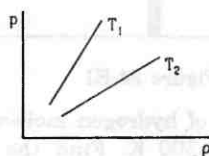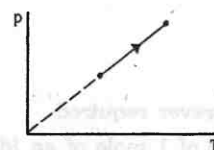    (a) isothermal (b) isobaric (c) isochoric (d) none of these.



Figure 24-Q2

14. There is some liquid in a closed bottle. The amount of liquid is continuously decreasing. The vapour in the remaining part
    (a) must be saturated    (b) must be unsaturated
    (c) may be saturated    (d) there will be no vapour.

15. There is some liquid in a closed bottle. The amount of liquid remains constant as time passes. The vapour in the remaining part
    (a) must be saturated    (b) must be unsaturated
    (c) may be unsaturated    (d) there will be no vapour.

16. Vapour is injected at a uniform rate in a closed vessel which was initially evacuated. The pressure in the vessel
    (a) increases continuously
    (b) decreases continuously
    (c) first increases and then decreases
    (d) first increases and then becomes constant.

17. A vessel $A$ has volume $V$ and a vessel $B$ has volume $2V$. Both contain some water which has a constant volume. The pressure in the space above water is $p_a$ for vessel $A$ and $p_b$ for vessel $B$.
    (a) $p_a = p_b$.                (b) $p_a = 2p_b$.
    (c) $p_b = 2p_a$.              (d) $p_b = 4p_a$.

## OBJECTIVE II

1. Consider a collision between an oxygen molecule and a hydrogen molecule in a mixture of oxygen and hydrogen kept at room temperature. Which of the following are possible ?
   (a) The kinetic energies of both the molecules increase.
   (b) The kinetic energies of both the molecules decrease.

(c) The kinetic energy of the oxygen molecule increases and that of the hydrogen molecule decreases.

(d) The kinetic energy of the hydrogen molecule increases and that of the oxygen molecule decreases.

2. Consider a mixture of oxygen and hydrogen kept at room temperature. As compared to a hydrogen molecule an oxygen molecule hits the wall
   (a) with greater average speed
   (b) with smaller average speed
   (c) with greater average kinetic energy
   (d) with smaller average kinetic energy.

3. Which of the following quantities is zero on an average for the molecules of an ideal gas in equilibrium ?
   (a) kinetic energy      (b) momentum
   (c) density            (d) speed.

4. Keeping the number of moles, volume and temperature the same, which of the following are the same for all ideal gases ?

(a) rms speed of a molecule      (b) density
(c) pressure
(d) average magnitude of momentum.

5. The average momentum of a molecule in a sample of an ideal gas depends on
   (a) temperature      (b) number of moles
   (c) volume           (d) none of these.

6. Which of the following quantities is the same for all ideal gases at the same temperature ?
   (a) the kinetic energy of 1 mole
   (b) the kinetic energy of 1 g
   (c) the number of molecules in 1 mole
   (d) the number of molecules in 1 g

7. Consider the quantity $\frac{MkT}{pV}$ of an ideal gas where $M$ is the mass of the gas. It depends on the
   (a) temperature of the gas      (b) volume of the gas
   (c) pressure of the gas       (d) nature of the gas.

## EXERCISES

Use $R = 8.3$ J/mol–K wherever required.

1. Calculate the volume of 1 mole of an ideal gas at STP.

2. Find the number of molecules of an ideal gas in a volume of $1.000$ cm$^3$ at STP.

3. Find the number of molecules in 1 cm$^3$ of an ideal gas at $0°$C and at a pressure of $10^{-5}$ mm of mercury.

4. Calculate the mass of 1 cm$^3$ of oxygen kept at STP.

5. Equal masses of air are sealed in two vessels, one of volume $V_0$ and the other of volume $2V_0$. If the first vessel is maintained at a temperature 300 K and the other at 600 K, find the ratio of the pressures in the two vessels.

6. An electric bulb of volume 250 cc was sealed during manufacturing at a pressure of $10^{-3}$ mm of mercury at $27°$C. Compute the number of air molecules contained in the bulb. Avogadro constant $= 6 \times 10^{23}$ per mol, density of mercury $= 13600$ kg/m$^3$ and $g = 10$ m/s$^2$.

7. A gas cylinder has walls that can bear a maximum pressure of $1.0 \times 10^6$ Pa. It contains a gas at $8.0 \times 10^5$ Pa and 300 K. The cylinder is steadily heated. Neglecting any change in the volume, calculate the temperature at which the cylinder will break.

8. 2 g of hydrogen is sealed in a vessel of volume $0.02$ m$^3$ and is maintained at 300 K. Calculate the pressure in the vessel.

9. The density of an ideal gas is $1.25 \times 10^{-3}$ g/cm$^3$ at STP. Calculate the molecular weight of the gas.

10. The temperature and pressure at Simla are $15.0°$C and $72.0$ cm of mercury and at Kalka these are $35.0°$C and $76.0$ cm of mercury. Find the ratio of air density at Kalka to the air density at Simla.

11. Figure (24-E1) shows a cylindrical tube with adiabatic walls and fitted with a diathermic separator. The separator can be slid in the tube by an external mechanism. An ideal gas is injected in the two sides at equal pressures and equal temperatures. The separator remains in equilibrium at the middle. It is now slid to a position where it divides the tube in the ratio of 1:3. Find the ratio of the pressures in the two parts of the vessel.



Figure 24-E1

12. Find the rms speed of hydrogen molecules in a sample of hydrogen gas at 300 K. Find the temperature at which the rms speed is double the speed calculated in the previous part.

13. A sample of $0.177$ g of an ideal gas occupies 1000 cm$^3$ at STP. Calculate the rms speed of the gas molecules.

14. The average translational kinetic energy of air molecules is $0.040$ eV (1 eV $= 1.6 \times 10^{-19}$ J). Calculate the temperature of the air. Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K.

15. Consider a sample of oxygen at 300 K. Find the average time taken by a molecule to travel a distance equal to the diameter of the earth.

16. Find the average magnitude of linear momentum of a helium molecule in a sample of helium gas at $0°$C. Mass of a helium molecule $= 6.64 \times 10^{-27}$ kg and Boltzmann constant $= 1.38 \times 10^{-23}$ J/K.

17. The mean speed of the molecules of a hydrogen sample equals the mean speed of the molecules of a helium sample. Calculate the ratio of the temperature of the hydrogen sample to the temperature of the helium sample.

18. At what temperature the mean speed of the molecules of hydrogen gas equals the escape speed from the earth ?

19. Find the ratio of the mean speed of hydrogen molecules to the mean speed of nitrogen molecules in a sample containing a mixture of the two gases.

20. Figure (24-E2) shows a vessel partitioned by a fixed diathermic separator. Different ideal gases are filled in the two parts. The rms speed of the molecules in the left part equals the mean speed of the molecules in the right part. Calculate the ratio of the mass of a molecule in the left part to the mass of a molecule in the right part.



Figure 24-E2

21. Estimate the number of collisions per second suffered by a molecule in a sample of hydrogen at STP. The mean free path (average distance covered by a molecule between successive collisions) = $1.38 \times 10^{-5}$ cm.

22. Hydrogen gas is contained in a closed vessel at 1 atm (100 kPa) and 300 K. (a) Calculate the mean speed of the molecules. (b) Suppose the molecules strike the wall with this speed making an average angle of 45° with it. How many molecules strike each square metre of the wall per second?

23. Air is pumped into an automobile tyre's tube upto a pressure of 200 kPa in the morning when the air temperature is 20°C. During the day the temperature rises to 40°C and the tube expands by 2%. Calculate the pressure of the air in the tube at this temperature.

24. Oxygen is filled in a closed metal jar of volume $1.0 \times 10^{-3}$ m$^3$ at a pressure of $1.5 \times 10^5$ Pa and temperature 400 K. The jar has a small leak in it. The atmospheric pressure is $1.0 \times 10^5$ Pa and the atmospheric temperature is 300 K. Find the mass of the gas that leaks out by the time the pressure and the temperature inside the jar equalise with the surrounding.

25. An air bubble of radius 2.0 mm is formed at the bottom of a 3.3 m deep river. Calculate the radius of the bubble as it comes to the surface. Atmospheric pressure = $1.0 \times 10^5$ Pa and density of water = 1000 kg/m$^3$.

26. Air is pumped into the tubes of a cycle rickshaw at a pressure of 2 atm. The volume of each tube at this pressure is 0.002 m$^3$. One of the tubes gets punctured and the volume of the tube reduces to 0.0005 m$^3$. How many moles of air have leaked out? Assume that the temperature remains constant at 300 K and that the air behaves as an ideal gas.

27. 0.040 g of He is kept in a closed container initially at 100.0°C. The container is now heated. Neglecting the expansion of the container, calculate the temperature at which the internal energy is increased by 12 J.

28. During an experiment, an ideal gas is found to obey an additional law $pV^2$ = constant. The gas is initially at a temperature $T$ and volume $V$. Find the temperature when it expands to a volume $2V$.

29. A vessel contains 1.60 g of oxygen and 2.80 g of nitrogen. The temperature is maintained at 300 K and the volume of the vessel is 0.166 m$^3$. Find the pressure of the mixture.

30. A vertical cylinder of height 100 cm contains air at a constant temperature. The top is closed by a frictionless light piston. The atmospheric pressure is equal to 75 cm of mercury. Mercury is slowly poured over the piston. Find the maximum height of the mercury column that can be put on the piston.

31. Figure (24-E3) shows two vessels $A$ and $B$ with rigid walls containing ideal gases. The pressure, temperature and the volume are $p_A$, $T_A$, $V$ in the vessel $A$ and $p_B$, $T_B$, $V$ in the vessel $B$. The vessels are now connected through a small tube. Show that the pressure $p$ and the temperature $T$ satisfy

$$\frac{p}{T} = \frac{1}{2}\left(\frac{p_A}{T_A} + \frac{p_B}{T_B}\right)$$

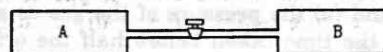when equilibrium is achieved.



Figure 24-E3

32. A container of volume 50 cc contains air (mean molecular weight = 28.8 g) and is open to atmosphere where the pressure is 100 kPa. The container is kept in a bath containing melting ice (0°C). (a) Find the mass of the air in the container when thermal equilibrium is reached. (b) The container is now placed in another bath containing boiling water (100°C). Find the mass of air in the container. (c) The container is now closed and placed in the melting-ice bath. Find the pressure of the air when thermal equilibrium is reached.

33. A uniform tube closed at one end, contains a pallet of mercury 10 cm long. When the tube is kept vertically with the closed end upward, the length of the air column trapped is 20 cm. Find the length of the air column trapped when the tube is inverted so that the closed end goes down. Atmospheric pressure = 75 cm of mercury.

34. A glass tube, sealed at both ends, is 100 cm long. It lies horizontally with the middle 10 cm containing mercury. The two ends of the tube contain air at 27°C and at a pressure 76 cm of mercury. The air column on one side is maintained at 0°C and the other side is maintained at 127°C. Calculate the length of the air column on the cooler side. Neglect the changes in the volume of mercury and of the glass.

35. An ideal gas is trapped between a mercury column and the closed end of a narrow vertical tube of uniform base containing the column. The upper end of the tube is open to the atmosphere. The atmospheric pressure equals 76 cm of mercury. The lengths of the mercury column and the trapped air column are 20 cm and 43 cm respectively. What will be the length of the air column when the tube is tilted slowly in a vertical plane through an angle of 60°? Assume the temperature to remain constant.

36. Figure (24-E4) shows a cylindrical tube of length 30 cm which is partitioned by a tight-fitting separator. The separator is very weakly conducting and can freely slide along the tube. Ideal gases are filled in the two parts of the vessel. In the beginning, the temperatures in the parts A and B are 400 K and 100 K respectively. The separator slides to a momentary equilibrium position shown in the figure. Find the final equilibrium position of the separator, reached after a long time.
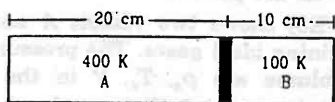


Figure 24-E4

37. A vessel of volume $V_0$ contains an ideal gas at pressure $p_0$ and temperature $T$. Gas is continuously pumped out of this vessel at a constant volume-rate $dV/dt = r$ keeping the temperature constant. The pressure of the gas being taken out equals the pressure inside the vessel. Find (a) the pressure of the gas as a function of time, (b) the time taken before half the original gas is pumped out.

38. One mole of an ideal gas undergoes a process

$$p = \frac{p_0}{1 + (V/V_0)^2}$$

where $p_0$ and $V_0$ are constants. Find the temperature of the gas when $V = V_0$.

39. Show that the internal energy of the air (treated as an ideal gas) contained in a room remains constant as the temperature changes between day and night. Assume that the atmospheric pressure around remains constant and the air in the room maintains this pressure by communicating with the surrounding through the windows etc.

40. Figure (24-E5) shows a cylindrical tube of radius 5 cm and length 20 cm. It is closed by a tight-fitting cork. The friction coefficient between the cork and the tube is 0·20. The tube contains an ideal gas at a pressure of 1 atm and a temperature of 300 K. The tube is slowly heated and it is found that the cork pops out when the temperature reaches 600 K. Let $dN$ denote the magnitude of the normal contact force exerted by a small length $dl$ of the cork along the periphery (see the figure). Assuming that the temperature of the gas is uniform at any instant, calculate $\dfrac{dN}{dl}$.
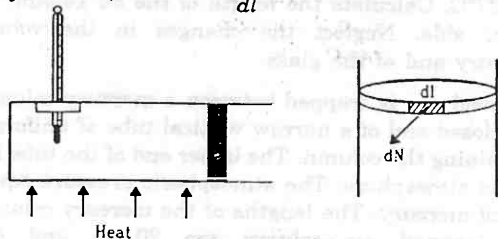


Figure 24-E5

41. Figure (24-E6) shows a cylindrical tube of cross-sectional area $A$ fitted with two frictionless pistons. The pistons are connected to each other by a metallic wire. Initially,

the temperature of the gas is $T_0$ and its pressure is $p_0$ which equals the atmospheric pressure. (a) What is the tension in the wire ? (b) What will be the tension if the temperature is increased to $2T_0$ ?
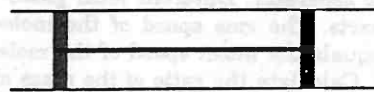


Figure 24-E6

42. Figure (24-E7) shows a large closed cylindrical tank containing water. Initially the air trapped above the water surface has a height $h_0$ and pressure $2p_0$ where $p_0$ is the atmospheric pressure. There is a hole in the wall of the tank at a depth $h_1$ below the top from which water comes out. A long vertical tube is connected as shown. (a) Find the height $h_2$ of the water in the long tube above the top initially. (b) Find the speed with which water comes out of the hole.(c) Find the height of the water in the long tube above the top when the water stops coming out of the hole.
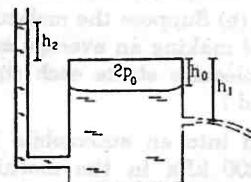


Figure 24-E7

43. An ideal gas is kept in a long cylindrical vessel fitted with a frictionless piston of cross-sectional area 10 cm² and weight 1 kg (figure 24-E8). The vessel itself is kept in a big chamber containing air at atmospheric pressure 100 kPa. The length of the gas column is 20 cm. If the chamber is now completely evacuated by an exhaust pump, what will be the length of the gas column ? Assume the temperature to remain constant throughout the process.
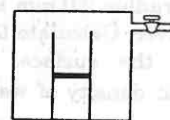


Figure 24-E8

44. An ideal gas is kept in a long cylindrical vessel fitted with a frictionless piston of cross-sectional area 10 cm² and weight 1 kg. The length of the gas column in the vessel is 20 cm. The atmospheric pressure is 100 kPa. The vessel is now taken into a spaceship revolving round the earth as a satellite. The air pressure in the spaceship is maintained at 100 kPa. Find the length of the gas column in the cylinder.

45. Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C at a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the

pressure inside the bulbs ? The volume of the connecting tube is negligible.

46. The weather report reads, "Temperature 20°C : Relative humidity 100%". What is the dew point ?

47. The condition of air in a closed room is described as follows. Temperature = 25°C, relative humidity = 60%, pressure = 104 kPa. If all the water vapour is removed from the room without changing the temperature, what will be the new pressure ? The saturation vapour pressure at 25°C = 3·2 kPa.

48. The temperature and the dew point in an open room are 20°C and 10°C. If the room temperature drops to 15°C, what will be the new dew point ?

49. Pure water vapour is trapped in a vessel of volume 10 cm³. The relative humidity is 40%. The vapour is compressed slowly and isothermally. Find the volume of the vapour at which it will start condensing.

50. A barometer tube is 80 cm long (above the mercury reservoir). It reads 76 cm on a particular day. A small amount of water is introduced in the tube and the reading drops to 75·4 cm. Find the relative humidity in the space above the mercury column if the saturation vapour pressure at the room temperature is 1·0 cm.

51. Using figure (24.6) of the text, find the boiling point of methyl alcohol at 1 atm (760 mm of mercury) and at 0·5 atm.

52. The human body has an average temperature of 98 °F. Assume that the vapour pressure of the blood in the veins behaves like that of pure water. Find the minimum atmospheric pressure which is necessary to prevent the blood from boiling. Use figure (24.6) of the text for the vapour pressures.

53. A glass contains some water at room temperature 20°C. Refrigerated water is added to it slowly. When the temperature of the glass reaches 10°C, small droplets condense on the outer surface. Calculate the relative humidity in the room. The boiling point of water at a pressure of 17·5 mm of mercury is 20°C and at 8·9 mm of mercury it is 10°C.

54. 50 m³ of saturated vapour is cooled down from 30°C to 20°C. Find the mass of the water condensed. The absolute humidity of saturated water vapour is 30 g/m³ at 30°C and 16 g/m³ at 20°C.

55. A barometer correctly reads the atmospheric pressure as 76 cm of mercury. Water droplets are slowly introduced into the barometer tube by a dropper. The height of the mercury column first decreases and then becomes constant. If the saturation vapour pressure at the atmospheric temperature is 0·80 cm of mercury, find the height of the mercury column when it reaches its minimum value.

56. 50 cc of oxygen is collected in an inverted gas jar over water. The atmospheric pressure is 99·4 kPa and the room temperature is 27°C. The water level in the jar is same as the level outside. The saturation vapour pressure at 27°C is 3·4 kPa. Calculate the number of moles of oxygen collected in the jar.

57. A faulty barometer contains certain amount of air and saturated water vapour. It reads 74·0 cm when the atmospheric pressure is 76·0 cm of mercury and reads 72·10 cm when the atmospheric pressure is 74·0 cm of mercury. Saturation vapour pressure at the air temperature = 1·0 cm of mercury. Find the length of the barometer tube above the mercury level in the reservoir.

58. On a winter day, the outside temperature is 0°C and relative humidity 40%. The air from outside comes into a room and is heated to 20°C. What is the relative humidity in the room ? The saturation vapour pressure at 0°C is 4·6 mm of mercury and at 20°C it is 18 mm of mercury.

59. The temperature and humidity of air are 27°C and 50% on a particular day. Calculate the amount of vapour that should be added to 1 cubic metre of air to saturate it. The saturation vapour pressure at 27°C = 3600 Pa.

60. The temperature and relative humidity in a room are 300 K and 20% respectively. The volume of the room is 50 m³. The saturation vapour pressure at 300 K is 3·3 kPa. Calculate the mass of the water vapour present in the room.

61. The temperature and the relative humidity are 300 K and 20% in a room of volume 50 m³. The floor is washed with water, 500 g of water sticking on the floor. Assuming no communication with the surrounding, find the relative humidity when the floor dries. The changes in temperature and pressure may be neglected. Saturation vapour pressure at 300 K = 3·3 kPa.

62. A bucket full of water is placed in a room at 15°C with initial relative humidity 40%. The volume of the room is 50 m³. (a) How much water will evaporate ? (b) If the room temperature is increased by 5°C how much more water will evaporate ? The saturation vapour pressure of water at 15°C and 20°C are 1·6 kPa and 2·4 kPa respectively.

□

## ANSWERS

### OBJECTIVE I

| 1. (d) | 2. (b) | 3. (a) | 4. (d) | 5. (a) | 6. (a) |
|---|---|---|---|---|---|
| 7. (c) | 8. (c) | 9. (b) | 10. (a) | 11. (c) | 12. (d) |
| 13. (c) | 14. (b) | 15. (a) | 16. (d) | 17. (a) | |

### OBJECTICE II

| 1. (c), (d) | 2. (b) | 3. (b) |
|---|---|---|
| 4. (c) | 5. (d) | 6. (a), (c) |
| 7. (d) | | |

## EXERCISES

1. $2.24 \times 10^{-2} \, m^3$
2. $2.685 \times 10^{19}$
3. $3.53 \times 10^{11}$
4. $1.43 \, mg$
5. $1:1$
6. $8.0 \times 10^{15}$
7. $375 \, K$
8. $1.24 \times 10^5 \, Pa$
9. $28.3 \, g/mol$
10. $0.987$
11. $3:1$
12. $1930 \, m/s$, $1200 \, K$
13. $1300 \, m/s$
14. $310 \, K$
15. $8.0 \, hour$
16. $8.0 \times 10^{-22} \, kg\text{-}m/s$
17. $1:2$
18. $11800 \, K$
19. $3.74$
20. $1.18$
21. $1.23 \times 10^{10}$
22. (a) $1780 \, m/s$ (b) $1.2 \times 10^{28}$
23. $209 \, kPa$
24. $0.16 \, g$
25. $2.2 \, mm$
26. $0.14$
27. $196°C$
28. $T/2$
29. $2250 \, N/m^2$
30. $25 \, cm$

32. (a) $0.058 \, g$ (b) $0.0468 \, g$ (c) $73.0 \, kPa$
33. $15 \, cm$
34. $36.5 \, cm$
35. $48 \, cm$
36. $10 \, cm$ from the left end
37. (a) $p = p_0 \, e^{-\gamma t/V_0}$ (b) $\dfrac{V_0 \ln 2}{\gamma}$
38. $\dfrac{p_0 V_0}{2R} \, mol^{-1}$
40. $1.25 \times 10^4 \, N/m$
41. (a) zero (b) $p_0 A$
42. (a) $\dfrac{p_0}{\rho g} - h_0$ (b) $\left[ \dfrac{2}{\rho} \left( p_0 + \rho g \, (h_1 - h_0) \right) \right]^{1/2}$ (c) $-h_1$
43. $2.2 \, m$
44. $22 \, cm$
45. $84 \, cm$ of mercury
46. $20°C$
47. $102 \, kPa$
48. $10°C$
49. $4.0 \, cm^3$
50. $60\%$
51. $65°C$, $48°C$
52. $50 \, mm$ of mercury
53. $51\%$
54. $700 \, g$
55. $75.2 \, cm$
56. $1.93 \times 10^{-3}$
57. $91.1 \, cm$
58. $9.5\%$
59. $13 \, g$
60. $238 \, g$
61. $62\%$
62. (a) $361 \, g$ (b) $296 \, g$

## QUESTIONS FOR SHORT ANSWER

1. Is heat a conserved quantity ?

2. The calorie is defined as 1 cal = 4·186 joule. Why not as 1 cal = 4 J to make the conversions easy ?

3. A calorimeter is kept in a wooden box to insulate it thermally from the surroundings. Why is it necessary ?

4. In a calorimeter, the heat given by the hot object is assumed to be equal to the heat taken by the cold object. Does it mean that heat of the two objects taken together remains constant ?

5. In Regnault's apparatus for measuring specific heat capacity of a solid, there is an inlet and an outlet in the steam chamber. The inlet is near the top and the outlet is near the bottom. Why is it better than the opposite choice where the inlet is near the bottom and the outlet is near the top ?

6. When a solid melts or a liquid boils, the temperature does not increase even when heat is supplied. Where does the energy go ?

7. What is the specific heat capacity of (a) melting ice (b) boiling water ?

8. A person's skin is more severely burnt when put in contact with 1 g of steam at 100°C than when put in contact with 1 g of water at 100°C. Explain.

9. The atmospheric temperature in the cities on sea-coast change very little. Explain.

10. Should a thermometer bulb have large heat capacity or small heat capacity ?

## OBJECTIVE I

1. The specific heat capacity of a body depends on
   (a) the heat given           (b) the temperature raised
   (c) the mass of the body    (d) the material of the body.

2. Water equivalent of a body is measured in
   (a) kg        (b) calorie       (c) kelvin       (d) m$^3$.

3. When a hot liquid is mixed with a cold liquid, the temperature of the mixture
   (a) first decreases then becomes constant
   (b) first increases then becomes constant
   (c) continuously increases
   (d) is undefined for some time and then becomes nearly constant.

4. Which of the following pairs represent units of the same physical quantity ?
   (a) kelvin and joule          (b) kelvin and calorie
   (c) newton and calorie        (d) joule and calorie.

5. Which of the following pairs of physical quantities may be represented in the same unit ?
   (a) heat and temperature      (b) temperature and mole
   (c) heat and work             (d) specific heat and heat.

6. Two bodies at different temperatures are mixed in a calorimeter. Which of the following quantities remains conserved ?
   (a) sum of the temperatures of the two bodies
   (b) total heat of the two bodies
   (c) total internal energy of the two bodies
   (d) internal energy of each body.

7. The mechanical equivalent of heat
   (a) has the same dimension as heat
   (b) has the same dimension as work
   (c) has the same dimension as energy
   (d) is dimensionless.

## OBJECTIVE II

1. The heat capacity of a body depends on
   (a) the heat given            (b) the temperature raised
   (c) the mass of the body     (d) the material of the body.

2. The ratio of specific heat capacity to molar heat capacity of a body
   (a) is a universal constant
   (b) depends on the mass of the body
   (c) depends on the molecular weight of the body
   (d) is dimensionless.

3. If heat is supplied to a solid, its temperature
   (a) must increase             (b) may increase
   (c) may remain constant       (d) may decrease.

4. The temperature of a solid object is observed to be constant during a period. In this period
   (a) heat may have been supplied to the body
   (b) heat may have been extracted from the body

   (c) no heat is supplied to the body
   (d) no heat is extracted from the body.

5. The temperature of an object is observed to rise in a period. In this period
   (a) heat is certainly supplied to it
   (b) heat is certainly not supplied to it
   (c) heat may have been supplied to it
   (d) work may have been done on it.

6. Heat and work are equivalent. This means,
   (a) when we supply heat to a body we do work on it
   (b) when we do work on a body we supply heat to it
   (c) the temperature of a body can be increased by doing work on it
   (d) a body kept at rest may be set into motion along a line by supplying heat to it.

# EXERCISES

1. An aluminium vessel of mass 0.5 kg contains 0.2 kg of water at 20°C. A block of iron of mass 0.2 kg at 100°C is gently put into the water. Find the equilibrium temperature of the mixture. Specific heat capacities of aluminium, iron and water are 910 J/kg-K, 470 J/kg-K and 4200 J/kg-K respectively.

2. A piece of iron of mass 100 g is kept inside a furnace for a long time and then put in a calorimeter of water equivalent 10 g containing 240 g of water at 20°C. The mixture attains an equilibrium temperature of 60°C. Find the temperature of the furnace. Specific heat capacity of iron = 470 J/kg-°C.

3. The temperatures of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C, and when B and C are mixed, it is 23°C. What will be the temperature when A and C are mixed?

4. Four 2 cm × 2 cm × 2 cm cubes of ice are taken out from a refrigerator and are put in 200 ml of a drink at 10°C. (a) Find the temperature of the drink when thermal equilibrium is attained in it. (b) If the ice cubes do not melt completely, find the amount melted. Assume that no heat is lost to the outside of the drink and that the container has negligible heat capacity. Density of ice = 900 kg/m$^3$, density of the drink = 1000 kg/m$^3$, specific heat capacity of the drink = 4200 J/kg-K, latent heat of fusion of ice = 3.4 × 10$^5$ J/kg.

5. Indian style of cooling drinking water is to keep it in a pitcher having porous walls. Water comes to the outer surface very slowly and evaporates. Most of the energy needed for evaporation is taken from the water itself and the water is cooled down. Assume that a pitcher contains 10 kg of water and 0.2 g of water comes out per second. Assuming no backward heat transfer from the atmosphere to the water, calculate the time in which the temperature decreases by 5°C. Specific heat capacity of water = 4200 J/kg-°C and latent heat of vaporization of water = 2.27 × 10$^6$ J/kg.

6. A cube of iron (density = 8000 kg/m$^3$, specific heat capacity = 470 J/kg-K) is heated to a high temperature and is placed on a large block of ice at 0°C. The cube melts the ice below it, displaces the water and sinks. In the final equilibrium position, its upper surface just goes inside the ice. Calculate the initial temperature of the cube. Neglect any loss of heat outside the ice and the cube. The density of ice = 900 kg/m$^3$ and the latent heat of fusion of ice = 3.36 × 10$^5$ J/kg.

7. 1 kg of ice at 0°C is mixed with 1 kg of steam at 100°C. What will be the composition of the system when thermal equilibrium is reached? Latent heat of fusion of ice = 3.36 × 10$^5$ J/kg and latent heat of vaporization of water = 2.26 × 10$^6$ J/kg.

8. Calculate the time required to heat 20 kg of water from 10°C to 35°C using an immersion heater rated 1000 W. Assume that 80% of the power input is used to heat the water. Specific heat capacity of water = 4200 J/kg-K.

9. On a winter day the temperature of the tap water is 20°C whereas the room temperature is 5°C. Water is stored in a tank of capacity 0.5 m$^3$ for household use. If it were possible to use the heat liberated by the water to lift a 10 kg mass vertically, how high can it be lifted as the water comes to the room temperature? Take g = 10 m/s$^2$.

10. A bullet of mass 20 g enters into a fixed wooden block with a speed of 40 m/s and stops in it. Find the change in internal energy during the process.

11. A 50 kg man is running at a speed of 18 km/h. If all the kinetic energy of the man can be used to increase the temperature of water from 20°C to 30°C, how much water can be heated with this energy?

12. A brick weighing 4.0 kg is dropped into a 1.0 m deep river from a height of 2.0 m. Assuming that 80% of the gravitational potential energy is finally converted into thermal energy, find this thermal energy in calorie.

13. A van of mass 1500 kg travelling at a speed of 54 km/h is stopped in 10 s. Assuming that all the mechanical energy lost appears as thermal energy in the brake mechanism, find the average rate of production of thermal energy in cal/s.

14. A block of mass 100 g slides on a rough horizontal surface. If the speed of the block decreases from 10 m/s to 5 m/s, find the thermal energy developed in the process.

15. Two blocks of masses 10 kg and 20 kg moving at speeds of 10 m/s and 20 m/s respectively in opposite directions, approach each other and collide. If the collision is completely inelastic, find the thermal energy developed in the process.

16. A ball is dropped on a floor from a height of 2.0 m. After the collision it rises up to a height of 1.5 m. Assume that 40% of the mechanical energy lost goes as thermal energy into the ball. Calculate the rise in the temperature of the ball in the collision. Heat capacity of the ball is 800 J/K.

17. A copper cube of mass 200 g slides down on a rough inclined plane of inclination 37° at a constant speed. Assume that any loss in mechanical energy goes into the copper block as thermal energy. Find the increase in the temperature of the block as it slides down through 60 cm. Specific heat capacity of copper = 420 J/kg-K.

18. A metal block of density 6000 kg/m$^3$ and mass 1.2 kg is suspended through a spring of spring constant 200 N/m. The spring-block system is dipped in water kept in a vessel. The water has a mass of 260 g and the block is at a height 40 cm above the bottom of the vessel. If the support to the spring is broken, what will be the rise in the temperature of the water. Specific heat capacity of the block is 250 J/kg-K and that of water is 4200 J/kg-K. Heat capacities of the vessel and the spring are negligible.

# ANSWERS

## OBJECTIVE I

1. (d)    2. (a)    3. (d)    4. (d)    5. (c)    6. (c)

7. (d)

## OBJECTIVE II

1. (c), (d)              2. (c)                3. (b), (c)

4. (a), (b)             5. (c), (d)          6. (c)

## EXERCISES

1. 25°C

2. 950°C

3. 20·3°C

4. (a) 0°C (b) 25 g

5. 7·7 min

6. 80°C

7. 665 g steam and 1·335 kg water

8. 44 min

9. 315 km

10. 16 J

11. 15 g

12. 23 cal

13. 4000 cal/s

14. 3·75 J

15. 3000 J

16. $2 \cdot 5 \times 10^{-3}$°C

17. $8 \cdot 6 \times 10^{-3}$°C

18. 0·003°C

□

10. *A sample of 100 g water is slowly heated from 27°C to 87°C. Calculate the change in the entropy of the water. Specific heat capacity of water = 4200 J/kg-K.*

**Solution :** The heat supplied to increase the temperature of the sample from $T$ to $T + \Delta T$ is

$$\Delta Q = ms\,\Delta T,$$

where $m = 100\ g = 0.1\ kg$ and $C = 4200$ J/kg-K.

The change in entropy during this process is

$$\Delta S = \frac{\Delta Q}{T} = ms\,\frac{\Delta T}{T}.$$

The total change in entropy as the temperature rises from $T_1$ to $T_2$ is,

$$S_2 - S_1 = \int_{T_1}^{T_2} ms\,\frac{dT}{T}$$

$$= ms\,\ln\frac{T_2}{T_1}.$$

Putting $T_1 = 27°C = 300$ K and $T_2 = 87°C = 360$ K,

$$S_2 - S_1 = (0.1\ kg)\,(4200\ \text{J/kg-K})\ln\frac{360}{300}$$

$$= 76.6\ \text{J/K}.$$

11. *A heat engine operates between a cold reservoir at temperature $T_2 = 300$ K and a hot reservoir at temperature $T_1$. It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle. What could be the minimum temperature of the hot reservoir?*

**Solution :** The work done by the engine in a cycle is
$$W = 200\ J - 120\ J = 80\ J.$$

The efficiency of the engine is

$$\eta = \frac{W}{Q} = \frac{80\ J}{200\ J} = 0.40.$$

From Carnot's theorem, no engine can have an efficiency greater than that of a Carnot engine.

Thus,           $0.40 \le 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{300\ K}{T_1}$

or,           $\dfrac{300\ K}{T_1} \le 1 - 0.40 = 0.60$

or,           $T_1 \ge \dfrac{300\ K}{0.60}$

or,           $T_1 \ge 500\ K.$

The minimum temperature of the hot reservoir may be 500 K.

□

# QUESTIONS FOR SHORT ANSWER

1. Should the internal energy of a system necessarily increase if heat is added to it?

2. Should the internal energy of a system necessarily increase if its temperature is increased?

3. A cylinder containing a gas is lifted from the first floor to the second floor. What is the amount of work done on the gas? What is the amount of work done by the gas? Is the internal energy of the gas increased? Is the temperature of the gas increased?

4. A force $F$ is applied on a block of mass $M$. The block is displaced through a distance $d$ in the direction of the force. What is the work done by the force on the block? Does the internal energy change because of this work?

5. The outer surface of a cylinder containing a gas is rubbed vigorously by a polishing machine. The cylinder and its gas become warm. Is the energy transferred to the gas heat or work?

6. When we rub our hands they become warm. Have we supplied heat to the hands?

7. A closed bottle contains some liquid. The bottle is shaken vigorously for 5 minutes. It is found that the temperature of the liquid is increased. Is heat transferred to the liquid? Is work done on the liquid? Neglect expansion on heating.

8. The final volume of a system is equal to the initial volume in a certain process. Is the work done by the system necessarily zero? Is it necessarily nonzero?

9. Can work be done by a system without changing its volume?

10. An ideal gas is pumped into a rigid container having diathermic walls so that the temperature remains constant. In a certain time interval, the pressure in the container is doubled. Is the internal energy of the contents of the container also doubled in the interval?

11. When a tyre bursts, the air coming out is cooler than the surrounding air. Explain.

12. When we heat an object, it expands. Is work done by the object in this process? Is heat given to the object equal to the increase in its internal energy?

13. When we stir a liquid vigorously, it becomes warm. Is it a reversible process?

14. What should be the condition for the efficiency of a Carnot engine to be equal to 1?

15. When an object cools down, heat is withdrawn from it. Does the entropy of the object decrease in this process? If yes, is it a violation of the second law of thermodynamics stated in terms of increase in entropy?

# OBJECTIVE I

1. The first law of thermodynamics is a statement of
   (a) conservation of heat
   (b) conservation of work
   (c) conservation of momentum
   (d) conservation of energy.

2. If heat is supplied to an ideal gas in an isothermal process,
   (a) the internal energy of the gas will increase
   (b) the gas will do positive work
   (c) the gas will do negative work
   (d) the said process is not possible.

3. Figure (26-Q1) shows two processes $A$ and $B$ on a system. Let $\Delta Q_1$ and $\Delta Q_2$ be the heat given to the system in processes $A$ and $B$ respectively. Then
   (a) $\Delta Q_1 > \Delta Q_2$ (b) $\Delta Q_1 = \Delta Q_2$ (c) $\Delta Q_1 < \Delta Q_2$ (d) $\Delta Q_1 \le \Delta Q_2$.
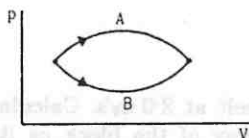


Figure 26-Q1

4. Refer to figure (26-Q1). Let $\Delta U_1$ and $\Delta U_2$ be the changes in internal energy of the system in the processes $A$ and $B$. Then
   (a) $\Delta U_1 > \Delta U_2$   (b) $\Delta U_1 = \Delta U_2$
   (c) $\Delta U_1 < \Delta U_2$   (d) $\Delta U_1 \ne \Delta U_2$.

5. Consider the process on a system shown in figure (26-Q2). During the process, the work done by the system
   (a) continuously increases
   (b) continuously decreases
   (c) first increases then decreases
   (d) first decreases then increases.
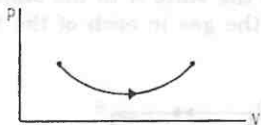


Figure 26-Q2

6. Consider the following two statements.
   (A) If heat is added to a system, its temperature must increase.

   (B) If positive work is done by a system in a thermodynamic process, its volume must increase.
   (a) Both A and B are correct.
   (b) A is correct but B is wrong.
   (c) B is correct but A is wrong.
   (d) Both A and B are wrong.

7. An ideal gas goes from the state $i$ to the state $f$ as shown in figure (26-Q3). The work done by the gas during the process
   (a) is positive    (b) is negative    (c) is zero
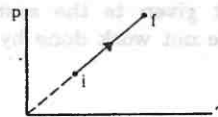   (d) cannot be obtained from this information.



Figure 26-Q3

8. Consider two processes on a system as shown in figure (26-Q4).


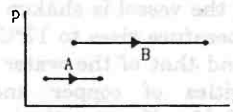
Figure 26-Q4

The volumes in the initial states are the same in the two processes and the volumes in the final states are also the same. Let $\Delta W_1$ and $\Delta W_2$ be the work done by the system in the processes $A$ and $B$ respectively.
   (a) $\Delta W_1 > \Delta W_2$.    (b) $\Delta W_1 = \Delta W_2$.    (c) $\Delta W_1 < \Delta W_2$.
   (d) Nothing can be said about the relation between $\Delta W_1$ and $\Delta W_2$.

9. A gas is contained in a metallic cylinder fitted with a piston. The piston is suddenly moved in to compress the gas and is maintained at this position. As time passes the pressure of the gas in the cylinder
   (a) increases            (b) decreases
   (c) remains constant
   (d) increases or decreases depending on the nature of the gas.

# OBJECTIVE II

1. The pressure $p$ and volume $V$ of an ideal gas both increase in a process.
   (a) Such a process is not possible.
   (b) The work done by the system is positive.
   (c) The temperature of the system must increase.

   (d) Heat supplied to the gas is equal to the change in internal energy.

2. In a process on a system, the initial pressure and volume are equal to the final pressure and volume.
   (a) The initial temperature must be equal to the final temperature.

(b) The initial internal energy must be equal to the final internal energy.

(c) The net heat given to the system in the process must be zero.

(d) The net work done by the system in the process must be zero.

3. A system can be taken from the initial state $p_1$, $V_1$ to the final state $p_2$, $V_2$ by two different methods. Let $\Delta Q$ and $\Delta W$ represent the heat given to the system and the work done by the system. Which of the following must be the same in both the methods ?
(a) $\Delta Q$     (b) $\Delta W$     (c) $\Delta Q + \Delta W$     (d) $\Delta Q - \Delta W$.

4. Refer to figure (26-Q5). Let $\Delta U_1$ and $\Delta U_2$ be the change in internal energy in processes $A$ and $B$ respectively, $\Delta Q$ be the net heat given to the system in process $A + B$ and $\Delta W$ be the net work done by the system in the process $A + B$.
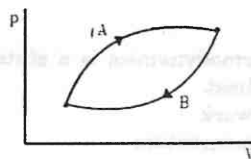


Figure 26-Q5

(a) $\Delta U_1 + \Delta U_2 = 0$.          (b) $\Delta U_1 - \Delta U_2 = 0$.
(c) $\Delta Q - \Delta W = 0$.             (d) $\Delta Q + \Delta W = 0$.

5. The internal energy of an ideal gas decreases by the same amount as the work done by the system.
(a) The process must be adiabatic.
(b) The process must be isothermal.
(c) The process must be isobaric.
(d) The temperature must decrease.

## EXERCISES

1. A thermally insulated, closed copper vessel contains water at 15°C. When the vessel is shaken vigorously for 15 minutes, the temperature rises to 17°C. The mass of the vessel is 100 g and that of the water is 200 g. The specific heat capacities of copper and water are 420 J/kg–K and 4200 J/kg–K respectively. Neglect any thermal expansion. (a) How much heat is transferred to the liquid–vessel system ? (b) How much work has been done on this system ? (c) How much is the increase in internal energy of the system ?

2. Figure (26-E1) shows a paddle wheel coupled to a mass of 12 kg through fixed frictionless pulleys. The paddle is immersed in a liquid of heat capacity 4200 J/K kept in an adiabatic container. Consider a time interval in which the 12 kg block falls slowly through 70 cm. (a) How much heat is given to the liquid ? (b) How much work is done on the liquid ? (c) Calculate the rise in the temperature of the liquid neglecting the heat capacity of the container and the paddle.
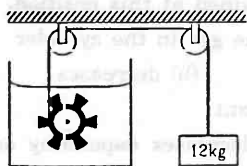


Figure 26-E1

3. A 100 kg block is started with a speed of 2·0 m/s on a long, rough belt kept fixed in a horizontal position. The coefficient of kinetic friction between the block and the belt is 0·20. (a) Calculate the change in the internal energy of the block–belt system as the block comes to a stop on the belt. (b) Consider the situation from a frame of reference moving at 2·0 m/s along the initial velocity of the block. As seen from this frame, the block is gently put on a moving belt and in due time the block starts moving with the belt at 2·0 m/s. Calculate the increase in the kinetic energy of the block as it stops slipping past the belt. (c) Find the work done in this frame by the external force holding the belt.

4. Calculate the change in internal energy of a gas kept in a rigid container when 100 J of heat is supplied to it.

5. The pressure of a gas changes linearly with volume from 10 kPa, 200 cc to 50 kPa, 50 cc. (a) Calculate the work done by the gas. (b) If no heat is supplied or extracted from the gas, what is the change in the internal energy of the gas ?

6. An ideal gas is taken from an initial state $i$ to a final state $f$ in such a way that the ratio of the pressure to the absolute temperature remains constant. What will be the work done by the gas ?

7. Figure (26-E2) shows three paths through which a gas can be taken from the state $A$ to the state $B$. Calculate the work done by the gas in each of the three paths.
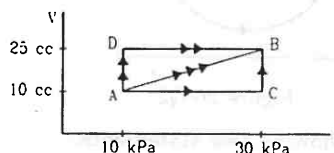


Figure 26-E2

8. When a system is taken through the process $abc$ shown in figure (26-E3), 80 J of heat is absorbed by the system and 30 J of work is done by it. If the system does 10 J
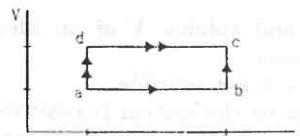


Figure 26-E3

of work during the process *adc*, how much heat flows into it during the process ?

9. 50 cal of heat should be supplied to take a system from the state *A* to the state *B* through the path *ACB* as shown in figure (26-E4). Find the quantity of heat to be supplied to take it from *A* to *B* via *ADB*.
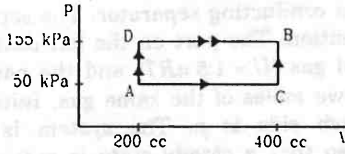


Figure 26-E4

10. Calculate the heat absorbed by a system in going through the cyclic process shown in figure (26-E5).
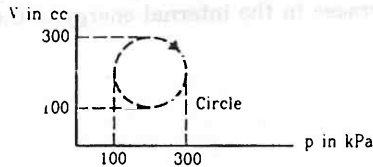


Figure 26-E5

11. A gas is taken through a cyclic process *ABCA* as shown in figure (26-E6). If 2·4 cal of heat is given in the process, what is the value of *J* ?
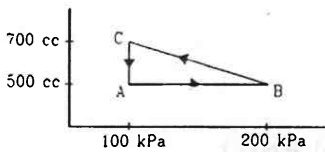


Figure 26-E6

12. A substance is taken through the process *abc* as shown in figure (26-E7). If the internal energy of the substance increases by 5000 J and a heat of 2625 cal is given to the system, calculate the value of *J*.
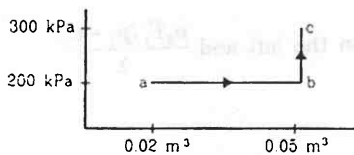


Figure 26-E7

13. A gas is taken along the path *AB* as shown in figure (26-E8). If 70 cal of heat is extracted from the gas in the
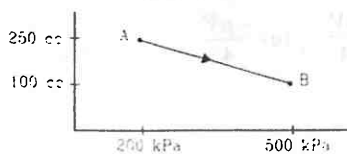


Figure 26-E8

process, calculate the change in the internal energy of the system.

14. The internal energy of a gas is given by $U = 1.5\,pV$. It expands from $100\ cm^3$ to $200\ cm^3$ against a constant pressure of $1.0 \times 10^5$ Pa. Calculate the heat absorbed by the gas in the process.

15. A gas is enclosed in a cylindrical vessel fitted with a frictionless piston. The gas is slowly heated for some time. During the process, 10 J of heat is supplied and the piston is found to move out 10 cm. Find the increase in the internal energy of the gas. The area of cross-section of the cylinder = $4\ cm^2$ and the atmospheric pressure = 100 kPa.

16. A gas is initially at a pressure of 100 kPa and its volume is $2.0\ m^3$. Its pressure is kept constant and the volume is changed from $2.0\ m^3$ to $2.5\ m^3$. Its volume is now kept constant and the pressure is increased from 100 kPa to 200 kPa. The gas is brought back to its initial state, the pressure varying linearly with its volume. (a) Whether the heat is supplied to or extracted from the gas in the complete cycle ? (b) How much heat was supplied or extracted ?

17. Consider the cyclic process *ABCA*, shown in figure (26-E9), performed on a sample of 2·0 mole of an ideal gas. A total of 1200 J of heat is withdrawn from the sample in the process. Find the work done by the gas during the part *BC*.
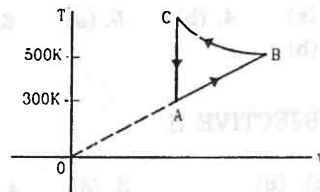


Figure 26-E9

18. Figure (26-E10) shows the variation in the internal energy *U* with the volume *V* of 2·0 mole of an ideal gas in a cyclic process *abcda*. The temperatures of the gas at *b* and *c* are 500 K and 300 K respectively. Calculate the heat absorbed by the gas during the process.
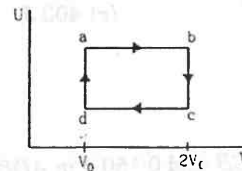


Figure 26-E10

19. Find the change in the internal energy of 2 kg of water as it is heated from 0°C to 4°C. The specific heat capacity of water is 4200 J/kg–K and its densities at 0°C and 4°C are 999·9 $kg/m^3$ and 1000 $kg/m^3$ respectively. Atmospheric pressure = $10^5$ Pa.

20. Calculate the increase in the internal energy of 10 g of water when it is heated from 0°C to 100°C and converted

into steam at 100 kPa. The density of steam = 0·6 kg/m³. Specific heat capacity of water = 4200 J/kg–°C and the latent heat of vaporization of water = 2·25 × 10⁶ J/kg.

21. Figure (26-E11) shows a cylindrical tube of volume $V$ with adiabatic walls containing an ideal gas. The internal energy of this ideal gas is given by $1.5 nRT$. The tube is divided into two equal parts by a fixed diathermic wall. Initially, the pressure and the temperature are $p_1$, $T_1$ on the left and $p_2$, $T_2$ on the right. The system is left for sufficient time so that the temperature becomes equal on the two sides. (a) How
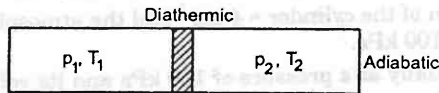


Figure 26-E11

much work has been done by the gas on the left part ? (b) Find the final pressures on the two sides. (c) Find the final equilibrium temperature. (d) How much heat has flown from the gas on the right to the gas on the left ?

22. An adiabatic vessel of total volume $V$ is divided into two equal parts by a conducting separator. The separator is fixed in this position. The part on the left contains one mole of an ideal gas ($U = 1.5 nRT$) and the part on the right contains two moles of the same gas. Initially, the pressure on each side is $p$. The system is left for sufficient time so that a steady state is reached. Find (a) the work done by the gas in the left part during the process, (b) the temperature on the two sides in the beginning, (c) the final common temperature reached by the gases, (d) the heat given to the gas in the right part and (e) the increase in the internal energy of the gas in the left part.

□

# ANSWERS

## OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (d) | 2. (b) | 3. (a) | 4. (b) | 5. (a) | 6. (c) |
| 7. (c) | 8. (c) | 9. (b) | | | |

## OBJECTIVE II

1. (b), (c)     2. (a). (b)     3. (d)     4. (a), (c)
5. (a), (d)

## EXERCISES

1. (a) zero     (b) 1764 J     (c) 1764 J
2. (a) zero     (b) 84 J     (c) 0·02°C
3. (a) 200 J     (b) 200 J     (c) 400 J
4. 100 J
5. (a) – 4·5 J     (b) 4·5 J
6. zero
7. 0·30 J in $AB$, 0·450 J in $ACB$ and 0·150 J in $ADB$
8. 60 J
9. 55 cal
10. 31·4 J

11. 4·17 J/cal
12. 4·19 J/cal
13. – 241 J
14. 25 J
15. 6 J
16. (a) extracted     (b) 25000 J
17. – 4520 J
18. 2300 J
19. (33600 – 0·02) J
20. 2·5 × 10⁴ J
21. (a) zero
    (b) $\dfrac{p_1 T_2 (p_1 + p_2)}{\lambda}$ on the left and $\dfrac{p_2 T_1 (p_1 + p_2)}{\lambda}$
        on the right
    (c) $\dfrac{T_1 T_2 (p_1 + p_2)}{\lambda}$
    (d) $\dfrac{3 p_1 p_2 (T_2 - T_1) V}{4\lambda}$ where $\lambda = p_1 T_2 + p_2 T_1$
22. (a) zero     (b) $\dfrac{pV}{(2 \text{ mol}) R}$, $\dfrac{pV}{(4 \text{ mol}) R}$
    (c) $\dfrac{pV}{(3 \text{ mol}) R}$     (d) $\dfrac{pV}{4}$     (e) $\dfrac{-pV}{4}$

□

## QUESTIONS FOR SHORT ANSWER

1. Does a gas have just two specific heat capacities or more than two ? Is the number of specific heat capacities of a gas countable ?

2. Can we define specific heat capacity at constant temperature ?

3. Can we define specific heat capacity for an adiabatic process ?

4. Does a solid also have two kinds of molar heat capacities $C_p$ and $C_v$ ? If yes, do we have $C_p > C_v$ ? $C_p - C_v = R$ ?

5. In a real gas the internal energy depends on temperature and also on volume. The energy increases when the gas expands isothermally. Looking into the derivation of $C_p - C_v = R$, find whether $C_p - C_v$ will be more than $R$, less than $R$ or equal to $R$ for a real gas.

6. Can a process on an ideal gas be both adiabatic and isothermal ?

7. Show that the slope of $p-V$ diagram is greater for an adiabatic process as compared to an isothermal process.

8. Is a slow process always isothermal ? Is a quick process always adiabatic ?

9. Can two states of an ideal gas be connected by an isothermal process as well as an adiabatic process ?

10. The ratio $C_p/C_v$ for a gas is 1·29. What is the degree of freedom of the molecules of this gas ?

## OBJECTIVE I

1. Work done by a sample of an ideal gas in a process $A$ is double the work done in another process $B$. The temperature rises through the same amount in the two processes. If $C_A$ and $C_B$ be the molar heat capacities for the two processes,
   (a) $C_A = C_B$         (b) $C_A < C_B$
   (c) $C_A > C_B$         (d) $C_A$ and $C_B$ cannot be defined.

2. For a solid with a small expansion coefficient,
   (a) $C_p - C_v = R$                 (b) $C_p - C_v$
   (c) $C_p$ is slightly greater than $C_v$
   (d) $C_p$ is slightly less than $C_v$.

3. The value of $C_p - C_v$ is 1·00 $R$ for a gas sample in state $A$ and is 1·08 $R$ in state $B$. Let $p_A$, $p_B$ denote the pressures and $T_A$ and $T_B$ denote the temperatures of the states $A$ and $B$ respectively. Most likely
   (a) $p_A < p_B$ and $T_A > T_B$         (b) $p_A > p_B$ and $T_A < T_B$
   (c) $p_A = p_B$ and $T_A < T_B$         (d) $p_A > p_B$ and $T_A = T_B$.

4. Let $C_v$ and $C_p$ denote the molar heat capacities of an ideal gas at constant volume and constant pressure respectively. Which of the following is a universal constant ?
   (a) $\dfrac{C_p}{C_v}$      (b) $C_p C_v$      (c) $C_p - C_v$      (d) $C_p + C_v$.

5. 70 calories of heat is required to raise the temperature of 2 mole of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required to raise the temperature of the same gas through the same range at constant volume is
   (a) 30 calories                    (b) 50 calories
   (c) 70 calories                    (d) 90 calories.



Figure 27-Q1

6. Figure (27-Q1) shows a process on a gas in which pressure and volume both change. The molar heat capacity for this process is $C$.
   (a) $C = 0$      (b) $C = C_v$      (c) $C > C_v$      (d) $C < C_v$.

7. The molar heat capacity for the process shown in figure (27-Q2) is
   (a) $C = C_p$      (b) $C = C_v$      (c) $C > C_v$      (d) $C = 0$.



Figure 27-Q2

8. In an isothermal process on an ideal gas, the pressure increases by 0·5%. The volume decreases by about
   (a) 0·25%      (b) 0·5%      (c) 0·7%      (d) 1%.

9. In an adiabatic process on a gas with $\gamma = 1·4$, the pressure is increased by 0·5%. The volume decreases by about
   (a) 0·36%      (b) 0·5%      (c) 0·7%      (d) 1%.

10. Two samples $A$ and $B$ are initially kept in the same state. The sample $A$ is expanded through an adiabatic process and the sample $B$ through an isothermal process. The final volumes of the samples are the same. The final pressures in $A$ and $B$ are $p_A$ and $p_B$ respectively.
    (a) $p_A > p_B$.         (b) $p_A = p_B$.         (c) $p_A < p_B$.
    (d) The relation between $p_A$ and $p_B$ cannot be deduced.

11. Let $T_a$ and $T_b$ be the final temperatures of the samples $A$ and $B$ respectively in the previous question.
    (a) $T_a < T_b$.         (b) $T_a = T_b$.         (c) $T_a > T_b$.
    (d) The relation between $T_a$ and $T_b$ cannot be deduced.

12. Let $\Delta W_a$ and $\Delta W_b$ be the work done by the systems $A$ and $B$ respectively in the previous question.
    (a) $\Delta W_a > \Delta W_b$.      (b) $\Delta W_a = \Delta W_b$.      (c) $\Delta W_a < \Delta W_b$.
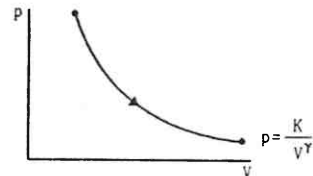
(d) The relation between $\Delta W_a$ and $\Delta W_b$ cannot be deduced.

13. The molar heat capacity of oxygen gas at STP is nearly $2.5\,R$. As the temperature is increased, it gradually increases and approaches $3.5\,R$. The most appropriate reason for this behaviour is that at high temperatures

(a) oxygen does not behave as an ideal gas
(b) oxygen molecules dissociate in atoms
(c) the molecules collide more frequently
(d) molecular vibrations gradually become effective.

## OBJECTIVE II

1. A gas kept in a container of finite conductivity is suddenly compressed. The process
(a) must be very nearly adiabatic
(b) must be very nearly isothermal
(c) may be very nearly adiabatic
(d) may be very nearly isothermal.

2. Let $Q$ and $W$ denote the amount of heat given to an ideal gas and the work done by it in an isothermal process.
(a) $Q = 0$.    (b) $W = 0$.    (c) $Q \neq W$.    (d) $Q = W$.

3. Let $Q$ and $W$ denote the amount of heat given to an ideal gas and the work done by it in an adiabatic process.
(a) $Q = 0$.    (b) $W = 0$.    (c) $Q = W$.    (d) $Q \neq W$.

4. Consider the processes $A$ and $B$ shown in figure (27-Q3). It is possible that
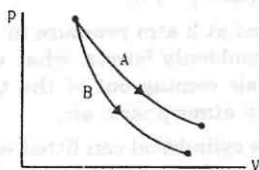


Figure 27-Q3

(a) both the processes are isothermal
(b) both the processes are adiabatic
(c) $A$ is isothermal and $B$ is adiabatic
(d) $A$ is adiabatic and $B$ is isothermal.

5. Three identical adiabatic containers $A$, $B$ and $C$ contain helium, neon and oxygen respectively at equal pressure. The gases are pushed to half their original volumes.
(a) The final temperatures in the three containers will be the same.
(b) The final pressures in the three containers will be the same.
(c) The pressures of helium and neon will be the same but that of oxygen will be different.
(d) The temperatures of helium and neon will be the same but that of oxygen will be different.

6. A rigid container of negligible heat capacity contains one mole of an ideal gas. The temperature of the gas increases by 1°C if $3.0$ cal of heat is added to it. The gas may be
(a) helium    (b) argon    (c) oxygen    (d) carbon dioxide.

7. Four cylinders contain equal number of moles of argon, hydrogen, nitrogen and carbon dioxide at the same temperature. The energy is minimum in
(a) argon (b) hydrogen (c) nitrogen (d) carbon dioxide.

## EXERCISES

1. A vessel containing one mole of a monatomic ideal gas (molecular weight = 20 g/mol) is moving on a floor at a speed of 50 m/s. The vessel is stopped suddenly. Assuming that the mechanical energy lost has gone into the internal energy of the gas, find the rise in its temperature.

2. 5 g of a gas is contained in a rigid container and is heated from 15°C to 25°C. Specific heat capacity of the gas at constant volume is $0.172$ cal/g–°C and the mechanical equivalent of heat is $4.2$ J/cal. Calculate the change in the internal energy of the gas.

3. Figure (27-E1) shows a cylindrical container containing oxygen ($\gamma = 1.4$) and closed by a 50 kg frictionless piston. The area of cross-section is 100 cm². atmospheric pressure is 100 kPa and $g$ is 10 m/s². The cylinder is slowly heated for some time. Find the amount of heat supplied to the gas if the piston moves out through a distance of 20 cm.
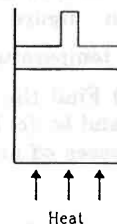


Heat

Figure 27-E1

4. The specific heat capacities of hydrogen at constant volume and at constant pressure are $2.4$ cal/g–°C and $3.4$ cal/g–°C respectively. The molecular weight of hydrogen is 2 g/mol and the gas constant $R = 8.3 \times 10^7$ erg/mol–°C. Calculate the value of $J$.

5. The ratio of the molar heat capacities of an ideal gas is $C_p/C_v = 7/6$. Calculate the change in internal energy of $1.0$ mole of the gas when its temperature is raised by

50 K (a) keeping the pressure constant, (b) keeping the volume constant and (c) adiabatically.

6. A sample of air weighing 1·18 g occupies $1·0 \times 10^3$ cm³ when kept at 300 K and $1·0 \times 10^5$ Pa. When 2·0 cal of heat is added to it at constant volume, its temperature increases by 1°C. Calculate the amount of heat needed to increase the temperature of air by 1°C at constant pressure if the mechanical equivalent of heat is $4·2 \times 10^7$ erg/cal. Assume that air behaves as an ideal gas.

7. An ideal gas expands from 100 cm³ to 200 cm³ at a constant pressure of $2·0 \times 10^5$ Pa when 50 J of heat is supplied to it. Calculate (a) the change in internal energy of the gas, (b) the number of moles in the gas if the initial temperature is 300 K, (c) the molar heat capacity $C_p$ at constant pressure and (d) the molar heat capacity $C_v$ at constant volume.

8. An amount $Q$ of heat is added to a monatomic ideal gas in a process in which the gas performs a work $Q/2$ on its surrounding. Find the molar heat capacity for the process.

9. An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation $p = kV$. Show that the molar heat capacity of the gas for the process is given by $C = C_v + \dfrac{R}{2}$.

10. An ideal gas $(C_p/C_v = \gamma)$ is taken through a process in which the pressure and the volume vary as $p = aV^b$. Find the value of $b$ for which the specific heat capacity in the process is zero.

11. Two ideal gases have the same value of $C_p/C_v = \gamma$. What will be the value of this ratio for a mixture of the two gases in the ratio $1 : 2$ ?

12. A mixture contains 1 mole of helium $(C_p = 2·5\,R, C_v = 1·5\,R)$ and 1 mole of hydrogen $(C_p = 3·5\,R, C_v = 2·5\,R)$. Calculate the values of $C_p$, $C_v$ and $\gamma$ for the mixture.

13. Half mole of an ideal gas $(\gamma = 5/3)$ is taken through the cycle $abcda$ as shown in figure (27-E2). Take $R = \dfrac{25}{3}$ J/mol-K (a) Find the temperature of the gas in the states $a$, $b$, $c$ and $d$. (b) Find the amount of heat supplied in the processes $ab$ and $bc$. (c) Find the amount of heat liberated in the processes $cd$ and $da$.
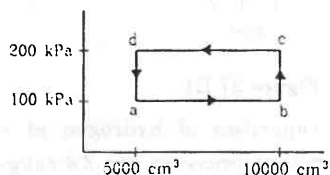


Figure 27-E2

14. An ideal gas $(\gamma = 1·67)$ is taken through the process $abc$ shown in figure (27-E3). The temperature at the point $a$ is 300 K. Calculate (a) the temperatures at $b$ and $c$, (b) the work done in the process, (c) the amount of heat supplied in the path $ab$ and in the path $bc$ and (d) the change in the internal energy of the gas in the process.
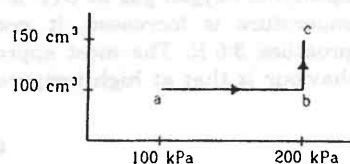


Figure 27-E3

15. In Joly's differential steam calorimeter, 3 g of an ideal gas is contained in a rigid closed sphere at 20°C. The sphere is heated by steam at 100°C and it is found that an extra 0·095 g of steam has condensed into water as the temperature of the gas becomes constant. Calculate the specific heat capacity of the gas in J/g-K. The latent heat of vaporization of water = 540 cal/g.

16. The volume of an ideal gas $(\gamma = 1·5)$ is changed adiabatically from 4·00 litres to 3·00 litres. Find the ratio of (a) the final pressure to the initial pressure and (b) the final temperature to the initial temperature.

17. An ideal gas at pressure $2·5 \times 10^5$ Pa and temperature 300 K occupies 100 cc. It is adiabatically compressed to half its original volume. Calculate (a) the final pressure, (b) the final temperature and (c) the work done by the gas in the process. Take $\gamma = 1·5$.

18. Air $(\gamma = 1·4)$ is pumped at 2 atm pressure in a motor tyre at 20°C. If the tyre suddenly bursts, what would be the temperature of the air coming out of the tyre. Neglect any mixing with the atmospheric air.

19. A gas is enclosed in a cylindrical can fitted with a piston. The walls of the can and the piston are adiabatic. The initial pressure, volume and temperature of the gas are 100 kPa, 400 cm³ and 300 K respectively. The ratio of the specific heat capacities of the gas is $C_p/C_v = 1·5$. Find the pressure and the temperature of the gas if it is (a) suddenly compressed (b) slowly compressed to 100 cm³.

20. The initial pressure and volume of a given mass of a gas $(C_p/C_v = \gamma)$ are $p_0$ and $V_0$. The gas can exchange heat with the surrounding. (a) It is slowly compressed to a volume $V_0/2$ and then suddenly compressed to $V_0/4$. Find the final pressure. (b) If the gas is suddenly compressed from the volume $V_0$ to $V_0/2$ and then slowly compressed to $V_0/4$, what will be the final pressure ?

21. Consider a given sample of an ideal gas $(C_p/C_v = \gamma)$ having initial pressure $p_0$ and volume $V_0$. (a) The gas is isothermally taken to a pressure $p_0/2$ and from there adiabatically to a pressure $p_0/4$. Find the final volume. (b) The gas is brought back to its initial state. It is adiabatically taken to a pressure $p_0/2$ and from there isothermally to a pressure $p_0/4$. Find the final volume.

22. A sample of an ideal gas $(\gamma = 1·5)$ is compressed adiabatically from a volume of 150 cm³ to 50 cm³. The initial pressure and the initial temperature are 150 kPa and 300 K. Find (a) the number of moles of the gas in the sample, (b) the molar heat capacity at constant volume, (c) the final pressure and temperature, (d) the

work done by the gas in the process and (e) the change in internal energy of the gas.

23. Three samples $A$, $B$ and $C$ of the same gas ($\gamma = 1.5$) have equal volumes and temperatures. The volume of each sample is doubled, the process being isothermal for $A$, adiabatic for $B$ and isobaric for $C$. If the final pressures are equal for the three samples, find the ratio of the initial pressures.

24. Two samples $A$ and $B$ of the same gas have equal volumes and pressures. The gas in sample $A$ is expanded isothermally to double its volume and the gas in $B$ is expanded adiabatically to double its volume. If the work done by the gas is the same for the two cases, show that $\gamma$ satisfies the equation $1 - 2^{1-\gamma} = (\gamma - 1)\ln 2$.

25. 1 litre of an ideal gas ($\gamma = 1.5$) at 300 K is suddenly compressed to half its original volume. (a) Find the ratio of the final pressure to the initial pressure. (b) If the original pressure is 100 kPa, find the work done by the gas in the process. (c) What is the change in internal energy? (d) What is the final temperature? (e) The gas is now cooled to 300 K keeping its pressure constant. Calculate the work done during the process. (f) The gas is now expanded isothermally to achieve its original volume of 1 litre. Calculate the work done by the gas. (g) Calculate the total work done in the cycle.

26. Figure (27-E4) shows a cylindrical tube with adiabatic walls and fitted with an adiabatic separator. The separator can be slid into the tube by an external mechanism. An ideal gas ($\gamma = 1.5$) is injected in the two sides at equal pressures and temperatures. The separator remains in equilibrium at the middle. It is now slid to a position where it divides the tube in the ratio 1 : 3. Find the ratio of the temperatures in the two parts of the vessel.



Figure 27-E4

27. Figure (27-E5) shows two rigid vessels $A$ and $B$, each of volume 200 cm$^3$ containing an ideal gas ($C_v = 12.5$ J/mol–K). The vessels are connected to a manometer tube containing mercury. The pressure in both the vessels is 75 cm of mercury and the temperature is 300 K. (a) Find the number of moles of the gas in each vessel. (b) 5.0 J of heat is supplied to the gas in the vessel $A$ and 10 J to the gas in the vessel $B$. Assuming no appreciable transfer of heat from $A$ to $B$ calculate the difference in the heights of mercury in the two sides of the manometer. Gas constant $R = 8.3$ J/mol–K.
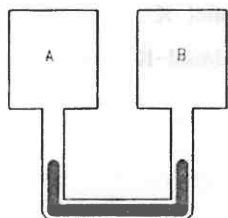


Figure 27-E5

28. Figure (27-E6) shows two vessels with adiabatic walls, one containing 0.1 g of helium ($\gamma = 1.67$, $M = 4$ g/mol) and the other containing some amount of hydrogen ($\gamma = 1.4$, $M = 2$ g/mol). Initially, the temperatures of the two gases are equal. The gases are electrically heated for some time during which equal amounts of heat are given to the two gases. It is found that the temperatures rise through the same amount in the two vessels. Calculate the mass of hydrogen.
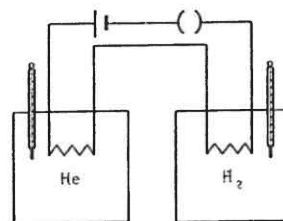


Figure 27-E6

29. Two vessels $A$ and $B$ of equal volume $V_0$ are connected by a narrow tube which can be closed by a valve. The vessels are fitted with pistons which can be moved to change the volumes. Initially, the valve is open and the vessels contain an ideal gas ($C_p/C_v = \gamma$) at atmospheric pressure $p_0$ and atmospheric temperature $T_0$. The walls of the vessel $A$ are diathermic and those of $B$ are adiabatic. The valve is now closed and the pistons are slowly pulled out to increase the volumes of the vessels to double the original value. (a) Find the temperatures and pressures in the two vessels. (b) The valve is now opened for sufficient time so that the gases acquire a common temperature and pressure. Find the new values of the temperature and the pressure.

30. Figure (27-E7) shows an adiabatic cylindrical tube of volume $V_0$ divided in two parts by a frictionless adiabatic separator. Initially, the separator is kept in the middle, an ideal gas at pressure $p_1$ and temperature $T_1$ is injected into the left part and another ideal gas at pressure $p_2$ and temperature $T_2$ is injected into the right part. $C_p/C_v = \gamma$ is the same for both the gases. The separator is slid slowly and is released at a position where it can stay in equilibrium. Find (a) the volumes of the two parts, (b) the heat given to the gas in the left part and (c) the final common pressure of the gases.



Figure 27-E7

31. An adiabatic cylindrical tube of cross-sectional area 1 cm$^2$ is closed at one end and fitted with a piston at the other end. The tube contains 0.03 g of an ideal gas. At 1 atm pressure and at the temperature of the surrounding, the length of the gas column is 40 cm. The piston is suddenly pulled out to double the length of the column. The pressure of the gas falls to 0.355 atm. Find the speed of sound in the gas at atmospheric temperature.

32. The speed of sound in hydrogen at 0°C is 1280 m/s. The density of hydrogen at STP is 0·089 kg/m³. Calculate the molar heat capacities $C_p$ and $C_v$ of hydrogen.

33. 4·0 g of helium occupies 22400 cm³ at STP. The specific heat capacity of helium at constant pressure is 5·0 cal/mol-K. Calculate the speed of sound in helium at STP.

34. An ideal gas having density $1·7 × 10^{-3}$ g/cm³ at a pressure $1·5 × 10^{5}$ Pa is filled in a Kundt's tube. When the gas is resonated at a frequency of 3·0 kHz, nodes are formed at a separation of 6·0 cm. Calculate the molar heat capacities $C_p$ and $C_v$ of the gas.

35. Standing waves of frequency 5·0 kHz are produced in a tube filled with oxygen at 300 K. The separation between the consecutive nodes is 3·3 cm. Calculate the specific heat capacities $C_p$ and $C_v$ of the gas.

□

# ANSWERS

## OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (c) | 3. (a) | 4. (c) | 5. (b) | 6. (c) |
| 7. (d) | 8. (b) | 9. (a) | 10. (c) | 11. (a) | 12. (c) |
| 13. (d) | | | | | |

## OBJECTIVE II

| | | |
|---|---|---|
| 1. (c), (d) | 2. (d) | 3. (a, d) |
| 4. (c) | 5. (c), (d) | 6. (a), (b) |
| 7. (a) | | |

## EXERCISES

1. 2·0 K
2. 36 J
3. 1050 J
4. $4·15 × 10^{7}$ erg/cal
5. 2490 J in all cases
6. 2·08 cal
7. (a) 30 J       (b) 0·008
   (c) 20·8 J/mol-K      (d) 12·5 J/mol-K
8. $3 R$
10. $-\gamma$
11. $\gamma$
12. $3 R, 2 R, 1·5$
13. (a) 120 K, 240 K, 480 K, 240 K
    (b) 1250 J. 1500 J     (c) 2500 J, 750 J
14. (a) 600 K, 900 K     (b) 10 J
    (c) 14·9 J, 24·9 J     (d) 29·8 J
15. 0·90 J/g-K

16. (a) 1·54     (b) 1·15
17. (a) $7·1 × 10^{5}$ Pa    (b) 424 K    (c) $-21$ J
18. 240 K
19. 800 kPa, 600 K in both cases
20. $2^{\gamma+1} p_0$ in both cases
21. $2^{(\gamma+1)/\gamma} V_0$ in each case
22. (a) 0·009      (b) $2 R = 16·6$ J/mol-K
    (c) 780 kPa, 520 K    (d) $-33$ J   (e) 33 J
23. $2 : 2\sqrt{2} : 1$
25. (a) $2\sqrt{2}$       (b) $-82$ J
    (c) 82 J        (d) 424 K
    (e) $-41·4$ J     (f) 103 J
    (g) $-23·4$ J
26. $\sqrt{3} : 1$
27. (a) 0·008       (b) 12·5 cm
28. 0·03 g
29. (a) $T_0, \dfrac{p_0}{2}$ in vessel $A$ and $T_0/2^{\gamma-1}$, $p_0/2^{\gamma}$ in vessel $B$
    (b) $T_0, p_0/2$
30. (a) $\dfrac{p_1^{1/\gamma} V_0}{A} \cdot \dfrac{p_2^{1/\gamma} V_0}{A}$    (b) zero
    (c) $(A/2)^{\gamma}$ where $A = p_1^{1/\gamma} + p_2^{1/\gamma}$
31. 447 m/s
32. 18·0 J/mol-K, 26·3 J/mol-K
33. 960 m/s
34. 26 J/mol-K, 17·7 J/mol-K
35. 29·0 J/mol-K, 20·7 J/mol-K

□

**Solution :** Suppose, the temperature of the water in the smaller vessel is $\theta$ at time $t$. In the next time interval $dt$, a heat $\Delta Q$ is transferred to it where

$$\Delta Q = \frac{KA}{L} (\theta_0 - \theta)\, dt. \qquad \ldots \text{(i)}$$

This heat increases the temperature of the water of mass $m$ to $\theta + d\theta$ where

$$\Delta Q = ms\, d\theta. \qquad \ldots \text{(ii)}$$

From (i) and (ii),

$$\frac{KA}{L} (\theta_0 - \theta)\, dt = ms\, d\theta$$

or,

$$dt = \frac{Lms}{KA} \frac{d\theta}{\theta_0 - \theta}$$

or,

$$\int_0^T dt = \frac{Lms}{KA} \int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta_0 - \theta}$$

where $T$ is the time required for the temperature of the water to become $\theta_2$.

Thus,

$$T = \frac{Lms}{KA} \ln \frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}.$$

13. *One mole of an ideal monatomic gas is kept in a rigid vessel. The vessel is kept inside a steam chamber whose tempeature is 97°C. Initially, the temperature of the gas is 5·0°C. The walls of the vessel have an inner surface of area 800 cm$^2$ and thickness 1·0 cm. If the temperature of the gas increases to 9·0°C in 5·0 seconds, find the thermal conductivity of the material of the walls.*

**Solution :** The initial temperature difference is 97°C − 5°C = 92°C and at 5·0 s the temperature difference becomes 97°C − 9°C = 88°C. As the change in the temperature difference is small, we work with the average temperature difference

$$\frac{92°C + 88°C}{2} = 90°C = 90 \text{ K}.$$

The rise in the temperature of the gas is

$$9·0°C − 5·0°C = 4°C = 4 \text{ K}.$$

The heat supplied to the gas in 5·0 s is

$$\Delta Q = nC_v \Delta T$$

$$= (1 \text{ mole}) \times \left( \frac{3}{2} \times 8·3 \ \frac{J}{\text{mol-K}} \right) \times (4 \text{ K})$$

$$= 49·8 \text{ J}.$$

If the thermal conductivity is $K$,

$$49·8 \text{ J} = \frac{K(800 \times 10^{-4} \text{ m}^2) \times (90 \text{ K})}{1·0 \times 10^{-2} \text{ m}} \times 5·0 \text{ s}$$

or,

$$K = \frac{49·8 \text{ J}}{3600 \text{ m-s-K}} = 0·014 \text{ J/m-s-K}.$$

14. *A monatomic ideal gas is contained in a rigid container of volume $V$ with walls of total inner surface area $A$, thickness $x$ and thermal conductivity $K$. The gas is at an initial temperature $T_0$ and pressure $p_0$. Find the pressure of the gas as a function of time if the temperature of the surrounding air is $T_s$. All temperatures are in absolute scale.*

**Solution :** As the volume of the gas is constant, a heat $\Delta Q$ given to the gas increases its temperature by $\Delta T = \Delta Q / C_v$. Also, for a monatomic gas, $C_v = \frac{3}{2} R$. If the temperature of the gas at time $t$ is $T$, the heat current into the gas is

$$\frac{\Delta Q}{\Delta t} = \frac{KA(T_s - T)}{x}$$

or,

$$\frac{\Delta T}{\Delta t} = \frac{2KA}{3xR} (T_s - T)$$

or,

$$\int_{T_0}^{T} \frac{dT}{T_s - T} = \int_0^t \frac{2KA}{3xR}\, dt$$

or,

$$\ln \frac{T_s - T_0}{T_s - T} = \frac{2KA}{3xR} t$$

or,

$$T_s - T = (T_s - T_0)\, e^{-\frac{2KA}{3xR} t}$$

or,

$$T = T_s - (T_s - T_0)\, e^{-\frac{2KA}{3xR} t}.$$

As the volume remains constant,

$$\frac{p}{T} = \frac{p_0}{T_0}$$

or,

$$p = \frac{p_0}{T_0} T$$

$$= \frac{p_0}{T_0} \left[ T_s - (T_s - T_0)\, e^{-\frac{2KA}{3xR} t} \right].$$

15. *Consider a cubical vessel of edge $a$ having a small hole in one of its walls. The total thermal resistance of the walls is $r$. At time $t = 0$, it contains air at atmospheric pressure $p_a$ and temperature $T_0$. The temperature of the surrounding air is $T_a (> T_0)$. Find the amount of the gas (in moles) in the vessel at time $t$. Take $C_v$ of air to be $5R/2$.*

**Solution :** As the gas can leak out of the hole, the pressure inside the vessel will be equal to the atmospheric pressure $p_a$. Let $n$ be the amount of the gas (moles) in the vessel at time $t$. Suppose an amount $\Delta Q$ of heat is given to the gas in time $dt$. Its temperature increases by $dT$ where

$$\Delta Q = nC_p dT.$$

If the temperature of the gas is $T$ at time $t$, we have

$$\frac{\Delta Q}{dt} = \frac{T_a - T}{r}$$

or,

$$(C_p r)n\, dT = (T_a - T)dt. \qquad \ldots \text{(i)}$$

We have,

$$p_a a^3 = nRT$$

or,

$$n\, dT + T\, dn = 0$$

or,

$$n\, dT = -T\, dn. \qquad \ldots \text{(ii)}$$

Also, $T = \dfrac{p_a a^3}{nR}$. ... (iii)

Using (ii) and (iii) in (i),

$$\dfrac{-C_p r p_a a^3}{nR} dn - \left(T_a - \dfrac{p_a a^3}{nR}\right) dt$$

or, $$\dfrac{dn}{nR\left(T_a - \dfrac{p_a a^3}{nR}\right)} = -\dfrac{dt}{C_p r p_a a^3}$$

or, $$\int_{n_0}^{n} \dfrac{dn}{nRT_a - p_a a^3} = -\int_{0}^{t} \dfrac{dt}{C_p r p_a a^3}$$

where $n_0 = \dfrac{p_a a^3}{RT_0}$ is the initial amount of the gas in the vessel. Thus,

$$\dfrac{1}{RT_a} \ln \dfrac{nRT_a - p_a a^3}{n_0 RT_a - p_a a^3} = -\dfrac{t}{C_p r p_a a^3}$$

or, $$nRT_a - p_a a^3 = (n_0 RT_a - p_a a^3) e^{\frac{RT_a}{C_p r p_a a^3} t}.$$

Writing $n_0 = \dfrac{p_a a^3}{RT_0}$ and $C_p = C_v + R = \dfrac{7R}{2}$,

$$n = \dfrac{p_a a^3}{RT_a}\left[1 + \left(\dfrac{T_a}{T_0} - 1\right) e^{-\frac{2 T_a}{7 r p_a a^3} t}\right].$$

**16** *A blackbody of surface area 1 cm$^2$ is placed inside an enclosure. The enclosure has a constant temperature 27°C and the blackbody is maintained at 327°C by heating it electrically. What electric power is needed to maintain the temperature? $\sigma = 6.0 \times 10^{-8}$ W/m$^2$-K$^4$.*

**Solution :** The area of the blackbody is $A = 10^{-4}$ m$^2$, its temperature is $T_1 = 327°C = 600$ K and the temperature of the enclosure is $T_2 = 27°C = 300$ K. The blackbody emits radiation at the rate of $A\sigma T_1^4$. The radiation falls on it (and gets absorbed) at the rate of $A\sigma T_2^4$. The net rate of loss of energy is $A\sigma(T_1^4 - T_2^4)$. The heater must supply this much of power. Thus, the power needed is $A\sigma(T_1^4 - T_2^4)$

$= (10^{-4}$ m$^2)(6.0 \times 10^{-8}$ W/m$^2$-K$^4)[(600$ K$)^4 - (300$ K$)^4]$

$= 0.73$ W.

**17.** *An electric heater emits 1000 W of thermal radiation. The coil has a surface area of 0.020 m$^2$. Assuming that the coil radiates like a blackbody, find its temperature. $\sigma = 6.00 \times 10^{-8}$ W/m$^2$-K$^4$.*

**Solution :** Let the temperature of the coil be $T$. The coil will emit radiation at a rate $A\sigma T^4$. Thus,

$1000$ W $= (0.020$ m$^2) \times (6.0 \times 10^{-8}$ W/m$^2$-K$^4) \times T^4$

or, $$T^4 = \dfrac{1000}{0.020 \times 6.00 \times 10^{-8}} \text{ K}^4$$

$= 8.33 \times 10^{11}$ K$^4$

or, $T = 955$ K.

**18.** *The earth receives solar radiation at a rate of 8.2 J/cm$^2$-minute. Assuming that the sun radiates like a blackbody, calculate the surface temperature of the sun. The angle subtended by the sun on the earth is 0.53° and the Stefan constant $\sigma = 5.67 \times 10^{-8}$ W/m$^2$-K$^4$.*
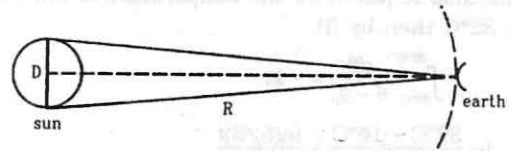
**Solution :**



Figure 28-W11

Let the diameter of the sun be $D$ and its distance from the earth be $R$. From the question,

$$\dfrac{D}{R} = 0.53 \times \dfrac{\pi}{180}$$

$= 9.25 \times 10^{-3}$. ... (i)

The radiation emitted by the surface of the sun per unit time is

$$4\pi\left(\dfrac{D}{2}\right)^2 \sigma T^4 = \pi D^2 \sigma T^4.$$

At distance $R$, this radiation falls on an area $4\pi R^2$ in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore,

$$\dfrac{\pi D^2 \sigma T^4}{4\pi R^2} = \dfrac{\sigma T^4}{4}\left(\dfrac{D}{R}\right)^2.$$

Thus, $$\dfrac{\sigma T^4}{4}\left(\dfrac{D}{R}\right)^2 = 8.2 \text{ J/cm}^2\text{-minute}$$

or, $$\dfrac{1}{4} \times \left(5.67 \times 10^{-8} \dfrac{W}{m^2\text{-}K^4}\right) T^4 \times (9.25 \times 10^{-3})^2$$

$$= \dfrac{8.2}{10^{-4} \times 60} \dfrac{W}{m^2}$$

or, $T = 5794$ K $\approx 5800$ K.

**19.** *The temperature of a body falls from 40°C to 36°C in 5 minutes when placed in a surrounding of constant temperature 16°C. Find the time taken for the temperature of the body to become 32°C.*

**Solution :** As the temperature differences are small, we can use Newton's law of cooling.

$$\dfrac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $$\dfrac{d\theta}{\theta - \theta_0} = -k dt$$ ... (i)

where $k$ is a constant, $\theta$ is the temperature of the body at time $t$ and $\theta_0 = 16°C$ is the temperature of the surrounding. We have,

$$\int_{40°C}^{36°C} \frac{d\theta}{\theta - \theta_0} = -k(5 \text{ min})$$

or, $\quad \ln \dfrac{36°C - 16°C}{40°C - 16°C} = -k(5 \text{ min})$

or, $\quad k = -\dfrac{\ln(5/6)}{5 \text{ min}}$.

If $t$ be the time required for the temperature to fall from 36°C to 32°C then by (i),

$$\int_{36°C}^{32°C} \frac{d\theta}{\theta - \theta_0} = -kt$$

or, $\quad \ln \dfrac{32°C - 16°C}{36°C - 16°C} = \dfrac{\ln(5/6)t}{5 \text{ min}}$

or, $\quad t = \dfrac{\ln(4/5)}{\ln(5/6)} \times 5 \text{ min}$

$\quad\quad\quad\quad = 6{\cdot}1 \text{ min}.$

**Alternative method**

The mean temperature of the body as it cools from 40°C to 36°C is $\dfrac{40°C + 36°C}{2} = 38°C$. The rate of decrease of temperature is $\dfrac{40°C - 36°C}{5 \text{ min}} = 0{\cdot}80°C/\text{min}$.

Newton's law of cooling is

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $\quad -0{\cdot}8°C/\text{min} = -k(38°C - 16°C) = -k(22°C)$

or, $\quad k = \dfrac{0{\cdot}8}{22} \text{ min}^{-1}$.

Let the time taken for the temperature to become 32°C be $t$.

During this period,

$$\frac{d\theta}{dt} = -\frac{36°C - 32°C}{t} = -\frac{4°C}{t}.$$

The mean temperature is $\dfrac{36°C + 32°C}{2} = 34°C$.

Now,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $\quad \dfrac{-4°C}{t} = -\dfrac{0{\cdot}8}{22} \times (34°C - 16°C)/\text{min}$

or, $\quad t = \dfrac{22 \times 4}{0{\cdot}8 \times 18} \text{ min} = 6{\cdot}1 \text{ min}.$

**20.** *A hot body placed in air is cooled down according to Newton's law of cooling, the rate of decrease of temperature being $k$ times the temperature difference from the surrounding. Starting from $t = 0$, find the time in which the body will lose half the maximum heat it can lose.*

*Solution :* We have,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

where $\theta_0$ is the temperature of the surrounding and $\theta$ is the temperature of the body at time $t$. Suppose $\theta = \theta_1$ at $t = 0$.

Then,

$$\int_{\theta_1}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k \int_0^t dt$$

or, $\quad \ln \dfrac{\theta - \theta_0}{\theta_1 - \theta_0} = -kt$

or, $\quad \theta - \theta_0 = (\theta_1 - \theta_0)e^{-kt}$.      ... (i)

The body continues to lose heat till its temperature becomes equal to that of the surrounding. The loss of heat in this entire period is

$$\Delta Q_m = ms(\theta_1 - \theta_0).$$

This is the maximum heat the body can lose. If the body loses half this heat, the decrease in its temperature will be,

$$\frac{\Delta Q_m}{2\,ms} = \frac{\theta_1 - \theta_0}{2}.$$

If the body loses this heat in time $t_1$, the temperature at $t_1$ will be

$$\theta_1 - \frac{\theta_1 - \theta_0}{2} = \frac{\theta_1 + \theta_0}{2}.$$

Putting these values of time and temperature in (i),

$$\frac{\theta_1 + \theta_0}{2} - \theta_0 = (\theta_1 - \theta_0)e^{-kt_1}$$

or, $\quad e^{-kt_1} = \dfrac{1}{2}$

or, $\quad t_1 = \dfrac{\ln 2}{k}$.

□

## QUESTIONS FOR SHORT ANSWER

1. The heat current is written as $\dfrac{\Delta Q}{\Delta t}$. Why don't we write $\dfrac{dQ}{dt}$ ?

2. Does a body at 20°C radiate in a room, where the room temperature is 30°C ? If yes, why does its temperature not fall further ?

3. Why does blowing over a spoonful of hot tea cools it ? Does evaporation play a role ? Does radiation play a role ?

4. On a hot summer day we want to cool our room by opening the refrigerator door and closing all the windows and doors. Will the process work ?

5. On a cold winter night you are asked to sit on a chair. Would you like to choose a metal chair or a wooden chair ? Both are kept in the same lawn and are at the same temperature.

6. Two identical metal balls one at $T_1 = 300$ K and the other at $T_2 = 600$ K are kept at a distance of 1 m in vacuum. Will the temperatures equalise by radiation ? Will the rate of heat gained by the colder sphere be proportional to $T_2^4 - T_1^4$ as may be expected from the Stefan's law ?

7. An ordinary electric fan does not cool the air, still it gives comfort in summer. Explain.

8. The temperature of the atmosphere at a high altitude is around 500°C. Yet an animal there would freeze to death and not boil. Explain.

9. Standing in the sun is more pleasant on a cold winter day than standing in shade. Is the temperature of air in the sun considerably higher than that of the air in shade ?

10. Cloudy nights are warmer than the nights with clean sky. Explain.

11. Why is a white dress more comfortable than a dark dress in summer ?

## OBJECTIVE I

1. The thermal conductivity of a rod depends on
   (a) length          (b) mass
   (c) area of cross-section    (d) material of the rod.

2. In a room containing air, heat can go from one place to another
   (a) by conduction only    (b) by convection only
   (c) by radiation only     (d) by all the three modes.

3. A solid at temperature $T_1$ is kept in an evacuated chamber at temperature $T_2 > T_1$. The rate of increase of temperature of the body is proportional to
   (a) $T_2 - T_1$          (b) $T_2^2 - T_1^2$
   (c) $T_2^3 - T_1^3$       (d) $T_2^4 - T_1^4$.

4. The thermal radiation emitted by a body is proportional to $T^n$ where $T$ is its absolute temperature. The value of $n$ is exactly 4 for
   (a) a blackbody          (b) all bodies
   (c) bodies painted black only   (d) polished bodies only.

5. Two bodies $A$ and $B$ having equal surface areas are maintained at temperatures 10°C and 20°C. The thermal radiation emitted in a given time by $A$ and $B$ are in the ratio
   (a) 1 : 1·15          (b) 1 : 2
   (c) 1 : 4            (d) 1 : 16.

6. One end of a metal rod is kept in a furnace. In steady state, the temperature of the rod
   (a) increases          (b) decreases
   (c) remains constant     (d) is nonuniform.

7. Newton's law of cooling is a special case of
   (a) Wien's displacement law    (b) Kirchoff's law
   (c) Stefan's law          (d) Planck's law.

8. A hot liquid is kept in a big room. Its temperature is plotted as a function of time. Which of the following curves may represent the plot ?
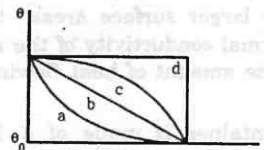


Figure 28-Q1

9. A hot liquid is kept in a big room. The logarithm of the numerical value of the temperature difference between the liquid and the room is plotted against time. The plot will be very nearly
   (a) a straight line       (b) a circular arc
   (c) a parabola          (d) an ellipse.

10. A body cools down from 65°C to 60°C in 5 minutes. It will cool down from 60°C to 55°C in
    (a) 5 minutes          (b) less than 5 minutes
    (c) more than 5 minutes
    (d) less than or more than 5 minutes depending on whether its mass is more than or less than 1 kg.

## OBJECTIVE II

1. One end of a metal rod is dipped in boiling water and the other is dipped in melting ice.
   (a) All parts of the rod are in thermal equilibrium with each other.
   (b) We can assign a temperature to the rod.
   (c) We can assign a temperature to the rod after steady

state is reached.
   (d) The state of the rod does not change after steady state is reached.

2. A blackbody does not
   (a) emit radiation       (b) absorb radiation
   (c) reflect radiation     (d) refract radiation.

3. In summer, a mild wind is often found on the shore of a calm river. This is caused due to
   (a) difference in thermal conductivity of water and soil
   (b) convection currents
   (c) conduction between air and the soil
   (d) radiation from the soil.

4. A piece of charcoal and a piece of shining steel of the same area are kept for a long time in an open lawn in bright sun.
   (a) The steel will absorb more heat than the charcoal.
   (b) The temperature of the steel will be higher than that of the charcoal.
   (c) If both are picked up by bare hands, the steel will be felt hotter than the charcoal.
   (d) If the two are picked up from the lawn and kept in a cold chamber, the charcoal will lose heat at a faster rate than the steel.

5. A heated body emits radiation which has maximum intensity near the frequency $v_0$. The emissivity of the material is $0.5$. If the absolute temperature of the body

is doubled,
   (a) the maximum intensity of radiation will be near the frequency $2v_0$
   (b) the maximum intensity of radiation will be near the frequency $v_0/2$
   (c) the total energy emitted will increase by a factor of 16
   (d) the total energy emitted will increase by a factor of 8.

6. A solid sphere and a hollow sphere of the same material and of equal radii are heated to the same temperature.
   (a) Both will emit equal amount of radiation per unit time in the biginning.
   (b) Both will absorb equal amount of radiation from the surrounding in the biginning.
   (c) The initial rate of cooling ($dT/dt$) will be the same for the two spheres.
   (d) The two spheres will have equal temperatures at any instant.

## EXERCISES

1. A uniform slab of dimension $10 \text{ cm} \times 10 \text{ cm} \times 1 \text{ cm}$ is kept between two heat reservoirs at temperatures $10°C$ and $90°C$. The larger surface areas touch the reservoirs. The thermal conductivity of the material is $0.80 \text{ W/m-°C}$. Find the amount of heat flowing through the slab per minute.

2. A liquid-nitrogen container is made of a 1 cm thick thermocoal sheet having thermal conductivity $0.025 \text{ J/m-s-°C}$. Liquid nitrogen at 80 K is kept in it. A total area of $0.80 \text{ m}^2$ is in contact with the liquid nitrogen. The atmospheric temperature is 300 K. Calculate the rate of heat flow from the atmosphere to the liquid nitrogen.

3. The normal body-temperature of a person is $97°F$. Calculate the rate at which heat is flowing out of his body through the clothes assuming the following values. Room temperature $= 47°F$, surface of the body under clothes $= 1.6 \text{ m}^2$, conductivity of the cloth $= 0.04 \text{ J/m-s-°C}$, thickness of the cloth $= 0.5 \text{ cm}$.

4. Water is boiled in a container having a bottom of surface area $25 \text{ cm}^2$, thickness $1.0 \text{ mm}$ and thermal conductivity $50 \text{ W/m-°C}$. 100 g of water is converted into steam per minute in the steady state after the boiling starts. Assuming that no heat is lost to the atmosphere, calculate the temperature of the lower surface of the bottom. Latent heat of vaporization of water $= 2.26 \times 10^6 \text{ J/kg}$.

5. One end of a steel rod ($K = 46 \text{ J/m-s-°C}$) of length $1.0 \text{ m}$ is kept in ice at $0°C$ and the other end is kept in boiling water at $100°C$. The area of cross-section of the rod is $0.04 \text{ cm}^2$. Assuming no heat loss to the atmosphere, find the mass of the ice melting per second. Latent heat of fusion of ice $= 3.36 \times 10^5 \text{ J/kg}$.

6. An icebox almost completely filled with ice at $0°C$ is dipped into a large volume of water at $20°C$. The box

has walls of surface area $2400 \text{ cm}^2$, thickness $2.0 \text{ mm}$ and thermal conductivity $0.06 \text{ W/m-°C}$. Calculate the rate at which the ice melts in the box. Latent heat of fusion of ice $= 3.4 \times 10^5 \text{ J/kg}$.

7. A pitcher with 1 mm thick porous walls contains 10 kg of water. Water comes to its outer surface and evaporates at the rate of $0.1 \text{ g/s}$. The surface area of the pitcher (one side) $= 200 \text{ cm}^2$. The room temperature $= 42°C$, latent heat of vaporization $= 2.27 \times 10^6 \text{ J/kg}$, and the thermal conductivity of the porous walls $= 0.80 \text{ J/m-s-°C}$. Calculate the temperature of water in the pitcher when it attains a constant value.

8. A steel frame ($K = 45 \text{ W/m-°C}$) of total length 60 cm and cross-sectional area $0.20 \text{ cm}^2$, forms three sides of a square. The free ends are maintained at $20°C$ and $40°C$. Find the rate of heat flow through a cross-section of the frame.

9. Water at $50°C$ is filled in a closed cylindrical vessel of height 10 cm and cross-sectional area $10 \text{ cm}^2$. The walls of the vessel are adiabatic but the flat parts are made of 1 mm thick aluminium ($K = 200 \text{ J/m-s-°C}$). Assume that the outside temperature is $20°C$. The density of water is $1000 \text{ kg/m}^3$, and the specific heat capacity of water $= 4200 \text{ J/kg-°C}$. Estimate the time taken for the temperature to fall by $1.0°C$. Make any simplifying assumptions you need but specify them.

10. The left end of a copper rod (length $= 20 \text{ cm}$, area of cross-section $= 0.20 \text{ cm}^2$) is maintained at $20°C$ and the right end is maintained at $80°C$. Neglecting any loss of heat through radiation, find (a) the temperature at a point 11 cm from the left end and (b) the heat current through the rod. Thermal conductivity of copper $= 385 \text{ W/m-°C}$.

11. The ends of a metre stick are maintained at 100°C and 0°C. One end of a rod is maintained at 25°C. Where should its other end be touched on the metre stick so that there is no heat current in the rod in steady state ?

12. A cubical box of volume 216 cm$^3$ is made up of 0·1 cm thick wood. The inside is heated electrically by a 100 W heater. It is found that the temperature difference between the inside and the outside surface is 5°C in steady state. Assuming that the entire electrical energy spent appears as heat, find the thermal conductivity of the material of the box.

13. Figure (28-E1) shows water in a container having 2·0 mm thick walls made of a material of thermal conductivity 0·50 W/m–°C. The container is kept in a melting-ice bath at 0°C. The total surface area in contact with water is 0·05 m$^2$. A wheel is clamped inside the water and is coupled to a block of mass $M$ as shown in the figure. As the block goes down, the wheel rotates. It is found that after some time a steady state is reached in which the block goes down with a constant speed of 10 cm/s and the temperature of the water remains constant at 1·0°C. Find the mass $M$ of the block. Assume that the heat flows out of the water only through the walls in contact. Take $g$ = 10 m/s$^2$.

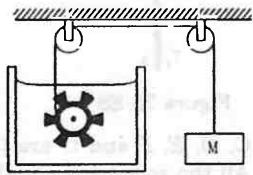

Figure 28-E1

14. On a winter day when the atmospheric temperature drops to –10°C, ice forms on the surface of a lake. (a) Calculate the rate of increase of thickness of the ice when 10 cm of ice is already formed. (b) Calculate the total time taken in forming 10 cm of ice. Assume that the temperature of the entire water reaches 0°C before the ice starts forming. Density of water = 1000 kg/m$^3$, latent heat of fusion of ice = 3·36 × 10$^5$ J/kg and thermal conductivity of ice = 1·7 W/m–°C. Neglect the expansion of water on freezing.

15. Consider the situation of the previous problem. Assume that the temperature of the water at the bottom of the lake remains constant at 4°C as the ice forms on the surface (the heat required to maintain the temperature of the bottom layer may come from the bed of the lake). The depth of the lake is 1·0 m. Show that the thickness of the ice formed attains a steady state maximum value. Find this value. The thermal conductivity of water = 0·50 W/m–°C. Take other relevant data from the previous problem.

16. Three rods of lengths 20 cm each and area of cross-section 1 cm$^2$ are joined to form a triangle $ABC$. The conductivities of the rods are $K_{AB}$ = 50 J/m–s–°C, $K_{BC}$ = 200 J/m–s–°C and $K_{AC}$ = 400 J/m–s–°C. The junctions $A$, $B$ and $C$ are maintained at 40°C, 80°C and

80°C respectively. Find the rate of heat flowing through the rods $AB$, $AC$ and $BC$.

17. A semicircular rod is joined at its end to a straight rod of the same material and the same cross-sectional area. The straight rod forms a diameter of the other rod. The junctions are maintained at different temperatures. Find the ratio of the heat transferred through a cross-section of the semicircular rod to the heat transferred through a cross-section of the straight rod in a given time.

18. A metal rod of cross-sectional area 1·0 cm$^2$ is being heated at one end. At one time, the temperature gradient is 5·0°C/cm at cross-section $A$ and is 2·5°C/cm at cross-section $B$. Calculate the rate at which the temperature is increasing in the part $AB$ of the rod. The heat capacity of the part $AB$ = 0·40 J/°C, thermal conductivity of the material of the rod = 200 W/m–°C. Neglect any loss of heat to the atmosphere.

19. Steam at 120°C is continuously passed through a 50 cm long rubber tube of inner and outer radii 1·0 cm and 1·2 cm. The room temperature is 30°C. Calculate the rate of heat flow through the walls of the tube. Thermal conductivity of rubber = 0·15 J/m–s–°C.

20. A hole of radius $r_1$ is made centrally in a uniform circular disc of thickness $d$ and radius $r_2$. The inner surface ( a cylinder of length $d$ and radius $r_1$) is maintained at a temperature $\theta_1$ and the outer surface (a cylinder of length $d$ and radius $r_2$) is maintained at a temperature $\theta_2(\theta_1 > \theta_2)$. The thermal conductivity of the material of the disc is $K$. Calculate the heat flowing per unit time through the disc.

21. A hollow tube has a length $l$, inner radius $R_1$ and outer radius $R_2$. The material has a thermal conductivity $K$. Find the heat flowing through the walls of the tube if (a) the flat ends are maintained at temperatures $T_1$ and $T_2(T_2 > T_1)$ (b) the inside of the tube is maintained at temperature $T_1$ and the outside is maintained at $T_2$.

22. A composite slab is prepared by pasting two plates of thicknesses $L_1$ and $L_2$ and thermal conductivities $K_1$ and $K_2$. The slabs have equal cross-sectional area. Find the equivalent conductivity of the slab.

23. Figure (28-E2) shows a copper rod joined to a steel rod. The rods have equal length and equal cross-sectional area. The free end of the copper rod is kept at 0°C and that of the steel rod is kept at 100°C. Find the temperature at the junction of the rods. Conductivity of copper = 390 W/m–°C and that of steel = 46 W/m–°C.
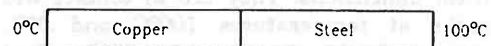


Figure 28-E2

24. An aluminium rod and a copper rod of equal length 1·0 m and cross-sectional area 1 cm$^2$ are welded together as shown in figure (28-E3). One end is kept at a temperature of 20°C and the other at 60°C. Calculate the amount of heat taken out per second from the hot end. Thermal conductivity of aluminium = 200 W/m–°C and of copper = 390 W/m–°C.

Figure 28-E3

25. Figure (28-E4) shows an aluminium rod joined to a copper rod. Each of the rods has a length of 20 cm and area of cross-section 0·20 cm $^2$. The junction is maintained at a constant temperature 40°C and the two ends are maintained at 80°C. Calculate the amount of heat taken out from the cold junction in one minute after the steady state is reached. The conductivities are $K_{Al}$ = 200 W/m-°C and $K_{Cu}$ = 400 W/m-°C.
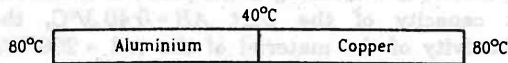


Figure 28-E4

26. Consider the situation shown in figure (28-E5). The frame is made of the same material and has a uniform cross-sectional area everywhere. Calculate the amount of heat flowing per second through a cross-section of the bent part if the total heat taken out per second from the end at 100°C is 130 J.
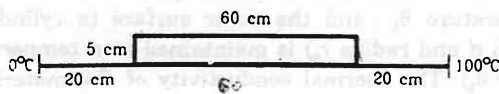


Figure 28-E5

27. Suppose the bent part of the frame of the previous problem has a thermal conductivity of 780 J/m-s-°C whereas it is 390 J/m-s-°C for the straight part. Calculate the ratio of the rate of heat flow through the bent part to the rate of heat flow through the straight part.

28. A room has a window fitted with a single 1·0 m × 2·0 m glass of thickness 2 mm. (a) Calculate the rate of heat flow through the closed window when the temperature inside the room is 32°C and that outside is 40°C. (b) The glass is now replaced by two glasspanes, each having a thickness of 1 mm and separated by a distance of 1 mm. Calculate the rate of heat flow under the same conditions of temperature. Thermal conductivity of window glass = 1·0 J/m-s-°C and that of air = 0·025 J/m-s-°C.

29. The two rods shown in figure (28-E6) have identical geometrical dimensions. They are in contact with two heat baths at temperatures 100°C and 0°C. The temperature of the junction is 70°C. Find the temperature of the junction if the rods are interchanged.
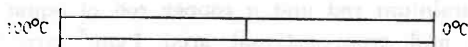


Figure 28-E6

30. The three rods shown in figure (28-E7) have identical geometrical dimensions. Heat flows from the hot end at a rate of 40 W in the arrangement (a). Find the rates of heat flow when the rods are joined as in arrangement (b) and in (c). Thermal conductivities of aluminium and copper are 200 W/m-°C and 400 W/m-°C respectively.
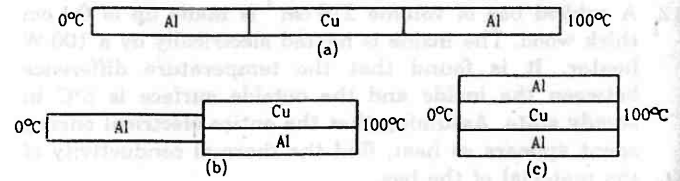


Figure 28-E7

31. Four identical rods AB, CD, CF and DE are joined as shown in figure (28-E8). The length, cross-sectional area and thermal conductivity of each rod are l, A and K respectively. The ends A, E and F are maintained at temperatures $T_1$, $T_2$ and $T_3$ respectively. Assuming no loss of heat to the atmosphere, find the temperature at B.
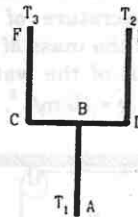


Figure 28-E8

32. Seven rods A, B, C, D, E, F and G are joined as shown in figure (28-E9). All the rods have equal cross-sectional area A and length l. The thermal conductivities of the rods are $K_A = K_C = K_0$, $K_B = K_D = 2K_0$, $K_E = 3K_0$, $K_F = 4K_0$ and $K_G = 5K_0$. The rod E is kept at a constant temperature $T_1$ and the rod G is kept at a constant temperature $T_2(T_2 > T_1)$. (a) Show that the rod F has a uniform temperature $T = (T_1 + 2T_2)/3$. (b) Find the rate of heat flow from the source which maintains the temperature $T_2$.
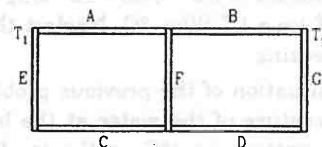


Figure 28-E9

33. Find the rate of heat flow through a cross-section of the rod shown in figure (28-E10) ($\theta_2 > \theta_1$). Thermal conductivity of the material of the rod is K.
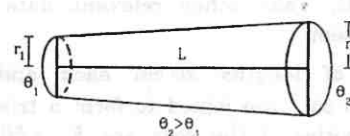


Figure 28-E10

34. A rod of negligible heat capacity has length 20 cm, area of cross-section $1.0$ cm$^2$ and thermal conductivity 200 W/m–°C. The temperature of one end is maintained at 0°C and that of the other end is slowly and linearly varied from 0°C to 60°C in 10 minutes. Assuming no loss of heat through the sides, find the total heat transmitted through the rod in these 10 minutes.

35. A hollow metallic sphere of radius 20 cm surrounds a concentric metallic sphere of radius 5 cm. The space between the two spheres is filled with a nonmetallic material. The inner and outer spheres are maintained at 50°C and 10°C respectively and it is found that 100 J of heat passes from the inner sphere to the outer sphere per second. Find the thermal conductivity of the material between the spheres.

36. Figure (28-E11) shows two adiabatic vessels, each containing a mass $m$ of water at different temperatures. The ends of a metal rod of length $L$, area of cross-section $A$ and thermal conductivity $K$, are inserted in the water as shown in the figure. Find the time taken for the difference between the temperatures in the vessels to become half of the original value. The specific heat capacity of water is $s$. Neglect the heat capacity of the rod and the container and any loss of heat to the atmosphere.



Figure 28-E11

37. Two bodies of masses $m_1$ and $m_2$ and specific heat capacities $s_1$ and $s_2$ are connected by a rod of length $l$, cross-sectional area $A$, thermal conductivity $K$ and negligible heat capacity. The whole system is thermally insulated. At time $t = 0$, the temperature of the first body is $T_1$ and the temperature of the second body is $T_2$ $(T_2 > T_1)$. Find the temperature difference between the two bodies at time $t$.

38. An amount $n$ (in moles) of a monatomic gas at an initial temperature $T_0$ is enclosed in a cylindrical vessel fitted with a light piston. The surrounding air has a temperature $T_s (> T_0)$ and the atmospheric pressure is $p_a$. Heat may be conducted between the surrounding and the gas through the bottom of the cylinder. The bottom has a surface area $A$, thickness $x$ and thermal conductivity $K$. Assuming all changes to be slow, find the distance moved by the piston in time $t$.

39. Assume that the total surface area of a human body is $1.6$ m$^2$ and that it radiates like an ideal radiator. Calculate the amount of energy radiated per second by the body if the body temperature is 37°C. Stefan constant $\sigma$ is $6.0 \times 10^{-8}$ W/m$^2$-K$^4$.

40. Calculate the amount of heat radiated per second by a body of surface area 12 cm$^2$ kept in thermal equilibrium in a room at temperature 20°C. The emissivity of the surface = $0.80$ and $\sigma = 6.0 \times 10^{-8}$ W/m$^2$-K$^4$.

41. A solid aluminium sphere and a solid copper sphere of twice the radius are heated to the same temperature and are allowed to cool under identical surrounding temperatures. Assume that the emissivity of both the spheres is the same. Find the ratio of (a) the rate of heat loss from the aluminium sphere to the rate of heat loss from the copper sphere and (b) the rate of fall of temperature of the aluminium sphere to the rate of fall of temperature of the copper sphere. The specific heat capacity of aluminium = 900 J/kg-°C and that of copper = 390 J/kg-°C. The density of copper = $3.4$ times the density of aluminium.

42. A 100 W bulb has tungsten filament of total length $1.0$ m and radius $4 \times 10^{-5}$ m. The emissivity of the filament is $0.8$ and $\sigma = 6.0 \times 10^{-8}$ W/m$^2$-K$^4$. Calculate the temperature of the filament when the bulb is operating at correct wattage.

43. A spherical ball of surface area 20 cm$^2$ absorbs any radiation that falls on it. It is suspended in a closed box maintained at 57°C. (a) Find the amount of radiation falling on the ball per second. (b) Find the net rate of heat flow to or from the ball at an instant when its temperature is 200°C. Stefan constant = $6.0 \times 10^{-8}$ W/m$^2$-K$^4$.

44. A spherical tungsten piece of radius $1.0$ cm is suspended in an evacuated chamber maintained at 300 K. The piece is maintained at 1000 K by heating it electrically. Find the rate at which the electrical energy must be supplied. The emissivity of tungsten is $0.30$ and the Stefan constant $\sigma$ is $6.0 \times 10^{-8}$ W/m$^2$-K$^4$.

45. A cubical block of mass $1.0$ kg and edge $5.0$ cm is heated to 227°C. It is kept in an evacuated chamber maintained at 27°C. Assuming that the block emits radiation like a blackbody, find the rate at which the temperature of the block will decrease. Specific heat capacity of the material of the block is 400 J/kg-K.

46. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at a constant temperature of 500 K by heating it electrically. A total of 210 W of electric power is needed to do it. When the surface of the copper sphere is completely blackened, 700 W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

47. A spherical ball $A$ of surface area 20 cm$^2$ is kept at the centre of a hollow spherical shell $B$ of area 80 cm$^2$. The surface of $A$ and the inner surface of $B$ emit as blackbodies. Assume that the thermal conductivity of the material of $B$ is very poor and that of $A$ is very high and that the air between $A$ and $B$ has been pumped out. The heat capacities of $A$ and $B$ are 42 J/°C and 82 J/°C respectively. Initially, the temperature of $A$ is 100°C and that of $B$ is 20°C. Find the rate of change of temperature of $A$ and that of $B$ at this instant. Explain the effects of the assumptions listed in the problem.

48. A cylindrical rod of length 50 cm and cross-sectional area 1 cm$^2$ is fitted between a large ice chamber at 0°C and an evacuated chamber maintained at 27°C as shown in figure (28-E12). Only small portions of the rod are inside the chambers and the rest is thermally insulated from the surrounding. The cross-section going into the

evacuated chamber is blackened so that it completely absorbs any radiation falling on it. The temperature of the blackened end is 17°C when steady state is reached. Stefan constant $\sigma = 6 \times 10^{-8}$ W/m$^2$-K$^4$. Find the thermal conductivity of the material of the rod.
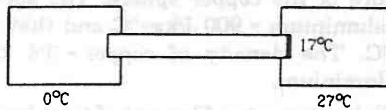


Figure 28-E12

49. One end of a rod of length 20 cm is inserted in a furnace at 800 K. The sides of the rod are covered with an insulating material and the other end emits radiation like a blackbody. The temperature of this end is 750 K in the steady state. The temperature of the surrounding air is 300 K. Assuming radiation to be the only important mode of energy transfer between the surrounding and the open end of the rod, find the thermal conductivity of the rod. Stefan constant $\sigma = 6.0 \times 10^{-8}$ W/m$^2$-K$^4$.

50. A calorimeter of negligible heat capacity contains 100 cc of water at 40°C. The water cools to 35°C in 5 minutes. The water is now replaced by K-oil of equal volume at 40°C. Find the time taken for the temperature to become 35°C under similar conditions. Specific heat capacities of water and K-oil are 4200 J/kg-K and 2100 J/kg-K respectively. Density of K-oil = 800 kg/m$^3$.

51. A body cools down from 50°C to 45°C in 5 minutes and to 40°C in another 8 minutes. Find the temperature of the surrounding.

52. A calorimeter contains 50 g of water at 50°C. The temperature falls to 45°C in 10 minutes. When the

calorimeter contains 100 g of water at 50°C, it takes 18 minutes for the temperature to become 45°C. Find the water equivalent of the calorimeter.

53. A metal ball of mass 1 kg is heated by means of a 20 W heater in a room at 20°C. The temperature of the ball becomes steady at 50°C. (a) Find the rate of loss of heat to the surrounding when the ball is at 50°C. (b) Assuming Newton's law of cooling, calculate the rate of loss of heat to the surrounding when the ball is at 30°C. (c) Assume that the temperature of the ball rises uniformly from 20°C to 30°C in 5 minutes. Find the total loss of heat to the surrounding during this period. (d) Calculate the specific heat capacity of the metal.

54. A metal block of heat capacity 80 J/°C placed in a room at 20°C is heated electrically. The heater is switched off when the temperature reaches 30°C. The temperature of the block rises at the rate of 2 °C/s just after the heater is switched on and falls at the rate of 0·2 °C/s just after the heater is switched off. Assume Newton's law of cooling to hold. (a) Find the power of the heater. (b) Find the power radiated by the block just after the heater is switched off. (c) Find the power radiated by the block when the temperature of the block is 25°C. (d) Assuming that the power radiated at 25°C respresents the average value in the heating process, find the time for which the heater was kept on.

55. A hot body placed in a surrounding of temperature $\theta_0$ obeys Newton's law of cooling $\dfrac{d\theta}{dt} = -k(\theta - \theta_0)$. Its temperature at $t = 0$ is $\theta_1$. The specific heat capacity of the body is $s$ and its mass is $m$. Find (a) the maximum heat that the body can lose and (b) the time starting from $t = 0$ in which it will lose 90% of this maximum heat.

□

ANSWERS

OBJECTIVE I

| 1. (d) | 2. (d) | 3. (d) | 4. (b) | 5. (a) | 6. (d) |
|--------|--------|--------|--------|--------|--------|
| 7. (c) | 8. (a) | 9. (a) | 10. (c) | | |

OBJECTIVE II

| 1. (d) | 2. (c), (d) | 3. (b) |
|--------|-------------|--------|
| 4. (c), (d) | 5. (a), (c) | 6. (a), (b) |

EXERCISES

1. 3840 J
2. 440 W
3. 356 J/s
4. 130°C

5. $5.5 \times 10^{-5}$ g
6. 1·5 kg/h
7. 28°C
8. 0·03 W
9. 0·035 s
10. (a) 53°C          (b) 2·31 J/s
11. 25 cm from the cold end
12. 0·92 W/m-°C
13. 12·5 kg
14. (a) $5.0 \times 10^{-7}$ m/s          (b) 27·5 hours
15. 89 cm
16. 1 W, 8 W, zero
17. 2 : $\pi$

18. 12·5°C/s

19. 233 J/s

20. $\dfrac{2\pi Kd(\theta_1 - \theta_2)}{\ln(r_2/r_1)}$

21. (a) $\dfrac{K\pi(R_2^2 - R_1^2)(T_2 - T_1)}{l}$    (b) $\dfrac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$

22. $\dfrac{K_1 K_2(L_1 + L_2)}{L_1 K_2 + L_2 K_1}$

23. 10·6°C

24. 2·36 J

25. 144 J

26. 60 J

27. 12 : 7

28. (a) 8000 J/s    (b) 381 J/s

29. 30°C

30. 75 W, 400 W

31. $\dfrac{3\,T_1 + 2(T_2 + T_3)}{7}$

32. (b) $\dfrac{4\,K_0 A(T_2 - T_1)}{3l}$

33. $\dfrac{K\pi r_1 r_2(\theta_2 - \theta_1)}{L}$

34. 1800 J

35. 3·0 W/m–°C

36. $\dfrac{Lms}{2KA}\ln 2$

37. $(T_2 - T_1)\,e^{-\lambda t}$ where $\lambda = \dfrac{KA(m_1 s_1 + m_2 s_2)}{lm_1 m_2 s_1 s_2}$

38. $\dfrac{nR}{P_a A}(T_s - T_0)(1 - e^{-\frac{P_a A K t}{nR\,lm}})$

39. 887 J

40. 0·42 J

41. (a) 1 : 4    (b) 2·9 : 1

42. 1700 K

43. (a) 1·4 J (b) 4·58 W from the ball

44. 22 W

45. 0·12°C/s

46. 0·3

47. 0·05°C/s and 0·01°C/s

48. 1·8 W/m–°C

49. 74 W/m–K

50. 2 min

51. 34°C

52. 12·5 g

53. (a) 20 W    (b) $\dfrac{20}{3}$ W    (c) 1000 J    (d) 500 J/kg–K

54. (a) 160 W    (b) 16 W    (c) 8 W    (d) 5·2 s

55. (a) $ms(\theta_1 - \theta_0)$ (b) $\dfrac{\ln 10}{k}$

□

# ELECTRIC FIELD AND POTENTIAL

## 29.1 WHAT IS ELECTRIC CHARGE ?

Matter is made of certain elementary particles. With the advancement in technology, we have discovered hundreds of elementary particles. Many of them are rare and of no concern to us in the present course. The three most common elementary particles are electrons, protons and neutrons having masses $m_e = 9 \cdot 10940 \times 10^{-31}$ kg, $m_p = 1 \cdot 67262 \times 10^{-27}$ kg and $m_n = 1 \cdot 67493 \times 10^{-27}$ kg. Because of their mass these particles attract each other by gravitational forces. Thus, an electron attracts another electron, placed 1 cm away, with a gravitational force

$$F = \frac{Gm_1 m_2}{r^2}$$

$$= \frac{(6 \cdot 67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2) \times (9 \cdot 1 \times 10^{-31} \text{ kg})^2}{(10^{-2} \text{ m})^2}$$

$$= 5 \cdot 5 \times 10^{-67} \text{ N}.$$

However, an electron is found to repel another electron at 1 cm with a force of $2 \cdot 3 \times 10^{-24}$ N. This extra force is called the *electric force*. The electric force is very large as compared to the gravitational force. The electrons must have some additional property, apart from their mass, which is responsible for the electric force. We call this property *charge*. Just as masses are responsible for the gravitational force, charges are responsible for the electric force. Two protons placed at a distance of 1 cm also repel each other with a force of $2 \cdot 3 \times 10^{-24}$ N. Thus, protons also have charge. Two neutrons placed at a distance of 1 cm attract each other with a force of $1 \cdot 9 \times 10^{-60}$ N which is equal to $\frac{Gm_1 m_2}{r^2}$. Thus, neutrons exert only gravitational force on each other and experience no electric force. The neutrons have mass but no charge.

### Two Kinds of Charges

As mentioned above, the electric force between two electrons is the same as the electric force between two protons placed at the same separation. We may guess that the amount of charge on an electron is the same as that on a proton. However, if a proton and an electron are placed 1 cm apart, they attract each other with a force of $2 \cdot 3 \times 10^{-24}$ N. Certainly this force is electric, but it is attractive and not repulsive. The charge on an electron repels the charge on another electron but attracts the charge on a proton. Thus, although the charge on an electron and that on a proton have the same strength, they are of two different nature. Also, if we pack a proton and an electron together in a small volume, the combination does not attract or repel another electron or proton placed at a distance. The net charge on the proton–electron system seems to be zero. It is, therefore, convenient to define one charge as positive and the other as negative. We arbitrarily call the charge on a proton as positive and that on an electron as negative. This assignment of positive and negative signs to the proton charge and the electron charge is purely a convention. It does not mean that the charge on an electron is "less" than the charge on a proton.

### Unit of Charge

The above discussion suggests that charge is a basic property associated with the elementary particles and its definition is as difficult as the definition of mass or time or length. We can measure the charge on a system by comparing it with the charge on a standard body but we do not know what exactly it is that we intend to measure. The SI unit of charge is coulomb abbreviated as C. 1 coulomb is the charge flowing through a wire in 1 $s$ if the electric current in it is 1 A. The charge on a proton is

$$e = 1 \cdot 60218 \times 10^{-19} \text{ C}.$$

The charge on an electron is the negative of this value.

### Charge is Quantized

If protons and electrons are the only charge carriers in the universe, all observable charges must

$$| \tau | = | \vec{p} \times \vec{E} |$$

$$= qlE \sin\theta.$$

This torque will tend to rotate the dipole back towards the electric field. Also, for small angular displacement $\sin\theta \approx \theta$ so that

$$\tau = -qlE\theta.$$

The moment of inertia of the system about the axis of rotation is

$$I = 2 \times m \left(\frac{l}{2}\right)^2 = \frac{ml^2}{2}.$$

Thus, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\frac{2qE}{ml}\theta = -\omega^2\theta$$

where $\omega^2 = \frac{2qE}{ml}$.

Thus, the motion is angular simple harmonic and the time period is $T = 2\pi \sqrt{\dfrac{ml}{2qE}}$.

□

# QUESTIONS FOR SHORT ANSWER

1. The charge on a proton is $+1.6 \times 10^{-19}$ C and that on an electron is $-1.6 \times 10^{-19}$ C. Does it mean that the electron has a charge $3.2 \times 10^{-19}$ C less than the charge of a proton ?

2. Is there any lower limit to the electric force between two particles placed at a separation of 1 cm ?

3. Consider two particles $A$ and $B$ having equal charges and placed at some distance. The particle $A$ is slightly displaced towards $B$. Does the force on $B$ increase as soon as the particle $A$ is displaced ? Does the force on the particle $A$ increase as soon as it is displaced ?

4. Can a gravitational field be added vectorially to an electric field to get a total field ?

5. Why does a phonograph-record attract dust particles just after it is cleaned ?

6. Does the force on a charge due to another charge depend on the charges present nearby ?

7. In some old texts it is mentioned that $4\pi$ lines of force originate from each unit positive charge. Comment on the statement in view of the fact that $4\pi$ is not an integer.

8. Can two equipotential surfaces cut each other ?

9. If a charge is placed at rest in an electric field, will its path be along a line of force ? Discuss the situation when the lines of force are straight and when they are curved.

10. Consider the situation shown in figure (29-Q1). What are the signs of $q_1$ and $q_2$ ? If the lines are drawn in proportion to the charge, what is the ratio $q_1/q_2$ ?
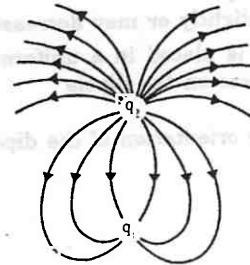


Figure 29-Q1

11. A point charge is taken from a point $A$ to a point $B$ in an electric field. Does the work done by the electric field depend on the path of the charge ?

12. It is said that the separation between the two charges forming an electric dipole should be small. Small compared to what ?

13. The number of electrons in an insulator is of the same order as the number of electrons in a conductor. What is then the basic difference between a conductor and an insulator ?

14. When a charged comb is brought near a small piece of paper, it attracts the piece. Does the paper become charged when the comb is brought near it ?

# OBJECTIVE I

1. Figure (29-Q2) shows some of the electric field lines corresponding to an electric field. The figure suggests that
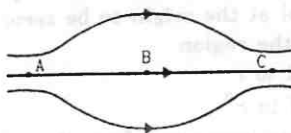


Figure 29-Q2

(a) $E_A > E_B > E_C$     (b) $E_A = E_B = E_C$
(c) $E_A = E_C > E_B$     (d) $E_A = E_C < E_B$.

2. When the separation between two charges is increased, the electric potential energy of the charges
(a) increases     (b) decreases
(c) remains the same     (d) may increase or decrease.

3. If a positive charge is shifted from a low-potential region to a high-potential region, the electric potential energy

(a) increases         (b) decreases

(c) remains the same     (d) may increase or decrease.

4. Two equal positive charges are kept at points $A$ and $B$. The electric potential at the points between $A$ and $B$ (excluding these points) is studied while moving from $A$ to $B$. The potential
   (a) continuously increases
   (b) continuously decreases
   (c) increases then decreases
   (d) decreases then increases.

5. The electric field at the origin is along the positive $X$-axis. A small circle is drawn with the centre at the origin cutting the axes at points $A$, $B$, $C$ and $D$ having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$, $(0, -a)$ respectively. Out of the points on the periphery of the circle, the potential is minimum at
   (a) $A$        (b) $B$        (c) $C$        (d) $D$.

6. If a body is charged by rubbing it, its weight
   (a) remains precisely constant
   (b) increases slightly
   (c) decreases slightly
   (d) may increase slightly or may decrease slightly.

7. An electric dipole is placed in a uniform electric field. The net electric force on the dipole
   (a) is always zero
   (b) depends on the orientation of the dipole

(c) can never be zero
(d) depends on the strength of the dipole.

8. Consider the situation of figure (29-Q3). The work done in taking a point charge from $P$ to $A$ is $W_A$, from $P$ to $B$ is $W_B$ and from $P$ to $C$ is $W_C$.
   (a) $W_A < W_B < W_C$.        (b) $W_A > W_B > W_C$.
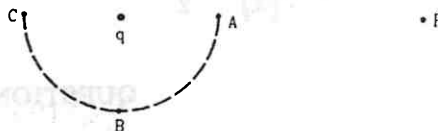   (c) $W_A = W_B = W_C$.        (d) None of these.



Figure 29-Q3

9. A point charge $q$ is rotated along a circle in the electric field generated by another point charge $Q$. The work done by the electric field on the rotating charge in one complete revolution is
   (a) zero        (b) positive        (c) negative
   (d) zero if the charge $Q$ is at the centre and nonzero otherwise.

## OBJECTIVE II

1. Mark out the correct options.
   (a) The total charge of the universe is constant.
   (b) The total positive charge of the universe is constant.
   (c) The total negative charge of the universe is constant.
   (d) The total number of charged particles in the universe is constant.

2. A point charge is brought in an electric field. The electric field at a nearby point
   (a) will increase if the charge is positive
   (b) will decrease if the charge is negative
   (c) may increase if the charge is positive
   (d) may decrease if the charge is negative.

3. The electric field and the electric potential at a point are $E$ and $V$ respectively.
   (a) If $E = 0$, $V$ must be zero.
   (b) If $V = 0$, $E$ must be zero.
   (c) If $E \neq 0$, $V$ cannot be zero.
   (d) If $V \neq 0$, $E$ cannot be zero.

4. The electric potential decreases uniformly from 120 V to 80 V as one moves on the $X$-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin
   (a) must be equal to 20 V/cm
   (b) may be equal to 20 V/cm
   (c) may be greater than 20 V/cm
   (d) may be less than 20 V/cm.

5. Which of the following quantities do not depend on the choice of zero potential or zero potential energy ?

(a) potential at a point
(b) potential difference between two points
(c) potential energy of a two-charge system
(d) change in potential energy of a two-charge system.

6. An electric dipole is placed in an electric field generated by a point charge.
   (a) The net electric force on the dipole must be zero.
   (b) The net electric force on the dipole may be zero.
   (c) The torque on the dipole due to the field must be zero.
   (d) The torque on the dipole due to the field may be zero.

7. A proton and an electron are placed in a uniform electric field.
   (a) The electric forces acting on them will be equal.
   (b) The magnitudes of the forces will be equal.
   (c) Their accelerations will be equal.
   (d) The magnitudes of their accelerations will be equal.

8. The electric field in a region is directed outward and is proportional to the distance $r$ from the origin. Taking the electric potential at the origin to be zero,
   (a) it is uniform in the region
   (b) it is proportional to $r$
   (c) it is proportional to $r^2$
   (d) it increases as one goes away from the origin.

## EXERCISES

1. Find the dimensional formula of $\varepsilon_0$.

2. A charge of 1·0 C is placed at the top of the your college building and another equal charge at the top of your house. Take the separation between the two charges to be 2·0 km. Find the force exerted by the charges on each other. How many times of your weight is this force ?

3. At what separation should two equal charges, 1·0 C each, be placed so that the force between them equals the weight of a 50 kg person ?

4. Two equal charges are placed at a separation of 1·0 m. What should be the magnitude of the charges so that the force between them equals the weight of a 50 kg person ?

5. Find the electric force between two protons separated by a distance of 1 fermi (1 fermi = $10^{-15}$ m). The protons in a nucleus remain at a separation of this order.

6. Two charges $2.0 \times 10^{-6}$ C and $1.0 \times 10^{-6}$ C are placed at a separation of 10 cm. Where should a third charge be placed such that it experiences no net force due to these charges ?

7. Suppose the second charge in the previous problem is $-1.0 \times 10^{-6}$ C. Locate the position where a third charge will not experience a net force.

8. Two charged particles are placed at a distance 1·0 cm apart. What is the minimum possible magnitude of the electric force acting on each charge ?

9. Estimate the number of electrons in 100 g of water. How much is the total negative charge on these electrons ?

10. Suppose all the electrons of 100 g water are lumped together to form a negatively charged particle and all the nuclei are lumped together to form a positively charged particle. If these two particles are placed 10·0 cm away from each other, find the force of attraction between them. Compare it with your weight.

11. Consider a gold nucleus to be a sphere of radius 6·9 fermi in which protons and neutrons are distributed. Find the force of repulsion between two protons situated at largest separation. Why do these protons not fly apart under this repulsion ?

12. Two insulating small spheres are rubbed against each other and placed 1 cm apart. If they attract each other with a force of 0·1 N, how many electrons were transferred from one sphere to the other during rubbing ?

13. NaCl molecule is bound due to the electric force between the sodium and the chlorine ions when one electron of sodium is transferred to chlorine. Taking the separation between the ions to be $2.75 \times 10^{-8}$ cm, find the force of attraction between them. State the assumptions (if any) that you have made.

14. Find the ratio of the electric and gravitational forces between two protons.

15. Suppose an attractive nuclear force acts between two protons which may be written as $F = Ce^{-kr}/r^2$. (a) Write down the dimensional formulae and appropriate SI units of $C$ and $k$. (b) Suppose that $k = 1$ fermi$^{-1}$ and that the repulsive electric force between the protons is just balanced by the attractive nuclear force when the separation is 5 fermi. Find the value of $C$.

16. Three equal charges, $2.0 \times 10^{-6}$ C each, are held fixed at the three corners of an equilateral triangle of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest two.

17. Four equal charges $2.0 \times 10^{-6}$ C each are fixed at the four corners of a square of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest three.

18. A hydrogen atom contains one proton and one electron. It may be assumed that the electron revolves in a circle of radius 0·53 angstrom (1 angstrom = $10^{-10}$ m and is abbreviated as Å) with the proton at the centre. The hydrogen atom is said to be in the ground state in this case. Find the magnitude of the electric force between the proton and the electron of a hydrogen atom in its ground state.

19. Find the speed of the electron in the ground state of a hydrogen atom. The description of ground state is given in the previous problem.

20. Ten positively charged particles are kept fixed on the X-axis at points $x = 10$ cm, 20 cm, 30 cm, ..., 100 cm. The first particle has a charge $1.0 \times 10^{-8}$ C, the second $8 \times 10^{-8}$ C, the third $27 \times 10^{-8}$ C and so on. The tenth particle has a charge $1000 \times 10^{-8}$ C. Find the magnitude of the electric force acting on a 1 C charge placed at the origin.

21. Two charged particles having charge $2.0 \times 10^{-8}$ C each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.

22. Two identical balls, each having a charge of $2.00 \times 10^{-7}$ C and a mass of 100 g, are suspended from a common point by two insulating strings each 50 cm long. The balls are held at a separation 5·0 cm apart and then released. Find (a) the electric force on one of the charged balls (b) the components of the resultant force on it along and perpendicular to the string (c) the tension in the string (d) the acceleration of one of the balls. Answers are to be obtained only for the instant just after the release.

23. Two identical pith balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each, the separation between the suspension points being 5 cm. In equilibrium, the separation between the balls is 3 cm. Find the mass of each ball and the tension in the strings. The charge on each ball has a magnitude $2.0 \times 10^{-8}$ C.

24. Two small spheres, each having a mass of 20 g, are suspended from a common point by two insulating strings of length 40 cm each. The spheres are identically charged and the separation between the balls at

equilibrium is found to be 4 cm. Find the charge on each sphere.

25. Two identical pith balls, each carrying a charge $q$, are suspended from a common point by two strings of equal length $l$. Find the mass of each ball if the angle between the strings is $2\theta$ in equilibrium.

26. A particle having a charge of $2.0 \times 10^{-4}$ C is placed directly below and at a separation of 10 cm from the bob of a simple pendulum at rest. The mass of the bob is 100 g. What charge should the bob be given so that the string becomes loose.

27. Two particles $A$ and $B$ having charges $q$ and $2q$ respectively are placed on a smooth table with a separation $d$. A third particle $C$ is to be clamped on the table in such a way that the particles $A$ and $B$ remain at rest on the table under electrical forces. What should be the charge on $C$ and where should it be clamped ?

28. Two identically charged particles are fastened to the two ends of a spring of spring constant 100 N/m and natural length 10 cm. The system rests on a smooth horizontal table. If the charge on each particle is $2.0 \times 10^{-8}$ C, find the extension in the length of the spring. Assume that the extension is small as compared to the natural length. Justify this assmuption after you solve the problem.

29. A particle $A$ having a charge of $2.0 \times 10^{-6}$ C is held fixed on a horizontal table. A second charged particle of mass 80 g stays in equilibrium on the table at a distance of 10 cm from the first charge. The coefficient of friction between the table and this second particle is $\mu = 0.2$. Find the range within which the charge of this second particle may lie.

30. A particle $A$ having a charge of $2.0 \times 10^{-6}$ C and a mass of 100 g is placed at the bottom of a smooth inclined plane of inclination 30°. Where should another particle $B$, having same charge and mass, be placed on the incline so that it may remain in equilibrium ?

31. Two particles $A$ and $B$, each having a charge $Q$, are placed a distance $d$ apart. Where should a particle of charge $q$ be placed on the perpendicular bisector of $AB$ so that it experiences maximum force ? What is the magnitude of this maximum force ?

32. Two particles $A$ and $B$, each carrying a charge $Q$, are held fixed with a separation $d$ between them. A particle $C$ having mass $m$ and charge $q$ is kept at the middle point of the line $AB$. (a) If it is displaced through a distance $x$ perpendicular to $AB$, what would be the electric force experienced by it. (b) Assuming $x \ll d$, show that this force is proportional to $x$. (c) Under what conditions will the particle $C$ execute simple harmonic motion if it is released after such a small displacement ? Find the time period of the oscillations if these conditions are satisfied.

33. Repeat the previous problem if the particle $C$ is displaced through a distance $x$ along the line $AB$.

34. The electric force experienced by a charge of $1.0 \times 10^{-6}$ C is $1.5 \times 10^{-3}$ N. Find the magnitude of the electric field at the position of the charge.

35. Two particles $A$ and $B$ having charges of $+ 2.00 \times 10^{-6}$ C and of $- 4.00 \times 10^{-6}$ C respectively are held fixed at a separation of 20.0 cm. Locate the point(s) on the line $AB$ where (a) the electric field is zero (b) the electric potential is zero.

36. A point charge produces an electric field of magnitude 5.0 N/C at a distance of 40 cm from it. What is the magnitude of the charge ?

37. A water particle of mass 10.0 mg and having a charge of $1.50 \times 10^{-6}$ C stays suspended in a room. What is the magnitude of electric field in the room ? What is its direction ?

38. Three identical charges, each having a value $1.0 \times 10^{-8}$ C, are placed at the corners of an equilateral triangle of side 20 cm. Find the electric field and potential at the centre of the triangle.

39. Positive charge $Q$ is distributed uniformly over a circular ring of radius $R$. A particle having a mass $m$ and a negative charge $q$, is placed on its axis at a distance $x$ from the centre. Find the force on the particle. Assuming $x \ll R$, find the time period of oscillation of the particle if it is released from there.

40. A rod of length $L$ has a total charge $Q$ distributed uniformly along its length. It is bent in the shape of a semicircle. Find the magnitude of the electric field at the centre of curvature of the semicircle.

41. A 10 cm long rod carries a charge of $+ 50$ μC distributed uniformly along its length. Find the magnitude of the electric field at a point 10 cm from both the ends of the rod.

42. Consider a uniformly charged ring of radius $R$. Find the point on the axis where the electric field is maximum.

43. A wire is bent in the form of a regular hexagon and a total charge $q$ is distributed uniformly on it. What is the electric field at the centre ? You may answer this part without making any numerical calculations.

44. A circular wire-loop of radius $a$ carries a total charge $Q$ distributed uniformly over its length. A small length $dL$ of the wire is cut off. Find the electric field at the centre due to the remaining wire.

45. A positive charge $q$ is placed in front of a conducting solid cube at a distance $d$ from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

46. A pendulum bob of mass 80 mg and carrying a charge of $2 \times 10^{-8}$ C is at rest in a uniform, horizontal electric field of 20 kV/m. Find the tension in the thread.

47. A particle of mass $m$ and charge $q$ is thrown at a speed $u$ against a uniform electric field $E$. How much distance will it travel before coming to momentary rest ?

48. A particle of mass 1 g and charge $2.5 \times 10^{-4}$ C is released from rest in an electric field of $1.2 \times 10^{4}$ N/C. (a) Find the electric force and the force of gravity acting on this particle. Can one of these forces be neglected in comparison with the other for approximate analysis ? (b) How long will it take for the particle to travel a distance of 40 cm ? (c) What will be the speed of the particle after travelling this distance ? (d) How much is the work done by the electric force on the particle during this period ?

49. A ball of mass 100 g and having a charge of $4.9 \times 10^{-5}$ C is released from rest in a region where a horizontal electric field of $2.0 \times 10^{4}$ N/C exists. (a) Find the resultant force acting on the ball. (b) What will be the path of the ball ? (c) Where will the ball be at the end of 2 s ?

50. The bob of a simple pendulum has a mass of 40 g and a positive charge of $4.0 \times 10^{-6}$ C. It makes 20 oscillations in 45 s. A vertical electric field pointing upward and of magnitude $2.5 \times 10^{4}$ N/C is switched on. How much time will it now take to complete 20 oscillations ?

51. A block of mass $m$ and having a charge $q$ is placed on a smooth horizontal table and is connected to a wall through an unstressed spring of spring constant $k$ as shown in figure (29-E1). A horizontal electric field $E$ parallel to the spring is switched on. Find the amplitude of the resulting SHM of the block.
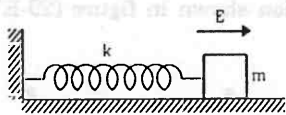


Figure 29-E1

52. A block of mass $m$ containing a net positive charge $q$ is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure (29-E2). The distance of the block from the wall is $d$. A horizontal electric field $E$ towards right is switched on. Assuming elastic collisions (if any) find the time period of the resulting oscillatory motion. Is it a simple harmonic motion ?
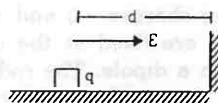


Figure 29-E2

53. A uniform electric field of 10 N/C exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 50 cm.

54. 12 J of work has to be done against an existing electric field to take a charge of 0.01 C from $A$ to $B$. How much is the potential difference $V_B - V_A$ ?

55. Two equal charges, $2.0 \times 10^{-7}$ C each, are held fixed at a separation of 20 cm. A third charge of equal magnitude is placed midway between the two charges. It is now moved to a point 20 cm from both the charges. How much work is done by the electric field during the process ?

56. An electric field of 20 N/C exists along the $X$-axis in space. Calculate the potential difference $V_B - V_A$ where the points $A$ and $B$ are given by,
   (a) $A = ( 0, 0 ); B = ( 4\, m, 2\, m )$
   (b) $A = ( 4\, m, 2\, m ); B = ( 6\, m, 5\, m )$
   (c) $A = ( 0, 0 ); B = ( 6\, m, 5\, m )$.
   Do you find any relation between the answers of parts (a), (b) and (c) ?

57. Consider the situation of the previous problem. A charge of $- 2.0 \times 10^{-4}$ C is moved from the point $A$ to the point $B$. Find the change in electrical potential energy $U_B - U_A$ for the cases (a), (b) and (c).

58. An electric field $\vec{E} = (\vec{i}\, 20 + \vec{j}\, 30)$ N/C exists in the space. If the potential at the origin is taken to be zero, find the potential at (2 m, 2 m).

59. An electric field $\vec{E} = \vec{i}\, Ax$ exists in the space, where $A = 10$ V/m$^2$. Take the potential at (10 m, 20 m) to be zero. Find the potential at the origin.

60. The electric potential existing in space is $V(x, y, z) = A(xy + yz + zx)$. (a) Write the dimensional formula of $A$. (b) Find the expression for the electric field. (c) If $A$ is 10 SI units, find the magnitude of the electric field at ( 1 m, 1 m, 1 m ).

61. Two charged particles, having equal charges of $2.0 \times 10^{-5}$ C each, are brought from infinity to within a separation of 10 cm. Find the increase in the electric potential energy during the process.

62. Some equipotential surfaces are shown in figure (29-E3). What can you say about the magnitude and the direction of the electric field ?
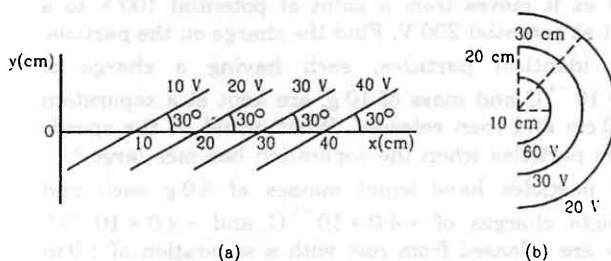


(a)                    (b)

Figure 29-E3

63. Consider a circular ring of radius $r$, uniformly charged with linear charge density $\lambda$. Find the electric potential at a point on the axis at a distance $x$ from the centre of the ring. Using this expression for the potential, find the electric field at this point.

64. An electric field of magnitude 1000 N/C is produced between two parallel plates having a separation of 2.0 cm as shown in figure (29-E4). (a) What is the potential difference between the plates ? (b) With what minimum speed should an electron be projected from the lower plate in the direction of the field so that it may reach the upper plate ? (c) Suppose the electron is projected from the lower plate with the speed calculated in part (b). The direction of projection makes an angle of 60° with the field. Find the maximum height reached by the electron.
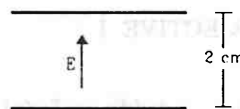


Figure 29-E4

**65.** A uniform field of $2.0$ N/C exists in space in $x$-direction. (a) Taking the potential at the origin to be zero, write an expression for the potential at a general point $(x, y, z)$. (b) At which points, the potential is 25 V ? (c) If the potential at the origin is taken to be 100 V, what will be the expression for the potential at a general point ? (d) What will be the potential at the origin if the potential at infinity is taken to be zero ? Is it practical to choose the potential at infinity to be zero ?

**66.** How much work has to be done in assembling three charged particles at the vertices of an equilateral triangle as shown in figure (29-E5) ?
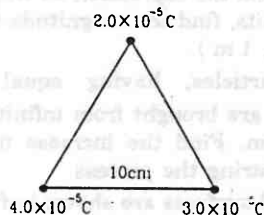


Figure 29-E5

**67.** The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle.

**68.** Two identical particles, each having a charge of $2.0 \times 10^{-4}$ C and mass of 10 g, are kept at a separation of 10 cm and then released. What would be the speeds of the particles when the separation becomes large ?

**69.** Two particles have equal masses of $5.0$ g each and opposite charges of $+4.0 \times 10^{-5}$ C and $-4.0 \times 10^{-5}$ C. They are released from rest with a separation of $1.0$ m between them. Find the speeds of the particles when the separation is reduced to 50 cm.

**70.** A sample of HCl gas is placed in an electric field of $2.5 \times 10^{4}$ N/C. The dipole moment of each HCl molecule is $3.4 \times 10^{-30}$ C-m. Find the maximum torque that can act on a molecule.

**71.** Two particles $A$ and $B$, having opposite charges $2.0 \times 10^{-6}$ C and $-2.0 \times 10^{-6}$ C, are placed at a separation of $1.0$ cm. (a) Write down the electric dipole moment of this pair. (b) Calculate the electric field at a point on the axis of the dipole $1.0$ cm away from the

centre. (c) Calculate the electric field at a point on the perpendicular bisector of the dipole and $1.0$ m away from the centre.

**72.** Three charges are arranged on the vertices of an equilateral triangle as shown in figure (29-E6). Find the dipole moment of the combination.
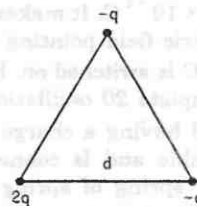


Figure 29-E6

**73.** Find the magnitude of the electric field at the point $P$ in the configuration shown in figure (29-E7) for $d \gg a$. Take $2qa = p$.
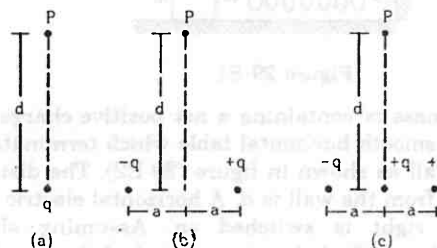


Figure 29-E7

**74.** Two particles, carrying charges $-q$ and $+q$ and having equal masses $m$ each, are fixed at the ends of a light rod of length $a$ to form a dipole. The rod is clamped at an end and is placed in a uniform electric field $E$ with the axis of the dipole along the electric field. The rod is slightly tilted and then released. Neglecting gravity find the time period of small oscillations.

**75.** Assume that each atom in a copper wire contributes one free electron. Estimate the number of free electrons in a copper wire having a mass of $6.4$ g (take the atomic weight of copper to be 64 g/mol).

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (d) | 3. (a) | 4. (d) | 5. (a) | 6. (d) |
| 7. (a) | 8. (c) | 9. (a) | | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (a) | 2. (c), (d) | 3. none |
| 4. (b), (c) | 5. (b), (d) | 6. (d) |
| 7. (b) | 8. (c) | |

## EXERCISES

1. $I^2 M^{-1} L^{-3} T^4$

2. $2 \cdot 25 \times 10^3$ N

3. $4 \cdot 3 \times 10^3$ m

4. $2 \cdot 3 \times 10^{-4}$ C

5. 230 N

6. 5·9 cm from the larger charge in between the two charges

7. 34·1 cm from the larger charge on the line joining the charge in the side of the smaller charge

8. $2 \cdot 3 \times 10^{-24}$ N

9. $3 \cdot 35 \times 10^{25}$, $5 \cdot 35 \times 10^6$ C

10. $2 \cdot 56 \times 10^{25}$ N

11. 1·2 N

12. $2 \times 10^{11}$

13. $3 \cdot 05 \times 10^{-9}$ N

14. $1 \cdot 23 \times 10^{36}$

15. (a) $ML^3 T^{-2}$, $L^{-1}$, $N\text{-m}^2$, $m^{-1}$     (b) $3 \cdot 4 \times 10^{-29}$ N-m$^2$

16. 24·9 N at 30° with the extended sides from the charge under consideration

17. 27·5 N at 45° with the extended sides of the square from the charge under consideration

18. $8 \cdot 2 \times 10^{-8}$ N

19. $2 \cdot 18 \times 10^6$ m/s

20. $4 \cdot 95 \times 10^5$ N

21. $3 \cdot 6 \times 10^{-6}$ N

22. (a) 0·144 N

    (b) zero, 0·095 N away from the other charge

    (c) 0·986 N and (d) 0·95 m/s$^2$ perpendicular to the string and going away from the other charge

23. 8·2 g, $8 \cdot 2 \times 10^{-2}$ N

24. $4 \cdot 17 \times 10^{-8}$ C

25. $\dfrac{q^2 \cot\theta}{16\pi\varepsilon_0 \, gl^2 \sin^2\theta}$

26. $5 \cdot 4 \times 10^{-9}$ C

27. $-(6 - 4\sqrt{2})\, q$, between $q$ and $2q$ at a distance of $(\sqrt{2} - 1)\, d$ from $q$

28. $3 \cdot 6 \times 10^{-6}$ m

29. between $\pm 8 \cdot 71 \times 10^{-8}$ C

30. 27 cm from the bottom

31. $d/2\sqrt{2}$, $3 \cdot 08 \dfrac{Qq}{4\pi\varepsilon_0 \, d^2}$

32. (a) $\dfrac{Qax}{2\pi\varepsilon_0 \left(x^2 + \dfrac{d^2}{4}\right)^{3/2}}$     (c) $\left[\dfrac{m\pi^3\varepsilon_0 d^3}{Qq}\right]^{\frac{1}{2}}$

33. time period $= \left[\dfrac{\pi^3\varepsilon_0 \, md^3}{2Qq}\right]^{\frac{1}{2}}$

34. $1 \cdot 5 \times 10^3$ N/C

35. (a) 48·3 cm from $A$ along $BA$

    (b) 20 cm from $A$ along $BA$ and $\dfrac{20}{3}$ cm from $A$ along $AB$

36. $8 \cdot 9 \times 10^{-11}$ C

37. 65·3 N/C, upward

38. zero, $2 \cdot 3 \times 10^3$ V

39. $\left[\dfrac{16\pi^3\varepsilon_0 \, mR^3}{Qq}\right]^{1/2}$

40. $\dfrac{Q}{2\varepsilon_0 L^2}$

41. $5 \cdot 2 \times 10^7$ N/C

42. $R/\sqrt{2}$

43. zero

44. $\dfrac{QdL}{8\pi^2\varepsilon_0 \, a^3}$

45. $\dfrac{q}{4\pi\varepsilon_0 \, d^2}$ towards the charge $q$

46. $8 \cdot 8 \times 10^{-4}$ N

47. $\dfrac{mu^2}{2qE}$

48. (a) 3·0 N, $9 \cdot 8 \times 10^{-3}$ N,     (b) $1 \cdot 63 \times 10^{-2}$ s

    (c) 49·0 m/s     (d) 1·20 J

49. (a) 1·4 N making an angle of 45° with $\vec{g}$ and $\vec{E}$

    (b) straight line along the resultant force

    (c) 28 m from the starting point on the line of motion

50. 52 s

51. $qE/k$

52. $\sqrt{\dfrac{8\,md}{qE}}$

53. 5 V

54. 1200 volts

55. $3 \cdot 6 \times 10^{-3}$ J

56. (a) − 80 V     (b) − 40 V     (c) −120 V

57. 0·016 J, 0·008 J, 0·024 J

58. −100 V

59. 500 V

60. (a) $MT^{-3} I^{-1}$     (b) $-A\{\vec{i}(y+z) + \vec{j}(z+x) + \vec{k}(x+y)\}$

    (c) 35 N/C

61. 36 J

62. (a) 200 V/m making an angle 120° with the $X$-axis

    (b) radially outward, decreasing with distance as $E = \dfrac{6 \text{ V-m}}{r^2}$.

63. $\dfrac{r\lambda}{2\varepsilon_0(r^2 + x^2)^{1/2}}$, $\dfrac{r\lambda x}{2\varepsilon_0(r^2 + x^2)^{3/2}}$

64. (a) 20 V      (b) $2.65 \times 10^{6}$ m/s    (c) 0.50 cm

65. (a) $-$ (2.0 V/m) $x$

     (b) points on the plane $x = -12.5$ m

     (c) 100 V $-$ ( 2.0 V/m ) $x$

     (d) infinity

66. 234 J

67. 0.1 C

68. 600 m/s

69. 54 m/s for each particle

70. $8.5 \times 10^{-3}$ N-m

71. (a) $2.0 \times 10^{-8}$ C-m      (b) 360 N/C      (c) 180 N/C

72. $qd\sqrt{3}$, along the bisector of the angle at $2q$, away from the triangle

73. (a) $\dfrac{q}{4\pi\varepsilon_0 d^2}$      (b) $\dfrac{p}{4\pi\varepsilon_0 d^3}$      (c) $\dfrac{1}{4\pi\varepsilon_0 d^3}\sqrt{q^2 d^2 + p^2}$

74. $2\pi\sqrt{\dfrac{ma}{qE}}$

75. $6 \times 10^{22}$

□

and due to the charge $Q_2 + q = \dfrac{Q_2 + q}{2A\varepsilon_0}$ (upward).

The net electric field at $P$ due to all the four charged surfaces is (in the downward direction)

$$\frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}.$$

As the point $P$ is inside the conductor, this field should be zero. Hence,

$$Q_1 - q - Q_2 - q = 0$$

or,

$$q = \frac{Q_1 - Q_2}{2}. \qquad \ldots \text{(i)}$$

Thus,

$$Q_1 - q = \frac{Q_1 + Q_2}{2} \qquad \ldots \text{(ii)}$$

and

$$Q_2 + q = \frac{Q_1 + Q_2}{2}.$$

Using these equations, the distribution shown in the figure (30-W6) can be redrawn as in figure (30-W7).



$(Q_1 + Q_2)/2$
$(Q_1 - Q_2)/2$
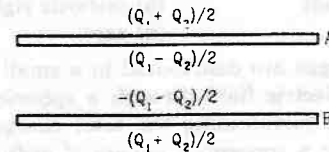$-(Q_1 - Q_2)/2$
$(Q_1 + Q_2)/2$

**Figure 30-W7**

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.

□

# QUESTIONS FOR SHORT ANSWER

1. A small plane area is rotated in an electric field. In which orientation of the area is the flux of electric field through the area maximum ? In which orientation is it zero ?

2. A circular ring of radius $r$ made of a nonconducting material is placed with its axis parallel to a uniform electric field. The ring is rotated about a diameter through 180°. Does the flux of electric field change ? If yes, does it decrease or increase ?

3. A charge $Q$ is uniformly distributed on a thin spherical shell. What is the field at the centre of the shell ? If a point charge is brought close to the shell, will the field at the centre change ? Does your answer depend on whether the shell is conducting or nonconducting ?

4. A spherical shell made of plastic, contains a charge $Q$ distributed uniformly over its surface. What is the electric field inside the shell ? If the shell is hammered to deshape it without altering the charge, will the field inside be changed ? What happens if the shell is made of a metal ?

5. A point charge $q$ is placed in a cavity in a metal block. If a charge $Q$ is brought outside the metal, will the charge $q$ feel an electric force ?

6. A rubber balloon is given a charge $Q$ distributed uniformly over its surface. Is the field inside the balloon zero everywhere if the balloon does not have a spherical surface ?

7. It is said that any charge given to a conductor comes to its surface. Should all the protons come to the surface ? Should all the electrons come to the surface ? Should all the free electrons come to the surface ?

# OBJECTIVE I

1. A charge $Q$ is uniformly distributed over a large plastic plate. The electric field at a point $P$ close to the centre of the plate is 10 V/m. If the plastic plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge $Q$, the electric field at the point $P$ will become
   (a) zero     (b) 5 V/m     (c) 10 V/m     (d) 20 V/m.

2. A metallic particle having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the particle will be
   (a) towards the plate       (b) away from the plate
   (c) parallel to the plate      (d) zero.

3. A thin, metallic spherical shell contains a charge $Q$ on it. A point charge $q$ is placed at the centre of the shell and another charge $q_1$ is placed outside it as shown in figure (30-Q1). All the three charges are positive. The
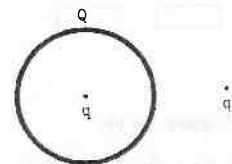


**Figure 30-Q1**

force on the charge at the centre is
(a) towards left            (b) towards right
(c) upward                  (d) zero.

4. Consider the situation of the previous problem. The force on the central charge due to the shell is
(a) towards left            (b) towards right
(c) upward                  (d) zero.

5. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V-m. The flux over a concentric sphere of radius 20 cm will be
(a) 25 V-m    (b) 50 V-m    (c) 100 V-m    (d) 200 V-m.

6. Figure (30-Q2a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of length $L$ moves towards left at a small but constant speed $v$. At $t = 0$, the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in figure (30-Q2b) represents the flux of the electric field through the cube as the rod goes through it ?
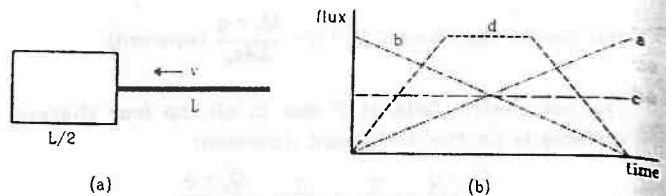


Figure 30-Q2

7. A charge $q$ is placed at the centre of the open end of a cylindrical vessel (figure 30-Q3). The flux of the electric field through the surface of the vessel is
(a) zero          (b) $q/\varepsilon_0$          (c) $q/2\varepsilon_0$          (d) $2q/\varepsilon_0$.
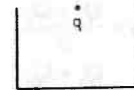


Figure 30-Q3

# OBJECTIVE II

1. Mark the correct options:
(a) Gauss's law is valid only for symmetrical charge distributions.
(b) Gauss's law is valid only for charges placed in vacuum.
(c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

2. A positive point charge $Q$ is brought near an isolated metal cube.
(a) The cube becomes negatively charged.
(b) The cube becomes positively charged.
(c) The interior becomes positively charged and the surface becomes negatively charged.
(d) The interior remains charge free and the surface gets nonuniform charge distribution.

3. A large nonconducting sheet $M$ is given a uniform charge density. Two uncharged small metal rods $A$ and $B$ are placed near the sheet as shown in figure (30-Q4).
(a) $M$ attracts $A$.            (b) $M$ attracts $B$.
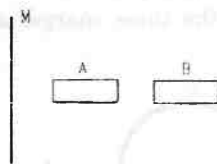(c) $A$ attracts $B$.            (d) $B$ attracts $A$.



Figure 30-Q4

4. If the flux of the electric field through a closed surface is zero,

(a) the electric field must be zero everywhere on the surface
(b) the electric field may be zero everywhere on the surface
(c) the charge inside the surface must be zero
(d) the charge in the vicinity of the surface must be zero.

5. An electric dipole is placed at the centre of a sphere. Mark the correct options:
(a) The flux of the electric field through the sphere is zero.
(b) The electric field is zero at every point of the sphere.
(c) The electric field is not zero anywhere on the sphere.
(d) The electric field is zero on a circle on the sphere.

6. Figure (30-Q5) shows a charge $q$ placed at the centre of a hemisphere. A second charge $Q$ is placed at one of the positions $A$, $B$, $C$ and $D$. In which position(s) of this second charge, the flux of the electric field through the hemisphere remains unchanged ?
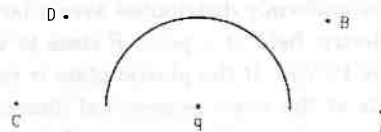(a) $A$          (b) $B$          (c) $C$          (d) $D$.



Figure 30-Q5

7. A closed surface $S$ is constructed around a conducting wire connected to a battery and a switch (figure 30-Q6). As the switch is closed, the free electrons in the wire start moving along the wire. In any time interval, the number of electrons entering the closed surface $S$ is equal to the number of electrons leaving it. On closing

the switch, the flux of the electric field through the closed surface
(a) is increased
(b) is decreased
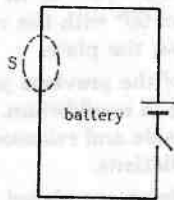(c) remains unchanged
(d) remains zero.



Figure 30-Q6

8. Figure (30-Q7) shows a closed surface which intersects a conducting sphere. If a positive charged is placed at

the point P, the flux of the electric field through the closed surface
(a) will remain zero
(b) will become positive
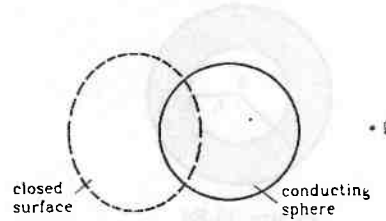(c) will become negative
(d) will become undefined.



Figure 30-Q7

## EXERCISES

1. The electric field in a region is given by $\vec{E} = \frac{3}{5} E_0 \vec{i} + \frac{4}{5} E_0 \vec{j}$ with $E_0 = 2.0 \times 10^3$ N/C. Find the flux of this field through a rectangular surface of area $0.2 \, m^2$ parallel to the $Y$–$Z$ plane.

2. A charge $Q$ is uniformly distributed over a rod of length $l$. Consider a hypothetical cube of edge $l$ with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

3. Show that there can be no net-charge in a region in which the electric field is uniform at all points.

4. The electric field in a region is given by $\vec{E} = \frac{E_0 x}{l} \vec{i}$. Find the charge contained inside a cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3$ N/C, $l = 2$ cm and $a = 1$ cm.

5. A charge $Q$ is placed at the centre of a cube. Find the flux of the electric field through the six surfaces of the cube.

6. A charge $Q$ is placed at a distance $a/2$ above the centre of a horizontal, square surface of edge $a$ as shown in figure (30-E1). Find the flux of the electric field through the square surface.
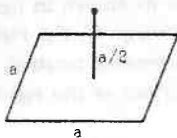


Figure 30-E1

7. Find the flux of the electric field through a spherical surface of radius $R$ due to a charge of $10^{-7}$ C at the centre and another equal charge at a point $2R$ away from the centre (figure 30-E2).
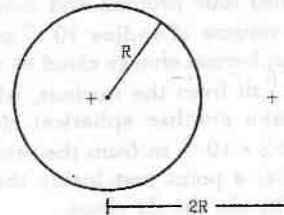


Figure 30-E2

8. A charge $Q$ is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure 30-E3).
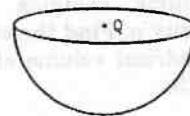


Figure 30-E3

9. A spherical volume contains a uniformly distributed charge of density $2.0 \times 10^{-4}$ C/m³. Find the electric field at a point inside the volume at a distance 4.0 cm from the centre.

10. The radius of a gold nucleus ($Z = 79$) is about $7.0 \times 10^{-15}$ m. Assume that the positive charge is distributed uniformly throughout the nuclear volume. Find the strength of the electric field at (a) the surface of the nucleus and (b) at the middle point of a radius. Remembering that gold is a conductor, is it justified to assume that the positive charge is uniformly distributed over the entire volume of the nucleus and does not come to the outer surface ?

11. A charge $Q$ is distributed uniformly within the material of a hollow sphere of inner and outer radii $r_1$ and $r_2$ (figure 30-E4). Find the electric field at a point $P$ a

distance $x$ away from the centre for $r_1 < x < r_2$. Draw a rough graph showing the electric field as a function of $x$ for $0 < x < 2r_2$ (figure 30-E4).
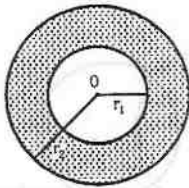


Figure 30-E4

12. A charge $Q$ is placed at the centre of an uncharged, hollow metallic sphere of radius $a$. (a) Find the surface charge density on the inner surface and on the outer surface. (b) If a charge $q$ is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces? (c) Find the electric field inside the sphere at a distance $x$ from the centre in the situations (a) and (b).

13. Consider the following very rough model of a beryllium atom. The nucleus has four protons and four neutrons confined to a small volume of radius $10^{-15}$ m. The two $1s$ electrons make a spherical charge cloud at an average distance of $1.3 \times 10^{-11}$ m from the nucleus, whereas the two $2s$ electrons make another spherical cloud at an average distance of $5.2 \times 10^{-11}$ m from the nucleus. Find the electric field at (a) a point just inside the $1s$ cloud and (b) a point just inside the $2s$ cloud.

14. Find the magnitude of the electric field at a point 4 cm away from a line charge of density $2 \times 10^{-8}$ C/m.

15. A long cylindrical wire carries a positive charge of linear density $2.0 \times 10^{-8}$ C/m. An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron. Note that it is independent of the radius.

16. A long cylindrical volume contains a uniformly distributed charge of density $\rho$. Find the electric field at a point $P$ inside the cylindrical volume at a distance $x$ from its axis (figure 30-E5)
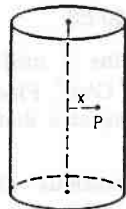


Figure 30-E5

17. A nonconducting sheet of large surface area and thickness $d$ contains uniform charge distribution of density $\rho$. Find the electric field at a point $P$ inside the plate, at a distance $x$ from the central plane. Draw a qualitative graph of $E$ against $x$ for $0 < x < d$.

18. A charged particle having a charge of $-2.0 \times 10^{-6}$ C is placed close to a nonconducting plate having a surface charge density $4.0 \times 10^{-6}$ C/m². Find the force of attraction between the particle and the plate.

19. One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged nonconducting plate and the other end is fastened to a small ball having a mass of 10 g and a charge of $4.0 \times 10^{-6}$ C. In equilibrium, the thread makes an angle of 60° with the vertical. Find the surface charge density on the plate.

20. Consider the situation of the previous problem. (a) Find the tension in the string in equilibrium. (b) Suppose the ball is slightly pushed aside and released. Find the time period of the small oscillations.

21. Two large conducting plates are placed parallel to each other with a separation of 2·00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2·00 microseconds. Find the surface charge density on the inner surfaces.

22. Two large conducting plates are placed parallel to each other and they carry equal and opposite charges with surface density $\sigma$ as shown in figure (30-E6). Find the electric field (a) at the left of the plates, (b) in between the plates and (c) at the right of the plates.
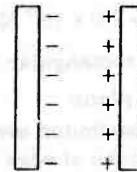


Figure 30-E6

23. Two conducting plates $X$ and $Y$, each having large surface area $A$ (on one side), are placed parallel to each other as shown in figure (30-E7). The plate $X$ is given a charge $Q$ whereas the other is neutral. Find (a) the surface charge density at the inner surface of the plate $X$, (b) the electric field at a point to the left of the plates, (c) the electric field at a point in between the plates and (d) the electric field at a point to the right of the plates.



Figure 30-E7

21. Three identical metal plates with large surface areas are kept parallel to each other as shown in figure (30-E8). The leftmost plate is given a charge $Q$, the rightmost a charge $-2Q$ and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.
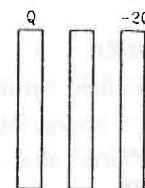


Figure 30-E8

# ANSWERS

## OBJECTIVE I

1. (c)    2. (a)    3. (d)    4. (b)    5. (a)    6. (d)
7. (c)

## OBJECTIVE II

1. (d)    2. (d)    3. all
4. (b), (c)    5. (a), (c)    6. (a), (c)
7. (c), (d)    8. (b)

## EXERCISES

1. $240 \frac{N\text{-}m^2}{C}$

2. $Q/(2\varepsilon_0)$

4. $2 \cdot 2 \times 10^{-12}$ C

5. $Q/\varepsilon_0$

6. $Q/(6\varepsilon_0)$

7. $1 \cdot 1 \times 10^4 \frac{N\text{-}m^2}{C}$

8. $Q/(2\varepsilon_0)$

9. $3 \cdot 0 \times 10^5$ N/C

10. (a) $2 \cdot 32 \times 10^{21}$ N/C (b) $1 \cdot 16 \times 10^{21}$ N/C

11. $\dfrac{Q(x^3 - r_1^3)}{4\pi\varepsilon_0 \, x^2 (r_2^3 - r_1^3)}$

12. (a) $-\dfrac{Q}{4\pi a^2}$, $\dfrac{Q}{4\pi a^2}$   (b) $-\dfrac{Q}{4\pi a^2}$, $\dfrac{Q+q}{4\pi a^2}$

    (c) $\dfrac{Q}{4\pi\varepsilon_0 \, x^2}$ in both situations

13. (a) $3 \cdot 4 \times 10^{13}$ N/C   (b) $1 \cdot 1 \times 10^{12}$ N/C

14. $9 \times 10^5$ N/C

15. $2 \cdot 88 \times 10^{-17}$ J

16. $\rho \, x/(2\varepsilon_0)$

17. $\rho \, x/\varepsilon_0$

18. $0 \cdot 45$ N

19. $7 \cdot 5 \times 10^{-7}$ C/m$^2$

20. (a) $0 \cdot 20$ N (b) $0 \cdot 45$ s

21. $0 \cdot 505 \times 10^{-12}$ C/m$^2$

22. (a) zero    (b) $\sigma/\varepsilon_0$    (c) zero

23. (a) $\dfrac{Q}{2A}$    (b) $\dfrac{Q}{2A\varepsilon_0}$ towards left    (c) $\dfrac{Q}{2A\varepsilon_0}$ **towards right**

    (d) $\dfrac{Q}{2A\varepsilon_0}$ towards right

24. $-Q/2$

□

## OBJECTIVE I

1. A capacitor of capacitance $C$ is charged to a potential $V$. The flux of the electric field through a closed surface enclosing the capacitor is

   (a) $\dfrac{CV}{\varepsilon_0}$    (b) $\dfrac{2CV}{\varepsilon_0}$    (c) $\dfrac{CV}{2\varepsilon_0}$    (d) zero.

2. Two capacitors each having capacitance $C$ and breakdown voltage $V$ are joined in series. The capacitance and the breakdown voltage of the combination will be

   (a) $2C$ and $2V$         (b) $C/2$ and $V/2$
   (c) $2C$ and $V/2$        (d) $C/2$ and $2V$.

3. If the capacitors in the previous question are joined in parallel, the capacitance and the breakdown voltage of the combination will be

   (a) $2C$ and $2V$         (b) $C$ and $2V$
   (c) $2C$ and $V$          (d) $C$ and $V$.

4. The equivalent capacitance of the combination shown in figure (31-Q1) is

   (a) $C$      (b) $2C$      (c) $C/2$      (d) none of these.
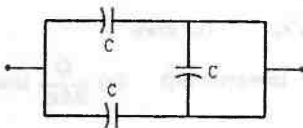


Figure 31-Q1

5. A dielectric slab is inserted between the plates of an isolated capacitor. The force between the plates will

   (a) increase               (b) decrease
   (c) remain unchanged       (d) become zero.

6. The energy density in the electric field created by a point charge falls off with the distance from the point charge as

   (a) $\dfrac{1}{r}$    (b) $\dfrac{1}{r^2}$    (c) $\dfrac{1}{r^3}$    (d) $\dfrac{1}{r^4}$.

7. A parallel-plate capacitor has plates of unequal area. The larger plate is connected to the positive terminal of the battery and the smaller plate to its negative terminal. Let $Q_+$ and $Q_-$ be the charges appearing on the positive and negative plates respectively.

   (a) $Q_+ > Q_-$.         (b) $Q_+ = Q_-$.         (c) $Q_+ < Q_-$.
   (d) The information is not sufficient to decide the relation between $Q_+$ and $Q_-$.

8. A thin metal plate $P$ is inserted between the plates of a parallel-plate capacitor of capacitance $C$ in such a way that its edges touch the two plates (figure 31-Q2). The capacitance now becomes

   (a) $C/2$         (b) $2C$         (c) $0$         (d) $\infty$.
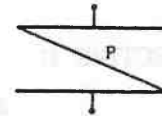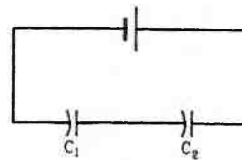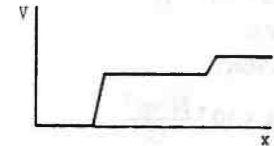


Figure 31-Q2

9. Figure (31-Q3) shows two capacitors connected in series and joined to a battery. The graph shows the variation in potential as one moves from left to right on the branch containing the capacitors.

   (a) $C_1 > C_2$.      (b) $C_1 = C_2$.      (c) $C_1 < C_2$.
   (d) The information is not sufficient to decide the relation between $C_1$ and $C_2$.



(a)                                    (b)

Figure 31-Q3

10. Two metal plates having charges $Q, -Q$ face each other at some separation and are dipped into an oil tank. If the oil is pumped out, the electric field between the plates will

    (a) increase             (b) decrease
    (c) remain the same      (d) become zero.

11. Two metal spheres of capacitances $C_1$ and $C_2$ carry some charges. They are put in contact and then separated. The final charges $Q_1$ and $Q_2$ on them will satisfy

    (a) $\dfrac{Q_1}{Q_2} < \dfrac{C_1}{C_2}$    (b) $\dfrac{Q_1}{Q_2} = \dfrac{C_1}{C_2}$    (c) $\dfrac{Q_1}{Q_2} > \dfrac{C_1}{C_2}$    (d) $\dfrac{Q_1}{Q_2} = \dfrac{C_2}{C_1}$.

12. Three capacitors of capacitances $6\,\mu F$ each are available. The minimum and maximum capacitances, which may be obtained are

    (a) $6\,\mu F$, $18\,\mu F$         (b) $3\,\mu F$, $12\,\mu F$
    (c) $2\,\mu F$, $12\,\mu F$         (d) $2\,\mu F$, $18\,\mu F$.

## OBJECTIVE II

1. The capacitance of a capacitor does not depend on

   (a) the shape of the plates
   (b) the size of the plates
   (c) the charges on the plates
   (d) the separation between the plates.

2. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities will remain the same ?

   (a) the electric field in the capacitor
   (b) the charge on the capacitor

(c) the potential difference between the plates

(d) the stored energy in the capacitor.

3. A dielectric slab is inserted between the plates of a capacitor. The charge on the capacitor is $Q$ and the magnitude of the induced charge on each surface of the dielectric is $Q'$.

(a) $Q'$ may be larger than $Q$.

(b) $Q'$ must be larger than $Q$.

(c) $Q'$ must be equal to $Q$.

(d) $Q'$ must be smaller than $Q$.

4. Each plate of a parallel plate capacitor has a charge $q$ on it. The capacitor is now connected to a battery. Now,

(a) the facing surfaces of the capacitor have equal and opposite charges.

(b) the two plates of the capacitor have equal and opposite charges.

(c) the battery supplies equal and opposite charges to the two plates.

(d) the outer surfaces of the plates have equal charges.

5. The separation between the plates of a charged parallel-plate capacitor is increased. Which of the following quantities will change ?

(a) charge on the capacitor

(b) potential difference across the capacitor

(c) energy of the capacitor

(d) energy density between the plates.

6. A parallel-plate capacitor is connected to a battery. A metal sheet of negligible thickness is placed between the plates. The sheet remains parallel to the plates of the capacitor.

(a) The battery will supply more charge.

(b) The capacitance will increase.

(c) The potential difference between the plates will increase.

(d) Equal and opposite charges will appear on the two faces of the metal plate.

7. Following operations can be performed on a capacitor:

$X$ – connect the capacitor to a battery of emf $\mathcal{E}$.

$Y$ – disconnect the battery.

$Z$ – reconnect the battery with polarity reversed.

$W$ – insert a dielectric slab in the capacitor.

(a) In $XYZ$ (perform $X$, then $Y$, then $Z$) the stored electric energy remains unchanged and no thermal energy is developed.

(b) The charge appearing on the capacitor is greater after the action $XWY$ than after the action $XYW$.

(c) The electric energy stored in the capacitor is greater after the action $WXY$ than after the action $XYW$.

(d) The electric field in the capacitor after the action $XW$ is the same as that after $WX$.

## EXERCISES

1. When $1.0 \times 10^{12}$ electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Calculate the capacitance of the two-conductor system.

2. The plates of a parallel-plate capacitor are made of circular discs of radii $5.0$ cm each. If the separation between the plates is $1.0$ mm, what is the capacitance ?

3. Suppose, one wishes to construct a $1.0$ farad capacitor using circular discs. If the separation between the discs be kept at $1.0$ mm, what would be the radius of the discs ?

4. A parallel-plate capacitor having plate area 25 cm$^2$ and separation $1.00$ mm is connected to a battery of $6.0$ V. Calculate the charge flown through the battery. How much work has been done by the battery during the process ?

5. A parallel-plate capacitor has plate area $25.0$ cm$^2$ and a separation of $2.00$ mm between the plates. The capacitor is connected to a battery of $12.0$ V. (a) Find the charge on the capacitor. (b) The plate separation is decreased to $1.00$ mm. Find the extra charge given by the battery to the positive plate.

6. Find the charges on the three capacitors connected to a battery as shown in figure (31-E1). Take $C_1 = 2.0\,\mu\text{F}$, $C_2 = 4.0\,\mu\text{F}$, $C_3 = 6.0\,\mu\text{F}$ and $V = 12$ volt.



Figure 31-E1

7. Three capacitors having capacitances $20\,\mu\text{F}$, $30\,\mu\text{F}$ and $40\,\mu\text{F}$ are connected in series with a 12 V battery. Find the charge on each of the capacitors. How much work has been done by the battery in charging the capacitors ?

8. Find the charge appearing on each of the three capacitors shown in figure (31-E2).



Figure 31-E2

9. Take $C_1 = 4.0\,\mu\text{F}$ and $C_2 = 6.0\,\mu\text{F}$ in figure (31-E3). Calculate the equivalent capacitance of the combination between the points indicated.
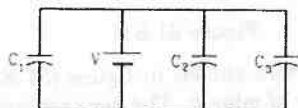


(a)                              (b)
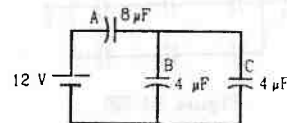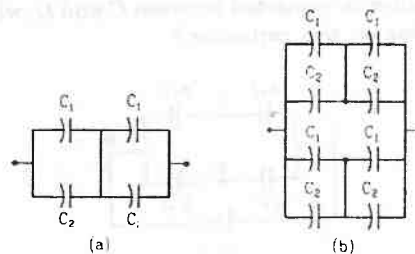
Figure 31-E3

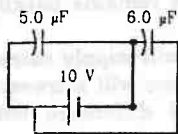10. Find the charge supplied by the battery in the arrangement shown in figure (31-E4).

Figure 31-E4

11. The outer cylinders of two cylindrical capacitors of capacitance 2·2 μF each, are kept in contact and the inner cylinders are connected through a wire. A battery of emf 10 V is connected as shown in figure (31-E5). Find the total charge supplied by the battery to the inner cylinders.
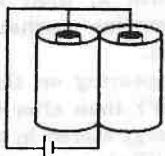
Figure 31-E5

12. Two conducting spheres of radii $R_1$ and $R_2$ are kept widely separated from each other. What are their individual capacitances ? If the spheres are connected by a metal wire, what will be the capacitance of the combination ? Think in terms of series–parallel connections.

13. Each of the capacitors shown in figure (31-E6) has a capacitance of 2 μF. Find the equivalent capacitance of the assembly between the points $A$ and $B$. Suppose, a battery of emf 60 volts is connected between $A$ and $B$. Find the potential difference appearing on the individual capacitors.
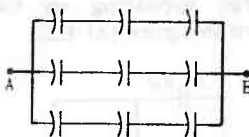
Figure 31-E6

14. It is required to construct a 10 μF capacitor which can be connected across a 200 V battery. Capacitors of capacitance 10 μF are available but they can withstand only 50 V. Design a combination which can yield the desired result.

15. Take the potential of the point $B$ in figure (31-E7) to be zero. (a) Find the potentials at the points $C$ and $D$. (b) If a capacitor is connected between $C$ and $D$, what charge will appear on this capacitor ?
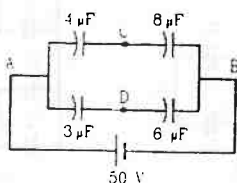
Figure 31-E7

16. Find the equivalent capacitance of the system shown in figure (31-E8) between the points $a$ and $b$.
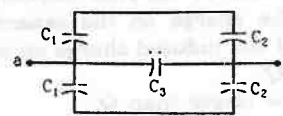
Figure 31-E8

17. A capacitor is made of a flat plate of area $A$ and a second plate having a stair-like structure as shown in figure (31-E9). The width of each stair is $a$ and the height is $b$. Find the capacitance of the assembly.
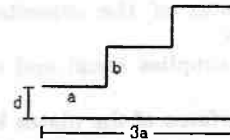
Figure 31-E9

18. A cylindrical capacitor is constructed using two coaxial cylinders of the same length 10 cm and of radii 2 mm and 4 mm. (a) Calculate the capacitance. (b) Another capacitor of the same length is constructed with cylinders of radii 4 mm and 8 mm. Calculate the capacitance.

19. A 100 pF capacitor is charged to a potential difference of 24 V. It is connected to an uncharged capacitor of capacitance 20 pF. What will be the new potential difference across the 100 pF capacitor ?

20. Each capacitor shown in figure (31-E10) has a capacitance of 5·0 μF. The emf of the battery is 50 V. How much charge will flow through $AB$ if the switch $S$ is closed ?
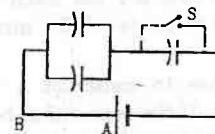
Figure 31-E10

21. The particle $P$ shown in figure (31-E11) has a mass of 10 mg and a charge of − 0·01 μC. Each plate has a surface area 100 cm$^2$ on one side. What potential difference $V$ should be applied to the combination to hold the particle $P$ in equilibrium ?

Figure 31-E11

22. Both the capacitors shown in figure (31-E12) are made of square plates of edge $a$. The separations between the plates of the capacitors are $d_1$ and $d_2$ as shown in the

Figure 31-E12

figure. A potential difference $V$ is applied between the points $a$ and $b$. An electron is projected between the plates of the upper capacitor along the central line. With what minimum speed should the electron be projected so that it does not collide with any plate ? Consider only the electric forces.

23. The plates of a capacitor are 2·00 cm apart. An electron-proton pair is released somewhere in the gap between the plates and it is found that the proton reaches the negative plate at the same time as the electron reaches the positive plate. At what distance from the negative plate was the pair released ?

24. Convince yourself that parts (a), (b) and (c) of figure (31-E13) are identical. Find the capacitance between the points $A$ and $B$ of the assembly.
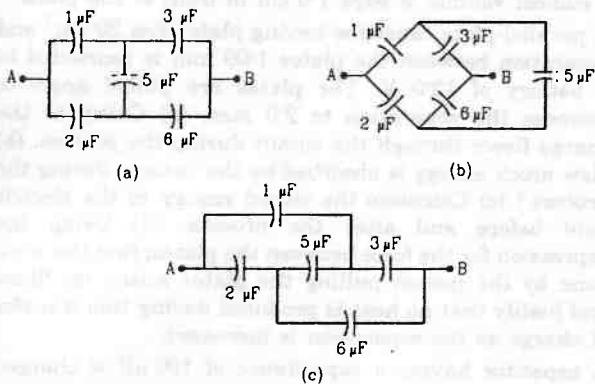


Figure 31-E13

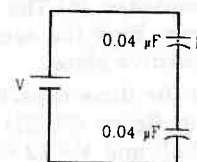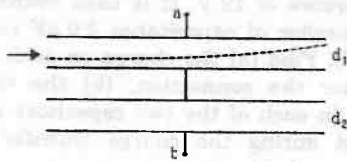25. Find the potential difference $V_a - V_b$ between the points $a$ and $b$ shown in each part of the figure (31-E14).
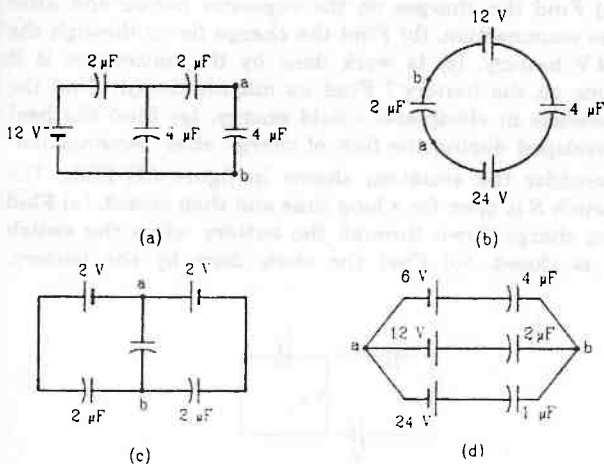


Figure 31-E14

26. Find the equivalent capacitances of the combinations shown in figure (31-E15) between the indicated points.
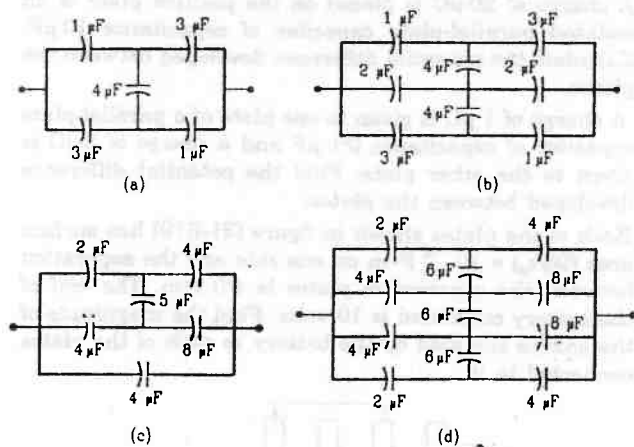


Figure 31-E15

27. Find the capacitance of the combination shown in figure (31-E16) between $A$ and $B$.
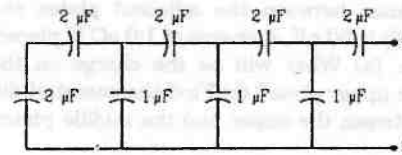


Figure 31-E16

28. Find the equivalent capacitance of the infinite ladder shown in figure (31-E17) between the points $A$ and $B$.
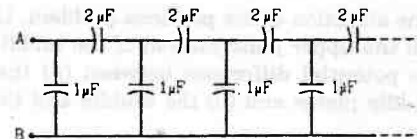


Figure 31-E17

29. A finite ladder is constructed by connecting several sections of $2\,\mu F$, $4\,\mu F$ capacitor combinations as shown in figure (31-E18). It is terminated by a capacitor of capacitance $C$. What value should be chosen for $C$, such that the equivalent capacitance of the ladder between the points $A$ and $B$ becomes independent of the number of sections in between ?
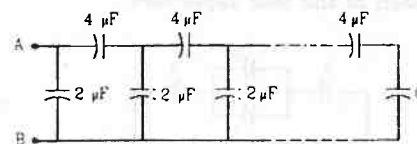


Figure 31-E18

30. A charge of $+ 2\,0 \times 10^{-8}$ C is placed on the positive plate and a charge of $- 1\,0 \times 10^{-8}$ C on the negative plate of a parallel-plate capacitor of capacitance $1·2 \times 10^{-3}\,\mu F$.

Calculate the potential difference developed between the plates.

31. A charge of 20 μC is placed on the positive plate of an isolated parallel-plate capacitor of capacitance 10 μF. Calculate the potential difference developed between the plates.

32. A charge of 1 μC is given to one plate of a parallel-plate capacitor of capacitance 0·1 μF and a charge of 2 μC is given to the other plate. Find the potential difference developed between the plates.

33. Each of the plates shown in figure (31-E19) has surface area $(96/\varepsilon_0) \times 10^{-12}$ F–m on one side and the separation between the consecutive plates is 4·0 mm. The emf of the battery connected is 10 volts. Find the magnitude of the charge supplied by the battery to each of the plates connected to it.
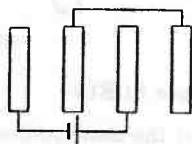


Figure 31-E19

34. The capacitance between the adjacent plates shown in figure (31-E20) is 50 nF. A charge of 1·0 μC is placed on the middle plate. (a) What will be the charge on the outer surface of the upper plate ? (b) Find the potential difference developed between the upper and the middle plates.
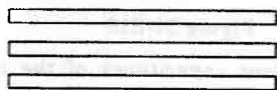


Figure 31-E20

35. Consider the situation of the previous problem. If 1·0 μC is placed on the upper plate instead of the middle, what will be the potential difference between (a) the upper and the middle plates and (b) the middle and the lower plates ?

36. Two capacitors of capacitances 20·0 pF and 50·0 pF are connected in series with a 6·00 V battery. Find (a) the potential difference across each capacitor and (b) the energy stored in each capacitor.

37. Two capacitors of capacitances 4·0 μF and 6·0 μF are connected in series with a battery of 20 V. Find the energy supplied by the battery.

38. Each capacitor in figure (31-E21) has a capacitance of 10 μF. The emf of the battery is 100 V. Find the energy stored in each of the four capacitors.
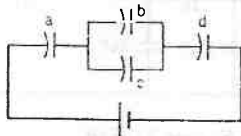


Figure 31-E21

39. A capacitor with stored energy 4·0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored in the two capacitors.

40. A capacitor of capacitance 2·0 μF is charged to a potential difference of 12 V. It is then connected to an uncharged capacitor of capacitance 4·0 μF as shown in figure (31-E22). Find (a) the charge on each of the two capacitors after the connection, (b) the electrostatic energy stored in each of the two capacitors and (c) the heat produced during the charge transfer from one capacitor to the other.
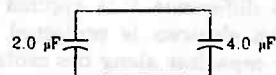


Figure 31-E22

41. A point charge Q is placed at the origin. Find the electrostatic energy stored outside the sphere of radius R centred at the origin.

42. A metal sphere of radius R is charged to a potential V. (a) Find the electrostatic energy stored in the electric field within a concentric sphere of radius 2 R. (b) Show that the electrostatic field energy stored outside the sphere of radius 2 R equals that stored within it.

43. A large conducting plane has a surface charge density $1·0 \times 10^{-4}$ C/m$^2$. Find the electrostatic energy stored in a cubical volume of edge 1·0 cm in front of the plane.

44. A parallel-plate capacitor having plate area 20 cm$^2$ and separation between the plates 1·00 mm is connected to a battery of 12·0 V. The plates are pulled apart to increase the separation to 2·0 mm. (a) Calculate the charge flown through the circuit during the process. (b) How much energy is absorbed by the battery during the process ? (c) Calculate the stored energy in the electric field before and after the process. (d) Using the expression for the force between the plates, find the work done by the person pulling the plates apart. (e) Show and justify that no heat is produced during this transfer of charge as the separation is increased.

45. A capacitor having a capacitance of 100 μF is charged to a potential difference of 24 V. The charging battery is disconnected and the capacitor is connected to another battery of emf 12 V with the positive plate of the capacitor joined with the positive terminal of the battery. (a) Find the charges on the capacitor before and after the reconnection. (b) Find the charge flown through the 12 V battery. (c) Is work done by the battery or is it done on the battery ? Find its magnitude. (d) Find the decrease in electrostatic field energy. (e) Find the heat developed during the flow of charge after reconnection.

46. Consider the situation shown in figure (31-E23). The switch S is open for a long time and then closed. (a) Find the charge flown through the battery when the switch S is closed. (b) Find the work done by the battery.
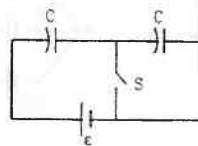


Figure 31-E23

(c) Find the change in energy stored in the capacitors.

(d) Find the heat developed in the system.

47. A capacitor of capacitance $5.00\,\mu F$ is charged to $24.0$ V and another capacitor of capacitance $6.0\,\mu F$ is charged to $12.0$ V. (a) Find the energy stored in each capacitor. (b) The positive plate of the first capacitor is now connected to the negative plate of the second and vice versa. Find the new charges on the capacitors. (c) Find the loss of electrostatic energy during the process. (d) Where does this energy go ?

48. A $5.0\,\mu F$ capacitor is charged to $12$ V. The positive plate of this capacitor is now connected to the negative terminal of a $12$ V battery and vice versa. Calculate the heat developed in the connecting wires.

49. The two square faces of a rectangular dielectric slab (dielectric constant $4.0$) of dimensions $20\,cm \times 20\,cm \times 1.0\,mm$ are metal-coated. Find the capacitance between the coated surfaces.

50. If the above capacitor is connected across a $6.0$ V battery, find (a) the charge supplied by the battery, (b) the induced charge on the dielectric and (c) the net charge appearing on one of the coated surfaces.

51. The separation between the plates of a parallel-plate capacitor is $0.500$ cm and its plate area is $100\,cm^2$. A $0.400$ cm thick metal plate is inserted into the gap with its faces parallel to the plates. Show that the capacitance of the assembly is independent of the position of the metal plate within the gap and find its value.

52. A capacitor stores $50\,\mu C$ charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of $100\,\mu C$ flows through the battery. Find the dielectric constant of the material inserted.

53. A parallel-plate capacitor of capacitance $5\,\mu F$ is connected to a battery of emf $6$ V. The separation between the plates is $2$ mm. (a) Find the charge on the positive plate. (b) Find the electric field between the plates. (c) A dielectric slab of thickness $1$ mm and dielectric constant $5$ is inserted into the gap to occupy the lower half of it. Find the capacitance of the new combination. (d) How much charge has flown through the battery after the slab is inserted ?

54. A parallel-plate capacitor has plate area $100\,cm^2$ and plate separation $1.0$ cm. A glass plate (dielectric constant $6.0$) of thickness $6.0$ mm and an ebonite plate (dielectric constant $4.0$) are inserted one over the other to fill the space between the plates of the capacitor. Find the new capacitance.

55. A parallel-plate capacitor having plate area $400\,cm^2$ and separation between the plates $1.0$ mm is connected to a power supply of $100$ V. A dielectric slab of thickness $0.5$ mm and dielectric constant $5.0$ is inserted into the gap. (a) Find the increase in electrostatic energy. (b) If the power supply is now disconnected and the dielectric slab is taken out, find the further increase in energy. (c) Why does the energy increase in inserting the slab as well as in taking it out ?

56. Find the capacitances of the capacitors shown in figure (31-E24). The plate area is $A$ and the separation between
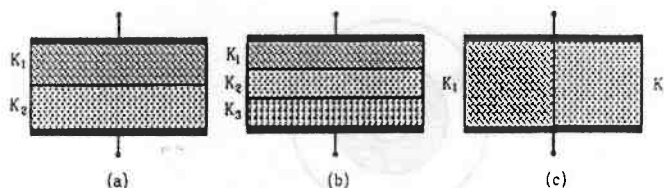


Figure 31-E24

the plates is $d$. Different dielectric slabs in a particular part of the figure are of the same thickness and the entire gap between the plates is filled with the dielectric slabs.

57. A capacitor is formed by two square metal-plates of edge $a$, separated by a distance $d$. Dielectrics of dielectric constants $K_1$ and $K_2$ are filled in the gap as shown in figure (31-E25). Find the capacitance.
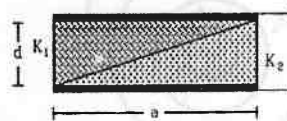


Figure 31-E25

58. Figure (31-E26) shows two identical parallel plate capacitors connected to a battery through a switch $S$. Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant $3$. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.
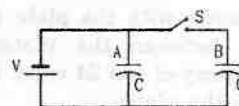


Figure 31-E26

59. A parallel-plate capacitor of plate area $A$ and plate separation $d$ is charged to a potential difference $V$ and then the battery is disconnected. A slab of dielectric constant $K$ is then inserted between the plates of the capacitor so as to fill the space between the plates. Find the work done on the system in the process of inserting the slab.

60. A capacitor having a capacitance of $100\,\mu F$ is charged to a potential difference of $50$ V. (a) What is the magnitude of the charge on each plate ? (b) The charging battery is disconnected and a dielectric of dielectric constant $2.5$ is inserted. Calculate the new potential difference between the plates. (c) What charge would have produced this potential difference in absence of the dielectric slab. (d) Find the charge induced at a surface of the dielectric slab.

61. A sphercial capacitor is made of two conducting spherical shells of radii $a$ and $b$. The space between the shells is filled with a dielectric of dielectric constant $K$ upto a
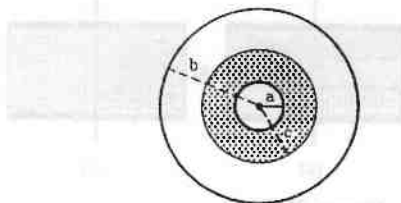
Figure 31-E27

radius c as shown in figure (31-E27). Calculate the capacitance.

62. Consider an assembly of three conducting concentric spherical shells of radii $a$, $b$ and $c$ as shown in figure (31-E28). Find the capacitance of the assembly between the points $A$ and $B$.
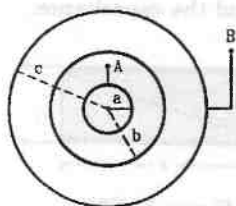


Figure 31-E28

63. Suppose the space between the two inner shells of the previous problem is filled with a dielectric of dielectric constant $K$. Find the capacitance of the system between $A$ and $B$.

64. An air-filled parallel-plate capacitor is to be constructed which can store $12\,\mu C$ of charge when operated at 1200 V. What can be the minimum plate area of the capacitor ? The dielectric strength of air is $3 \times 10^6$ V/m.

65. A parallel-plate capacitor with the plate area 100 cm$^2$ and the separation between the plates 1·0 cm is connected across a battery of emf 24 volts. Find the force of attraction between the plates.

66. Consider the situation shown in figure (31-E29). The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf $\mathcal{E}$. All surfaces are frictionless. Calculate the value of $M$ for which the dielectric slab will stay in equilibrium.
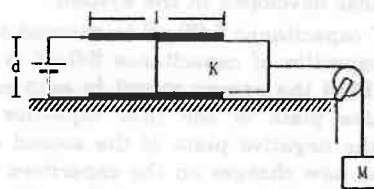


Figure 31-E29

67. Figure (31-E30) shows two parallel plate capacitors with fixed plates and connected to two batteries. The separation between the plates is the same for the two capacitors. The plates are rectangular in shape with width $b$ and lengths $l_1$ and $l_2$. The left half of the dielectric slab has a dielectric constant $K_1$ and the right half $K_2$. Neglecting any friction, find the ratio of the emf of the left battery to that of the right battery for which the dielectric slab may remain in equilibrium.
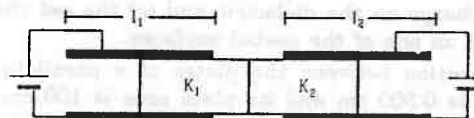


Figure 31-E30

68. Consider the situation shown in figure (31-E31). The plates of the capacitor have plate area $A$ and are clamped in the laboratory. The dielectric slab is released from rest with a length $a$ inside the capacitor. Neglecting any effect of friction or gravity, show that the slab will execute periodic motion and find its time period.
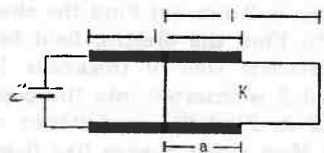


Figure 31-E31

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (d) | 2. (d) | 3. (c) | 4. (b) | 5. (c) | 6. (d) |
| 7. (b) | 8. (d) | 9. (c) | 10. (a) | 11. (b) | 12. (d) |

### OBJECTIVE II

1. (c)    2. (b)    3. (d)    4. (a), (c), (d)    5. (b), (c)
6. (d)    7. (b), (c), (d)

### EXERCISES

1. $1·6 \times 10^{-8}$ F
2. $6·95 \times 10^{-5}$ μF
3. 6 km
4. $1·33 \times 10^{-10}$ C, $8·0 \times 10^{-10}$ J
5. (a) $1·33 \times 10^{-10}$ C (b) $1·33 \times 10^{-10}$ C

6. 24 μC, 48 μC, 72 μC

7. 110 μC on each, $1.33 \times 10^{-3}$ J

8. 48 μC on the 8 μF capacitor and 24 μC on each of the 4 μF capacitors

9. (a) 5 μF  (b) 10 μF

10. 110 μC

11. 44 μC

12. $4\pi\varepsilon_0 R_1$, $4\pi\varepsilon_0 R_2$, $4\pi\varepsilon_0 (R_1 + R_2)$

13. 2 μF, 20 V

15. (a) 50/3 μV at each point (b) zero

16. $C_3 + \dfrac{2 C_1 C_2}{C_1 + C_2}$

17. $\dfrac{\varepsilon_0 A(3 d^2 + 6 bd + 2 b^2)}{3 d(d + b)(d + 2 b)}$

18. (a) 8 pF (b) same as in (a)

19. 20 V

20. $3.3 \times 10^{-4}$ C

21. 43 mV

22. $\left( \dfrac{Vea^2}{md_1(d_1 + d_2)} \right)^{1/2}$

23. $1.08 \times 10^{-8}$ cm

24. 2.25 μF

25. (a) $\dfrac{12}{11}$ V (b) $-8$ V (c) zero (d) $-10.3$ V

26. (a) $\dfrac{11}{6}$ μF (b) $\dfrac{11}{4}$ μF (c) 8 μF (d) 8 μF

27. 1 μF

28. 2 μF

29. 4 μF

30. 12.5 V

31. 1 V

32. 5 V

33. 0.16 μC

34. (a) 0.50 μC (b) 10 V

35. (a) 10 V (b) 10 V

36. (a) 1.71 V, 4.29 V (b) 184 pJ, 73.5 pJ

37. 960 μJ

38. 8 mJ in (a) and (d), 2 mJ in (b) and (c)

39. 2.0 J

40. (a) 8 μC, 16 μC (b) 16 μJ, 32 μJ, (c) 96 μJ

41. $\dfrac{Q^2}{8\pi\varepsilon_0 R}$

42. (a) $\pi\varepsilon_0 RV^2$

43. $5.6 \times 10^{-4}$ J

44. (a) $1.06 \times 10^{-10}$ C (b) $12.7 \times 10^{-10}$ J
   (c) $12.7 \times 10^{-10}$ J, $6.35 \times 10^{-10}$ J (d) $6.35 \times 10^{-10}$ J

45. (a) 2400 μC, 1200 μC (b) 1200 μC (c) 14.4 mJ
   (d) 21.6 mJ (e) 7.2 mJ

46. (a) $C\mathcal{E}/2$, (b) $C\mathcal{E}^2/2$ (c) $C\mathcal{E}^2/4$ (d) $C\mathcal{E}^2/4$

47. (a) 1.44 mJ, 0.432 mJ (b) 21.8 μC, 26.2 μC, (c) 1.77 mJ

48. 1.44 mJ

49. 1.42 nF

50. (a) 8.5 nC (b) 6.4 nC (c) 2.1 nC

51. 88 pF

52. 3

53. (a) 30 μC (b) $3 \times 10^3$ V/m (c) 8.3 μF (d) 20 μC

54. 44 pF

55. (a) 7.1 μJ (b) 35.4 μJ

56. (a) $\dfrac{2 K_1 K_2 \varepsilon_0 A}{d(K_1 + K_2)}$ (b) $\dfrac{3\varepsilon_0 A K_1 K_2 K_3}{d(K_1 K_2 + K_2 K_3 + K_3 K_1)}$
   (c) $\dfrac{\varepsilon_0 A}{2 d} (K_1 + K_2)$

57. $\dfrac{\varepsilon_0 K_1 K_2 a^2 \ln \dfrac{K_1}{K_2}}{(K_1 - K_2)d}$

58. 3 : 5

59. $\dfrac{\varepsilon_0 A V^2}{2 d} \left( \dfrac{1}{K} - 1 \right)$

60. (a) 5 mC (b) 20 V (c) 2 mC (d) 3 mC

61. $\dfrac{4\pi\varepsilon_0 Kabc}{Ka(b - c) + b(c - a)}$

62. $\dfrac{4\pi\varepsilon_0 ac}{c - a}$

63. $\dfrac{4\pi\varepsilon_0 Kabc}{Ka(c - b) + c(b - a)}$

64. 0.45 m$^2$

65. $2.5 \times 10^{-7}$ N

66. $\dfrac{\varepsilon_0 b \mathcal{E}^2 (K - 1)}{2 dg}$

67. $\sqrt{\dfrac{K_2 - 1}{K_1 - 1}}$

68. $8\sqrt{\dfrac{(l - a) lmd}{\varepsilon_0 A\mathcal{E}^2(K - 1)}}$

# QUESTIONS FOR SHORT ANSWER

1. Suppose you have three resistors each of value 30 Ω. List all the different resistances you can obtain using them.

2. A proton beam is going from east to west. Is there an electric current? If yes, in what direction?

3. In an electrolyte, the positive ions move from left to right and the negative ions from right to left. Is there a net current? If yes, in what direction?

4. In a TV tube, the electrons are accelerated from the rear to the front. What is the direction of the current?

5. The drift speed is defined as $v_d = \Delta l/\Delta t$ where $\Delta l$ is the distance drifted in a long time $\Delta t$. Why don't we define the drift speed as the limit of $\Delta l/\Delta t$ as $\Delta t \to 0$?

6. One of your friends argues that he has read in previous chapters that there can be no electric field inside a conductor. And hence there can be no current through it. What is the fallacy in this argument?

7. When a current is established in a wire, the free electrons drift in the direction opposite to the current. Does the number of free electrons in the wire continuously decrease?

8. A fan with copper winding in its motor consumes less power as compared to an otherwise similar fan having aluminium winding. Explain.

9. The thermal energy developed in a current-carrying resistor is given by $U = i^2Rt$ and also by $U = Vit$. Should we say that $U$ is proportional to $i^2$ or to $i$?

10. Consider a circuit containing an ideal battery connected to a resistor. Do "work done by the battery" and "the thermal energy developed" represent two names of the same physical quantity?

11. Is work done by a battery always equal to the thermal energy developed in electrical circuits? What happens if a capacitor is connected in the circuit?

12. A nonideal battery is connected to a resistor. Is work done by the battery equal to the thermal energy developed in the resistor? Does your answer change if the battery is ideal?

13. Sometimes it is said that "heat is developed" in a resistance when there is an electric current in it. Recall that heat is defined as the energy being transferred due to the temperature difference. Is the statement under quotes technically correct?

14. We often say "a current is going through the wire". What goes through the wire, the charge or the current?

15. Would you prefer a voltmeter or a potentiometer to measure the emf of a battery?

16. Does a conductor become charged when a current is passed through it?

17. Can the potential difference across a battery be greater than its emf?

# OBJECTIVE I

1. A metallic resistor is connected across a battery. If the number of collisions of the free electrons with the lattice is somehow decreased in the resistor (for example, by cooling it), the current will
   (a) increase                  (b) decrease
   (c) remain constant           (d) become zero.

2. Two resistors $A$ and $B$ have resistances $R_A$ and $R_B$ respectively with $R_A < R_B$. The resistivities of their materials are $\rho_A$ and $\rho_B$.
   (a) $\rho_A > \rho_B$.     (b) $\rho_A = \rho_B$.     (c) $\rho_A < \rho_B$.
   (d) The information is not sufficient to find the relation between $\rho_A$ and $\rho_B$.

3. The product of resistivity and conductivity of a cylindrical conductor depends on
   (a) temperature               (b) material
   (c) area of cross-section     (d) none of these.

4. As the temperature of a metallic resistor is increased, the product of its resistivity and conductivity
   (a) increases                 (b) decreases
   (c) remains constant          (d) may increase or decrease.

5. In an electric circuit containing a battery, the charge (assumed positive) inside the battery
   (a) always goes from the positive terminal to the negative terminal

   (b) may go from the positive terminal to the negative terminal
   (c) always goes from the negative terminal to the positive terminal
   (d) does not move.

6. A resistor of resistance $R$ is connected to an ideal battery. If the value of $R$ is decreased, the power dissipated in the resistor will
   (a) increase     (b) decrease     (c) remain unchanged.

7. A current passes through a resistor. Let $K_1$ and $K_2$ represent the average kinetic energy of the conduction electrons and the metal ions respectively.
   (a) $K_1 < K_2$.               (b) $K_1 = K_2$.
   (c) $K_1 > K_2$.               (d) Any of these three may occur.

8. Two resistors $R$ and $2R$ are connected in series in an electric circuit. The thermal energy developed in $R$ and $2R$ are in the ratio
   (a) 1 : 2     (b) 2 : 1     (c) 1 : 4     (d) 4 : 1.

9. Two resistances $R$ and $2R$ are connected in parallel in an electric circuit. The thermal energy developed in $R$ and $2R$ are in the ratio
   (a) 1 : 2     (b) 2 : 1     (c) 1 : 4     (d) 4 : 1.

10. A uniform wire of resistance 50 Ω is cut into 5 equal parts. These parts are now connected in parallel. The

equivalent resistance of the combination is

(a) 2 Ω     (b) 10 Ω     (c) 250 Ω     (d) 6250 Ω.

11. Consider the following two statements:
   (A) Kirchhoff's junction law follows from conservation of charge.
   (B) Kirchhoff's loop law follows from conservative nature of electric field.
   (a) Both A and B are correct.
   (b) A is correct but B is wrong.
   (c) B is correct but A is wrong.
   (d) Both A and B are wrong.

12. Two nonideal batteries are connected in series. Consider the following statements:
   (A) The equivalent emf is larger than either of the two emfs.
   (B) The equivalent internal resistance is smaller than either of the two internal resistances.
   (a) Each of A and B is correct.
   (b) A is correct but B is wrong.
   (c) B is correct but A is wrong.
   (d) Each of A and B is wrong.

13. Two nonideal batteries are connected in parallel. Consider the following statements:
   (A) The equivalent emf is smaller than either of the two emfs.
   (B) The equivalent internal resistance is smaller than either of the two internal resistances.

(a) Both A and B are correct.
(b) A is correct but B is wrong.
(c) B is correct but A is wrong.
(d) Both A and B are wrong.

14. The net resistance of an ammeter should be small to ensure that
   (a) it does not get overheated
   (b) it does not draw excessive current
   (c) it can measure large currents
   (d) it does not appreciably change the current to be measured.

15. The net resistance of a voltmeter should be large to ensure that
   (a) it does not get overheated
   (b) it does not draw excessive current
   (c) it can measure large potential differences
   (d) it does not appreciably change the potential difference to be measured.

16. Consider a capacitor-charging circuit. Let $Q_1$ be the charge given to the capacitor in a time interval of 10 ms and $Q_2$ be the charge given in the next time interval of 10 ms. Let 10 μC charge be deposited in a time interval $t_1$ and the next 10 μC charge is deposited in the next time interval $t_2$.

(a) $Q_1 > Q_2, t_1 > t_2$.            (b) $Q_1 > Q_2, t_1 < t_2$.
(c) $Q_1 < Q_2, t_1 > t_2$.            (d) $Q_1 < Q_2, t_1 < t_2$.

# OBJECTIVE II

1. Electrons are emitted by a hot filament and are accelerated by an electric field as shown in figure (32-Q1). The two stops at the left ensure that the electron beam has a uniform cross-section.
   (a) The speed of the electron is more at B than at A.
   (b) The electric current is from left to right.
   (c) The magnitude of the current is larger at B than at A.
   (d) The current density is more at B than at A.
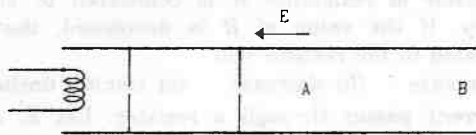


Figure 32-Q1

2. A capacitor with no dielectric is connected to a battery at $t = 0$. Consider a point A in the connecting wires and a point B in between the plates.
   (a) There is no current through A.
   (b) There is no current through B.
   (c) There is a current through A as long as the charging is not complete.
   (d) There is a current through B as long as the charging is not complete.

3. When no current is passed through a conductor,
   (a) the free electrons do not move

(b) the average speed of a free electron over a large period of time is zero
(c) the average velocity of a free electron over a large period of time is zero
(d) the average of the velocities of all the free electrons at an instant is zero.

4. Which of the following quantities do not change when a resistor connected to a battery is heated due to the current?
   (a) drift speed          (b) resistivity
   (c) resistance          (d) number of free electrons.

5. As the temperature of a conductor increases, its resistivity and conductivity change. The ratio of resistivity to conductivity
   (a) increases    (b) decreases    (c) remains constant
   (d) may increase or decrease depending on the actual temperature.

6. A current passes through a wire of nonuniform cross-section. Which of the following quantities are independent of the cross-section?
   (a) the charge crossing in a given time interval
   (b) drift speed
   (c) current density
   (d) free-electron density.

7. Mark out the correct options.
   (a) An ammeter should have small resistance.
   (b) An ammeter should have large resistance.

(c) A voltmeter should have small resistance.

(d) A voltmeter should have large resistance.

8. A capacitor of capacitance $500\,\mu F$ is connected to a battery through a $10\,k\Omega$ resistor. The charge stored on the capacitor in the first 5 s is larger than the charge stored in the next

(a) 5 s     (b) 50 s     (c) 500 s     (d) 500.

9. A capacitor $C_1$ of capacitance $1\,\mu F$ and a capacitor $C_2$ of capacitance $2\,\mu F$ are separately charged by a common battery for a long time. The two capacitors are then separately discharged through equal resistors. Both the discharge circuits are connected at $t = 0$.

(a) The current in each of the two discharging circuits is zero at $t = 0$.

(b) The currents in the two discharging circuits at $t = 0$ are equal but not zero.

(c) The currents in the two discharging circuits at $t = 0$ are unequal.

(d) $C_1$ loses 50% of its initial charge sooner than $C_2$ loses 50% of its initial charge.

## EXERCISES

1. The amount of charge passed in time $t$ through a cross-section of a wire is
$$Q(t) = At^2 + Bt + C.$$
(a) Write the dimensional formulae for $A$, $B$ and $C$.
(b) If the numerical values of $A$, $B$ and $C$ are 5, 3 and 1 respectively in SI units, find the value of the current at $t = 5$ s.

2. An electron gun emits $2.0 \times 10^{16}$ electrons per second. What electric current does this correspond to?

3. The electric current existing in a discharge tube is $2.0\,\mu A$. How much charge is transferred across a cross-section of the tube in 5 minutes?

4. The current through a wire depends on time as
$$i = i_0 + \alpha t,$$
where $i_0 = 10$ A and $\alpha = 4$ A/s. Find the charge crossed through a section of the wire in 10 seconds.

5. A current of $1.0$ A exists in a copper wire of cross-section $1.0\,mm^2$. Assuming one free electron per atom calculate the drift speed of the free electrons in the wire. The density of copper is $9000\,kg/m^3$.

6. A wire of length 1 m and radius $0.1$ mm has a resistance of $100\,\Omega$. Find the resistivity of the material.

7. A uniform wire of resistance $100\,\Omega$ is melted and recast in a wire of length double that of the original. What would be the resistance of the wire?

8. Consider a wire of length 4 m and cross-sectional area 1 $mm^2$ carrying a current of 2 A. If each cubic metre of the material contains $10^{29}$ free electrons, find the average time taken by an electron to cross the length of the wire.

9. What length of a copper wire of cross-sectional area $0.01\,mm^2$ will be needed to prepare a resistance of $1\,k\Omega$? Resistivity of copper $= 1.7 \times 10^{-8}\,\Omega$-m.

10. Figure (32-E1) shows a conductor of length $l$ having a circular cross-section. The radius of cross-section varies linearly from $a$ to $b$. The resistivity of the material is $\rho$. Assuming that $b - a \ll l$, find the resistance of the conductor.
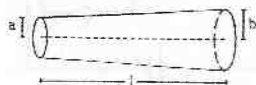


Figure 32-E1

11. A copper wire of radius $0.1$ mm and resistance $1\,k\Omega$ is connected across a power supply of 20 V. (a) How many electrons are transferred per second between the supply and the wire at one end? (b) Write down the current density in the wire.

12. Calculate the electric field in a copper wire of cross-sectional area $2.0\,mm^2$ carrying a current of 1 A. The resistivity of copper $= 1.7 \times 10^{-8}\,\Omega$-m.

13. A wire has a length of $2.0$ m and a resistance of $5.0\,\Omega$. Find the eleric field existing inside the wire if it carries a current of 10 A.

14. The resistances of an iron wire and a copper wire at 20°C are $3.9\,\Omega$ and $4.1\,\Omega$ respectively. At what temperature will the resistances be equal? Temperature coefficient of resistivity for iron is $5.0 \times 10^{-3}\,K^{-1}$ and for copper it is $4.0 \times 10^{-3}\,K^{-1}$. Neglect any thermal expansion.

15. The current in a conductor and the potential difference across its ends are measured by an ammeter and a voltmeter. The meters draw negligible currents. The ammeter is accurate but the voltmeter has a zero error (that is, it does not read zero when no potential difference is applied). Calculate the zero error if the readings for two different conditions are $1.75$ A, $14.4$ V and $2.75$ A, $22.4$ V.

16. Figure (32-E2) shows an arrangement to measure the emf $\mathcal{E}$ and internal resistance $r$ of a battery. The voltmeter has a very high resistance and the ammeter also has some resistance. The voltmeter reads $1.52$ V when the switch $S$ is open. When the switch is closed the voltmeter reading drops to $1.45$ V and the ammeter reads $1.0$ A. Find the emf and the internal resistance of the battery.
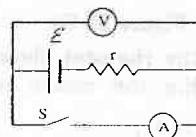


Figure 32-E2

17. The potential difference between the terminals of a battery of emf $6.0$ V and internal resistance $1\,\Omega$ drops to $5.8$ V when connected across an external resistor. Find the resistance of the external resistor.

18. The potential difference between the terminals of a 6·0 V battery is 7·2 V when it is being charged by a current of 2·0 A. What is the internal resistance of the battery ?

19. The internal resistance of an accumulator battery of emf 6 V is 10 Ω when it is fully discharged. As the battery gets charged up, its internal resistance decreases to 1 Ω. The battery in its completely discharged state is connected to a charger which maintains a constant potential difference of 9 V. Find the current through the battery (a) just after the connections are made and (b) after a long time when it is completely charged.

20. Find the value of $i_1/i_2$ in figure (32-E3) if (a) $R = 0·1$ Ω, (b) $R = 1$ Ω (c) $R = 10$ Ω. Note from your answers that in order to get more current from a combination of two batteries they should be joined in parallel if the external resistance is small and in series if the external resistance is large as compared to the internal resistances.
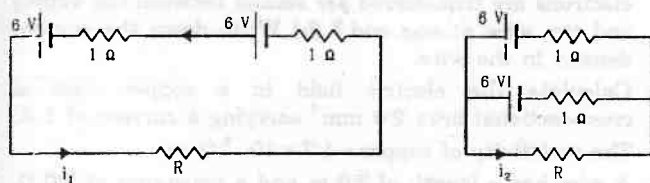


Figure 32-E3

21. Consider $N = n_1 n_2$ identical cells, each of emf $\mathcal{E}$ and internal resistance $r$. Suppose $n_1$ cells are joined in series to form a line and $n_2$ such lines are connected in parallel. The combination drives a current in an external resistance $R$. (a) Find the current in the external resistance. (b) Assuming that $n_1$ and $n_2$ can be continuously varied, find the relation between $n_1$, $n_2$, $R$ and $r$ for which the current in $R$ is maximum.

22. A battery of emf 100 V and a resistor of resistance 10 kΩ are joined in series. This system is used as a source to supply current to an external resistance $R$. If $R$ is not greater than 100 Ω, the current through it is constant upto two significant digits. Find its value. This is the basic principle of a *constant-current source.*

23. If the reading of ammeter $A_1$ in figure (32-E4) is 2·4 A, what will the ammeters $A_2$ and $A_3$ read ? Neglect the resistances of the ammeters.
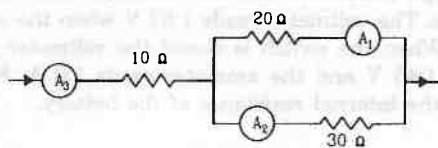


Figure 32-E4

24. The resistance of the rheostat shown in figure (32-E5) is 30 Ω. Neglecting the meter resistance, find the



Figure 32-E5

minimum and maximum currents through the ammeter as the rheostat is varied.
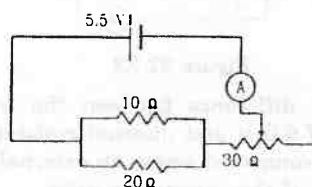
25. Three bulbs, each having a resistance of 180 Ω, are connected in parallel to an ideal battery of emf 60 V. Find the current delivered by the battery when (a) all the bulbs are switched on, (b) two of the bulbs are switched on and (c) only one bulb is switched on.

26. Suppose you have three resistors of 20 Ω, 50 Ω and 100 Ω. What minimum and maximum resistances can you obtain from these resistors ?

27. A bulb is made using two filaments. A switch selects whether the filaments are used individually or in parallel. When used with a 15 V battery, the bulb can be operated at 5 W, 10 W or 15 W. What should be the resistances of the filaments ?

28. Figure (32-E6) shows a part of a circuit. If a current of 12 mA exists in the 5 kΩ resistor, find the currents in the other three resistors. What is the potential difference between the points $A$ and $B$ ?
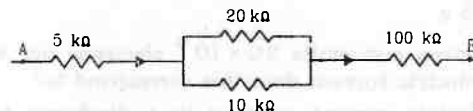


Figure 32-E6

29. An ideal battery sends a current of 5 A in a resistor. When another resistor of value 10 Ω is connected in parallel, the current through the battery is increased to 6 A. Find the resistance of the first resistor.

30. Find the equivalent resistance of the network shown in figure (32-E7) between the points $a$ and $b$.
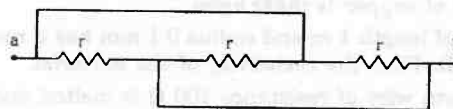


Figure 32-E7

31. A wire of resistance 15·0 Ω is bent to form a regular hexagon $ABCDEFA$. Find the equivalent resistance of the loop between the points (a) $A$ and $B$, (b) $A$ and $C$ and (c) $A$ and $D$.

32. Consider the circuit shown in figure (32-E8). Find the current through the 10 Ω resistor when the switch $S$ is (a) open (b) closed.
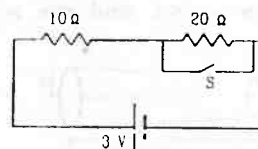


Figure 32-E8

33. Find the currents through the three resistors shown in figure (32-E9).
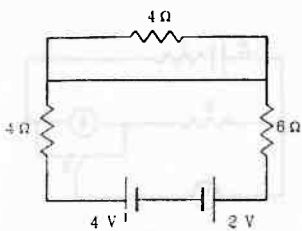


Figure 32-E9

34. Figure (32-E10) shows a part of an electric circuit. The potentials at the points $a$, $b$ and $c$ are 30 V, 12 V and 2 V respectively. Find the currents through the three resistors.
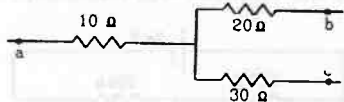


Figure 32-E10

35. Each of the resistors shown in figure (32-E11) has a resistance of 10 Ω and each of the batteries has an emf of 10 V. Find the currents through the resistors $a$ and $b$ in the two circuits.
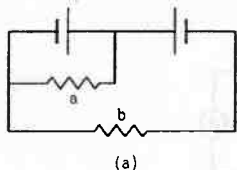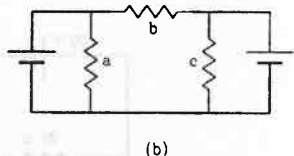


(a)                    (b)

Figure 32-E11

36. Find the potential difference $V_a - V_b$ in the circuits shown in figure (32-E12).
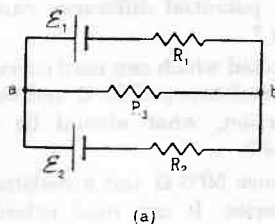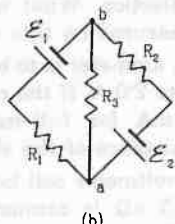


(a)                    (b)

Figure 32-E12

37. In the circuit shown in figure (32-E13), $\mathcal{E}_1 = 3$ V, $\mathcal{E}_2 = 2$ V, $\mathcal{E}_3 = 1$ V and $r_1 = r_2 = r_3 = 1$ Ω. Find the potential difference between the points $A$ and $B$ and the current through each branch.
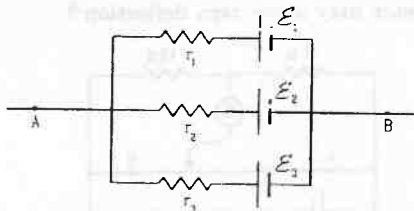


Figure 32-E13

38. Find the current through the 10 Ω resistor shown in figure (32-E14).
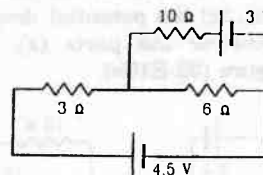


Figure 32-E14

39. Find the current in the three resistors shown in figure (32-E15).
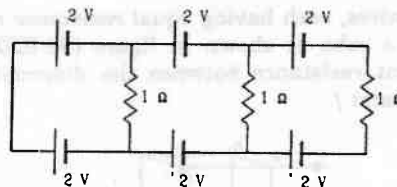


Figure 32-E15

40. What should be the value of $R$ in figure (32-E16) for which the current in it is zero ?
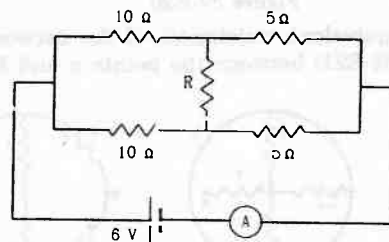


Figure 32-E16

41. Find the equivalent resistance of the circuits shown in figure (32-E17) between the points $a$ and $b$. Each resistor has a resistance $r$.
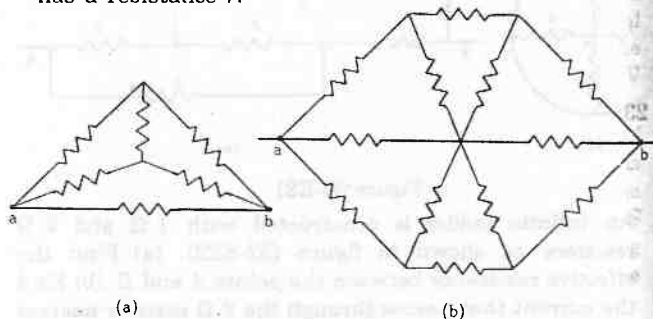


(a)                    (b)

Figure 32-E17

42. Find the current measured by the ammeter in the circuit shown in figure (32-E18).
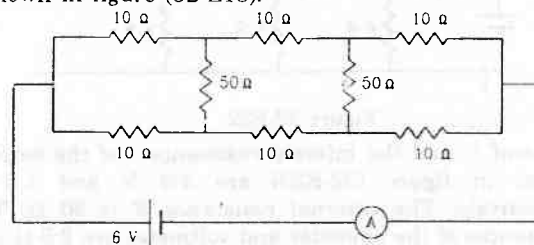


Figure 32-E18

43. Consider the circuit shown in figure (32-E19a). Find (a) the current in the circuit, (b) the potential drop across the 5 Ω resistor, (c) the potential drop across the 10 Ω resistor. (d) Answer the parts (a), (b) and (c) with reference to figure (32-E19b).



(a)                              (b)
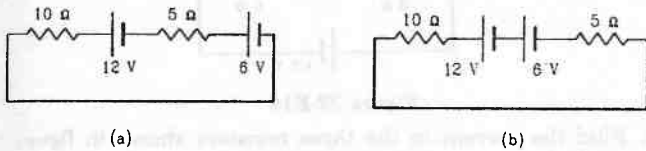
Figure 32-E19

44. Twelve wires, each having equal resistance $r$, are joined to form a cube as shown in figure (32-E20). Find the equivalent resistance between the diagonally opposite points $a$ and $f$.
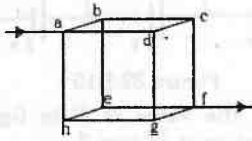


Figure 32-E20

45. Find the equivalent resistances of the networks shown in figure (32-E21) between the points $a$ and $b$.



(a)          (b)          (c)

(d)                    (e)
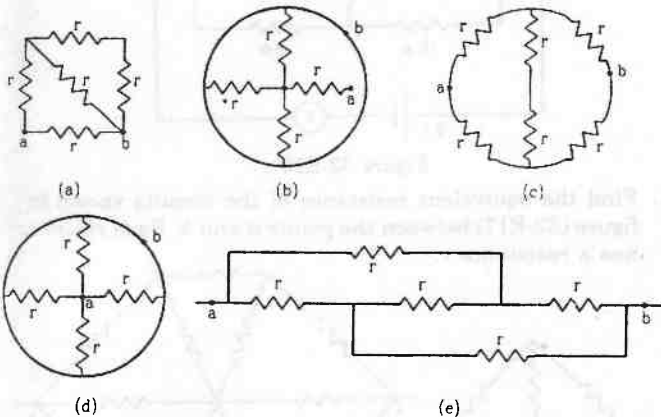
Figure 32-E21

46. An infinite ladder is constructed with 1 Ω and 2 Ω resistors as shown in figure (32-E22). (a) Find the effective resistance between the points $A$ and $B$. (b) Find the current that passes through the 2 Ω resistor nearest to the battery.
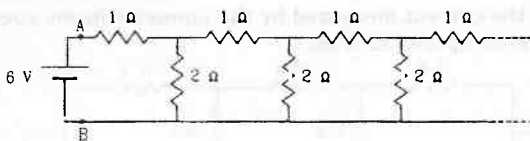


Figure 32-E22

47. The emf $\mathcal{E}$ and the internal resistance $r$ of the battery shown in figure (32-E23) are 4·3 V and 1·0 Ω respectively. The external resistance $R$ is 50 Ω. The resistances of the ammeter and voltmeter are 2·0 Ω and 200 Ω respectively. (a) Find the readings of the two

meters. (b) The switch is thrown to the other side. What will be the readings of the two meters now ?
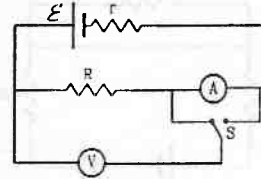


Figure 32-E23

48. A voltmeter of resistance 400 Ω is used to measure the potential difference across the 100 Ω resistor in the circuit shown in figure (32-E24). (a) What will be the reading of the voltmeter ? (b) What was the potential difference across 100 Ω before the voltmeter was connected ?
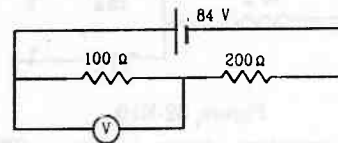


Figure 32-E24

49. The voltmeter shown in figure (32-E25) reads 18 V across the 50 Ω resistor. Find the resistance of the voltmeter.
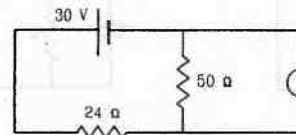


Figure 32-E25

50. A voltmeter consists of a 25 Ω coil connected in series with a 575 Ω resistor. The coil takes 10 mA for full scale deflection. What maximum potential difference can be measured on this voltmeter ?

51. An ammeter is to be constructed which can read currents upto 2·0 A. If the coil has a resistance of 25 Ω and takes 1 mA for full-scale deflection, what should be the resistance of the shunt used ?

52. A voltmeter coil has resistance 50·0 Ω and a resistor of 1·15 kΩ is connected in series. It can read potential differences upto 12 volts. If this same coil is used to construct an ammeter which can measure currents upto 2·0 A, what should be the resistance of the shunt used ?

53. The potentiometer wire $AB$ shown in figure (32-E26) is 40 cm long. Where should the free end of the galvanometer be connected on $AB$ so that the galvanometer may show zero deflection ?
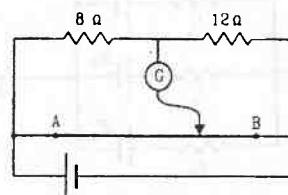


Figure 32-E26

**54.** The potentiometer wire $AB$ shown in figure (32-E27) is 50 cm long. When $AD = 30$ cm, no deflection occurs in the galvanometer. Find $R$.
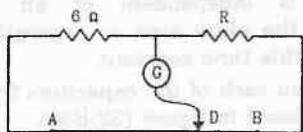


Figure 32-E27

**55.** A 6 volt battery of negligible internal resistance is connected across a uniform wire $AB$ of length 100 cm. The positive terminal of another battery of emf 4 V and internal resistance 1 $\Omega$ is joined to the point $A$ as shown in figure (32-E28). Take the potential at $B$ to be zero. (a) What are the potentials at the points $A$ and $C$ ? (b) At which point $D$ of the wire $AB$, the potential is equal to the potenial at $C$ ? (c) If the points $C$ and $D$ are connected by a wire, what will be the current through it ? (d) If the 4 V battery is replaced by 7·5 V battery, what would be the answers of parts (a) and (b) ?
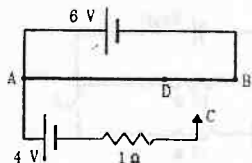


Figure 32-E28

**56.** Consider the potentiometer circuit arranged as in figure (32-E29). The potentiometer wire is 600 cm long. (a) At what distance from the point $A$ should the jockey touch the wire to get zero deflection in the galvanometer ? (b) If the jockey touches the wire at a distance of 560 cm from $A$, what will be the current in the galvanometer ?
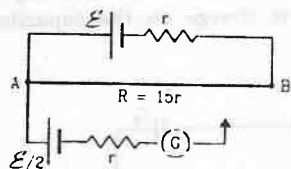


Figure 32-E29

**57.** Find the charge on the capacitor shown in figure (32-E30).
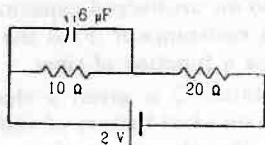


Figure 32-E30

**58.** (a) Find the current in the 20 $\Omega$ resistor shown in figure (32-E31). (b) If a capacitor of capacitance 4 $\mu$F is joined between the points $A$ and $B$, what would be the electrostatic energy stored in it in steady state ?
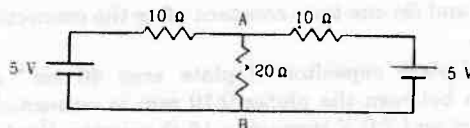


Figure 32-E31

**59.** Find the charges on the four capacitors of capacitances 1 $\mu$F, 2 $\mu$F, 3 $\mu$F and 4 $\mu$F shown in figure (32-E32).
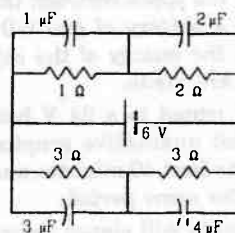


Figure 32-E32

**60.** Find the potential difference between the points $A$ and $B$ and between the points $B$ and $C$ of figure (32-E33) in steady state.
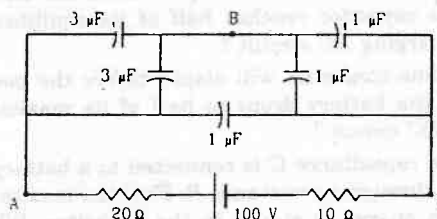


Figure 32-E33

**61.** A capacitance $C$, a resistance $R$ and an emf $\mathcal{E}$ are connected in series at $t = 0$. What is the maximum value of (a) the potential difference across the resistor, (b) the current in the circuit, (c) the potential difference across the capacitor, (d) the energy stored in the capacitor, (e) the power delivered by the battery and (f) the power converted into heat.

**62.** A parallel-plate capacitor with plate area 20 cm$^2$ and plate separation 1·0 mm is connected to a battery. The resistance of the circuit is 10 k$\Omega$. Find the time constant of the circuit.

**63.** A capacitor of capacitance 10 $\mu$F is connected to a battery of emf 2 V. It is found that it takes 50 ms for the charge on the capacitor to become 12·6 $\mu$C. Find the resistance of the circuit.

**64.** A 20 $\mu$F capacitor is joined to a battery of emf 6·0 V through a resistance of 100 $\Omega$. Find the charge on the capacitor 2·0 ms after the connections are made.

**65.** The plates of a capacitor of capacitance 10 $\mu$F, charged to 60 $\mu$C, are joined together by a wire of resistance 10 $\Omega$ at $t = 0$. Find the charge on the capacitor in the circuit at (a) $t = 0$, (b) $t = 30$ $\mu$s, (c) $t = 120$ $\mu$s and (d) $t = 1·0$ ms.

**66.** A capacitor of capacitance 8·0 $\mu$F is connected to a battery of emf 6·0 V through a resistance of 24 $\Omega$. Find

the current in the circuit (a) just after the connections are made and (b) one time constant after the connections are made.

67. A parallel-plate capacitor of plate area 40 cm$^2$ and separation between the plates 0·10 mm is connected to a battery of emf 2·0 V through a 16 Ω resistor. Find the electric field in the capacitor 10 ns after the connections are made.

68. A parallel-plate capacitor has plate area 20 cm$^2$, plate separation 1·0 mm and a dielectric slab of dielectric constant 5·0 filling up the space between the plates. This capacitor is joined to a battery of emf 6·0 V through a 100 kΩ resistor. Find the energy of the capacitor 8·9 μs after the connections are made.

69. A 100 μF capacitor is joined to a 24 V battery through a 1·0 MΩ resistor. Plot qualitative graphs (a) between current and time for the first 10 minutes and (b) between charge and time for the same period.

70. How many time constants will elapse before the current in a charging $RC$ circuit drops to half of its initial value? Answer the same question for a discharging $RC$ circuit.

71. How many time constants will elapse before the charge on a capacitor falls to 0·1% of its maximum value in a discharging $RC$ circuit?

72. How many time constants will elapse before the energy stored in the capacitor reaches half of its equilibrium value in a charging $RC$ circuit?

73. How many time constants will elapse before the power delivered by the battery drops to half of its maximum value in an $RC$ circuit?

74. A capacitor of capacitance $C$ is connected to a battery of emf $\mathcal{E}$ at $t = 0$ through a resistance $R$. Find the maximum rate at which energy is stored in the capacitor. When does the rate has this maximum value?

75. A capacitor of capacitance 12·0 μF is connected to a battery of emf 6·00 V and internal resistance 1 00 Ω through resistanceless leads. 12·0 μs after the connections are made, what will be (a) the current in the circuit, (b) the power delivered by the battery, (c) the power dissipated in heat and (d) the rate at which the energy stored in the capacitor is increasing.

76. A capacitance $C$ charged to a potential difference $V$ is discharged by connecting its plates through a resistance $R$. Find the heat dissipated in one time constant after the connections are made. Do this by calculating $\int i^2 R\, dt$ and also by finding the decrease in the energy stored in the capacitor.

77. By evaluating $\int i^2 R dt$, show that when a capacitor is charged by connecting it to a battery through a resistor, the energy dissipated as heat equals the energy stored in the capacitor.

78. A parallel-plate capacitor is filled with a dielectric material having resistivity ρ and dielectric constant $K$.

The capacitor is charged and disconnected from the charging source. The capacitor is slowly discharged through the dielectric. Show that the time constant of the discharge is independent of all geometrical parameters like the plate area or separation between the plates. Find this time constant.

79. Find the charge on each of the capacitors 0·20 ms after the switch $S$ is closed in figure (32-E34).
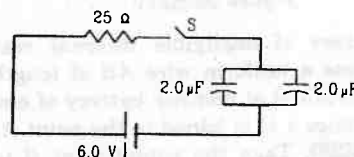


Figure 32-E34

80. The switch $S$ shown in figure (32-E35) is kept closed for a long time and is then opened at $t - 0$. Find the current in the middle 10 Ω resistor at $t = 1·0$ ms.
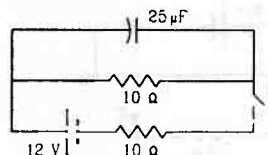


Figure 32-E35

81. A capacitor of capacitance 100 μF is connected across a battery of emf 6·0 V through a resistance of 20 kΩ for 4·0 s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 4·0 s after the battery is disconnected?

82. Consider the situation shown in figure (32-E36). The switch is closed at $t = 0$ when the capacitors are uncharged. Find the charge on the capacitor $C_1$ as a function of time $t$.
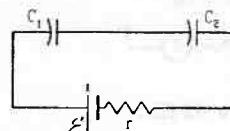


Figure 32-E36

83. A capacitor of capacitance $C$ is given a charge $Q$. At $t = 0$, it is connected to an uncharged capacitor of equal capacitance through a resistance $R$. Find the charge on the second capacitor as a function of time.

84. A capacitor of capacitance $C$ is given a charge $Q$. At $t = 0$, it is connected to an ideal battery of emf $\mathcal{E}$ through a resistance $R$. Find the charge on the capacitor at time $t$.

## ANSWERS

### OBJECTIVE I

1. (a)    2. (d)    3. (d)    4. (c)    5. (b)    6. (a)
7. (c)    8. (a)    9. (b)    10. (a)    11. (a)    12. (b)
13. (c)    14. (d)    15. (d)    16. (b)

### OBJECTIVE II

1. (a)        2. (b), (c)        3. (c), (d)
4. (d)        5. (a)        6. (a), (d)
7. (a), (d)        8. all        9. (b), (d)

### EXERCISES

1. (a) $IT^{-1}$, $I$, $IT$ (b) 53 A

2. $3.2 \times 10^{-3}$ A

3. $6.0 \times 10^{-4}$ C

4. 300 C

5. 0.074 mm/s

6. $\pi \times 10^{-6}$ Ω–m

7. 400 Ω

8. $3.2 \times 10^{4}$ s $\approx$ 8.9 hours

9. 0.6 km

10. $\dfrac{\rho l}{\pi ab}$

11. (a) $1.25 \times 10^{17}$ (b) $6.37 \times 10^{5}$ A/m$^2$

12. 8.5 mV/m

13. 25 V/m

14. 84.5°C

15. 0.4 V

16. 1.52 V, 0.07 Ω

17. 29 Ω

18. 0.6 Ω

19. (a) 0.3 A (b) 3 A

20. (a) 0.57 (b) 1 (c) 1.75

21. (a) $\dfrac{n.\mathcal{E}}{R + \dfrac{n_1 r}{n_2}}$ (b) $rn_1 = Rn_2$

22. 10 mA

23. 1.6 A, 4.0 A

24. 0.15 A, 0.83 A

25. (a) 1.0 A (b) 0.67 A (c) 0.33 A

26. 12.5 Ω, 170 Ω

27. 45 Ω, 22.5 Ω

28. 4 mA in 20 kΩ resistor, 8 mA in 10 kΩ resistor and 12 mA in 100 kΩ resistor, 1340 V

29. 2 Ω

30. $r/3$

31. (a) 2.08 Ω (b) 3.33 Ω (c) 3.75 Ω

32. (a) 0.1 A (b) 0.3 A

33. zero in the upper 4 Ω resistor and 0.2 A in the rest two

34. 1 A through 10 Ω, 0.4 Ω through 20 Ω and 0.6 A through 30 Ω

35. 1 A in $a$ and zero in $b$ in both the circuits

36. (a) $\dfrac{\dfrac{\mathcal{E}_1}{R_1} - \dfrac{\mathcal{E}_2}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$ (b) same as (a)

37. 2 V, $i_1 = 1$ A, $i_2 = 0$, $i_3 = -1$ A

38. zero

39. zero

40. any value of $R$ will do

41. (a) $r/2$ (b) $4\,r/5$

42. 0.4 A

43. (a) 1.2 A (b) 6 V (c) 12 V (d) same as the parts (a), (b) and (c)

44. $\dfrac{5}{6}\,r$

45. (a) $\dfrac{5}{8}\,r$ (b) $\dfrac{4}{3}\,r$ (c) $r$ (d) $\dfrac{r}{4}$ (e) $r$

46. (a) 2 Ω (b) 1.5 A

47. (a) 0.1 A 4.0 V (b) 0.08 A, 4.2 V

48. (a) 24 V (b) 28 V

49. 130 Ω

50. 6 V

51. $1.25 \times 10^{-2}$ Ω

52. 0.251 Ω

53. 16 cm from $A$

54. 4 Ω

55. (a) 6 V, 2 V (b) $AD = 66.7$ cm (c) zero (d) 6 V, $-1.5$ V, no such point $D$ exists.

56. (a) 320 cm (b) $\dfrac{3\mathcal{E}}{22\,r}$

57. 4 μC

58. (a) 0.2 A (b) 32 μJ

59. 2 μC, 8 μC, 9 μC and 12 μC

60. 25 V, 75 V

61. (a) $\mathcal{E}$ (b) $\dfrac{\mathcal{E}}{R}$ (c) $\mathcal{E}$ (d) $\dfrac{1}{2}C\mathcal{E}^2$ (e) $\dfrac{\mathcal{E}^2}{R}$ (f) $\dfrac{\mathcal{E}^2}{R}$

62. 0.18 μs

63. 5 kΩ

64. 76 μC

65. (a) 60 μC (b) 44 μC (c) 18 μC (d) 0.003 μC

66. (a) 0.25 A (b) 0.09 A

67. $1.7 \times 10^{-4}$ V/m

68. $6.3 \times 10^{-10}$ J

70. 0.69 in both cases

71. 6.9

72. 1.23

73. 0·69

74. $\dfrac{\mathcal{E}^2}{4R}$, $CR\ln 2$

75. (a) 2·21 A (b) 13·2 W (c) 4·87 W (d) 8·37 W

76. $\dfrac{1}{2}(1 - 1/e^2)CV^2$

78. $\varepsilon_0 \rho K$

79. 9·2 µC

80. 11 mA

81. 70 µC

82. $q = \mathcal{E}C(1 - e^{-t/rc})$, where $C = \dfrac{C_1 C_2}{C_1 + C_2}$.

83. $\dfrac{Q}{2}(1 - e^{-2t/RC})$

84. $C\mathcal{E}(1 - e^{-t/CR}) + Q\,e^{-t/CR}$

$= (12 \text{ V}) (0.4 \text{ A}) (30 \times 60 \text{ s}) = 8.64 \text{ kJ}.$

8. *A current of* 1 A *is passed through a dilute solution of sulphuric acid for some time to liberate* 1 g *of oxygen. How much hydrogen is liberated during this period? How long was the current passed? Faraday constant* = 96500 C/mole.

*Solution* : The relative atomic mass of oxygen = 16 and its valency = 2 so that the chemical equivalent $E = \dfrac{16}{2} = 8$. Chemical equivalent of hydrogen = 1.

$$\frac{m_{oxygen}}{m_{hydrogen}} = \frac{E_{oxygen}}{E_{hydrogen}} = \frac{8}{1}$$

or, $\quad m_{hydrogen} = \dfrac{m_{oxygen}}{8} = \dfrac{1}{8} = 0.125 \text{ g}.$

We have, 1 g of oxygen = $\dfrac{1}{8}$ gram-equivalent.

The charge needed to liberate $\dfrac{1}{8}$ gram-equivalent

$$= \frac{1}{8} \text{ faraday}$$

$$= \frac{96500}{8} \text{ C} = 1.12 \times 10^4 \text{ C}.$$

As the current is 1 A, the time taken is

$$t = \frac{Q}{i} = \frac{1.2 \times 10^4 \text{ C}}{1 \text{ A}}$$

$$= 1.2 \times 10^4 \text{ s}$$

$$= 3 \text{ hours } 20 \text{ minutes}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. If a constant potential difference is applied across a bulb, the current slightly decreases as time passes and then becomes constant. Explain.

2. Two unequal resistances $R_1$ and $R_2$ are connected across two identical batteries of emf $\mathcal{E}$ and internal resistance $r$ (figure 33-Q1). Can the thermal energies developed in $R_1$ and $R_2$ be equal in a given time. If yes, what will be the condition ?



Figure 33-Q1

3. When a current passes through a resistor, its temperature increases. Is it an adiabatic process ?

4. Apply the first law of thermodynamics to a resistor carrying a current $i$. Identify which of the quantities $\Delta Q$, $\Delta U$ and $\Delta W$ are zero, wich are positive and which are negative.

5. Do all the thermocouples have a neutral temperature ?

6. Is inversion temperature always double of the neutral temperature ? Does the unit of temperature have an effect in deciding this question ?

7. Is neutral temperature always the arithmetic mean of the inversion temperature and the temperature of the cold junction ? Does the unit of temperature have an effect in deciding this question ?

8. Do the electrodes in an electrolytic cell have fixed polarity like a battery ?

9. As temperature increases, the viscosity of liquids decreases considerably. Will this decrease the resistance of an electrolyte as the temperature increases ?

## OBJECTIVE I

1. Which of the following plots may represent the thermal energy produced in a resistor in a given time as a function of the electric current ?



Figure 33-Q2

2. A constant current $i$ is passed through a resistor. Taking the temperature coefficient of resistance into account, indicate which of the plots shown in figure (33-Q3) best
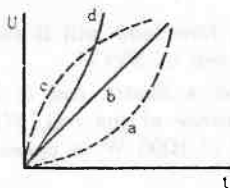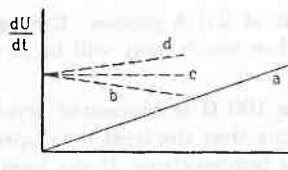


Figure 33-Q3

represents the rate of production of thermal energy in the resistor.

3. Consider the following statements regarding a thermocouple.
   (A) The neutral temperature does not depend on the temperature of the cold junction.
   (B) The inversion temperature does not depend on the temperature of the cold junction.
   (a) Both A and B are correct.
   (b) A is correct but B is wrong.
   (c) B is correct but A is wrong.
   (d) Both A and B are wrong.

4. The heat developed in a system is proportional to the current through it.
   (a) It cannot be Thomson heat.
   (b) It cannot be Peltier heat.
   (c) It cannot be Joule heat.
   (d) It can be any of the three heats mentioned above.

5. Consider the following two statements.
   (A) Free-electron density is different in different metals.
   (B) Free-electron density in a metal depends on temperature.
   Seebeck effect is caused

   (a) due to both A and B
   (b) due to A but not due to B
   (c) due to B but not due to A
   (d) neither due to A nor due to B.

6. Consider the statements A and B in the previous question. Peltier effect is caused
   (a) due to both A and B
   (b) due to A but not due to B
   (c) due to B but not due to A
   (d) neither due to A nor due to B.

7. Consider the statements A and B in question 5. Thomson effect is caused
   (a) due to both A and B
   (b) due to A but not due to B
   (c) due to B but not due to A
   (d) neither due to A nor due to B.

8. Faraday constant
   (a) depends on the amount of the electrolyte
   (b) depends on the current in the electrolyte
   (c) is a universal constant
   (d) depends on the amount of charge passed through the electrolyte.

## OBJECTIVE II

1. Two resistors having equal resistances are joined in series and a current is passed through the combination. Neglect any variation in resistance as the temperature changes. In a given time interval,
   (a) equal amounts of thermal energy must be produced in the resistors
   (b) unequal amounts of thermal energy may be produced
   (c) the temperature must rise equally in the resistors
   (d) the temperature may rise equally in the resistors.

2. A copper strip AB and an iron strip AC are joined at A. The junction A is maintained at 0°C and the free ends B and C are maintained at 100°C. There is a potential difference between
   (a) the two ends of the copper strip
   (b) the copper end and the iron end at the junction
   (c) the two ends of the iron strip
   (d) the free ends B and C.

3. The constants $a$ and $b$ for the pair silver–lead are $2.50 \, \mu V/°C$ and $0.012 \, \mu V/(°C)^2$ respectively. For a silver–lead thermocouple with colder junction at 0°C,

   (a) there will be no neutral temperature
   (b) there will be no inversion temperature
   (c) there will not be any thermo-emf even if the junctions are kept at different temperatures
   (d) there will be no current in the thermocouple even if the junctions are kept at different temperatures.

4. An electrolysis experiment is stopped and the battery terminals are reversed.
   (a) The electrolysis will stop.
   (b) The rate of liberation of material at the electrodes will increase.
   (c) The rate of liberation of material will remain the same.
   (d) Heat will be produced at a greater rate.

5. The electrochemical equivalent of a material depends on
   (a) the nature of the material
   (b) the current through the electrolyte containing the material
   (c) the amount of charge passed through the electrolyte
   (d) the amount of this material present in the electrolyte.

## EXERCISES

1. An electric current of 2.0 A passes through a wire of resistance 25 Ω. How much heat will be developed in 1 minute ?

2. A coil of resistance 100 Ω is connected across a battery of emf 6.0 V. Assume that the heat developed in the coil is used to raise its temperature. If the heat capacity of the coil is 4.0 J/K, how long will it take to raise the temperature of the coil by 15°C ?

3. The specification on a heater coil is 250 V, 500 W. Calculate the resistance of the coil. What will be the resistance of a coil of 1000 W to operate at the same voltage ?

4. A heater coil is to be constructed with a nichrome wire ($\rho = 1 \cdot 0 \times 10^{-6}$ $\Omega$–m) which can operate at 500 W when connected to a 250 V supply. (a) What would be the resistance of the coil ? (b) If the cross-sectional area of the wire is $0 \cdot 5$ mm$^2$, what length of the wire will be needed ? (c) If the radius of each turn is $4 \cdot 0$ mm, how many turns will be there in the coil ?

5. A bulb with rating 250 V, 100 W is connected to a power supply of 220 V situated 10 m away using a copper wire of area of cross-section 5 mm$^2$. How much power will be consumed by the connecting wires ? Resistivity of copper $= 1 \cdot 7 \times 10^{-8}$ $\Omega$–m.

6. An electric bulb, when connected across a power supply of 220 V, consumes a power of 60 W. If the supply drops to 180 V, what will be the power consumed ? If the supply is suddenly increased to 240 V, what will be the power consumed ?

7. A servo voltage stabiliser restricts the voltage output to 220 V ± 1%. If an electric bulb rated at 220 V, 100 W is connected to it, what will be the minimum and maximum power consumed by it ?

8. An electric bulb marked 220 V, 100 W will get fused if it is made to consume 150 W or more. What voltage fluctuation will the bulb withstand ?

9. An immersion heater rated 1000 W, 220 V is used to heat $0 \cdot 01$ m$^3$ of water. Assuming that the power is supplied at 220 V and 60% of the power supplied is used to heat the water, how long will it take to increase the temperature of the water from 15°C to 40°C ?

10. An electric kettle used to prepare tea, takes 2 minutes to boil 4 cups of water (1 cup contains 200 cc of water) if the room temperature is 25°C. (a) If the cost of power consumption is Rs. $1 \cdot 00$ per unit (1 unit = 1000 watt–hour), calculate the cost of boiling 4 cups of water. (b) What will be the corresponding cost if the room temperature drops to 5°C ?

11. The coil of an electric bulb takes 40 watts to start glowing. If more than 40 W is supplied, 60% of the extra power is converted into light and the remaining into heat. The bulb consumes 100 W at 220 V. Find the percentage drop in the light intensity at a point if the supply voltage changes from 220 V to 200 V.

12. The $2 \cdot 0$ $\Omega$ resistor shown in figure (33-E1) is dipped into a calorimeter containing water. The heat capacity of the calorimeter together with water is 2000 J/K. (a) If the circuit is active for 15 minutes, what would be the rise in the temperature of the water ? (b) Suppose the $6 \cdot 0$ $\Omega$ resistor gets burnt. What would be the rise in the temperature of the water in the next 15 minutes ?
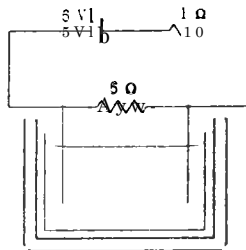


Figure 33-E1

13. The temperatures of the junctions of a bismuth-silver thermocouple are maintained at 0°C and $0 \cdot 001$°C. Find the thermo-emf (Seebeck emf) developed. For bismuth-silver, $a = -46 \times 10^{-6}$ V/deg and $b = -0 \cdot 48 \times 10$ V/deg$^2$.

14. Find the thermo-emf developed in a copper-silver thermocouple when the junctions are kept at 0°C and 40°C. Use the data in table (33.1).

15. Find the neutral temperature and inversion temperature of copper-iron thermocouple if the reference junction is kept at 0°C. Use the data in table (33.1).

16. Find the charge required to flow through an electrolyte to liberate one atom of (a) a monovalent material and (b) a divalent material.

17. Find the amount of silver liberated at cathode if $0 \cdot 500$ A of current is passed through AgNO$_3$ electrolyte for 1 hour. Atomic weight of silver is $107 \cdot 9$ g/mole.

18. An electroplating unit plates $3 \cdot 0$ g of silver on a brass plate in $3 \cdot 0$ minutes. Find the current used by the unit. The electrochemical equivalent of silver is $1 \cdot 12 \times 10^{-6}$ kg/C.

19. Find the time required to liberate 10 litre of hydrogen at STP in an electrolytic cell by a current of $5 \cdot 0$ A.

20. Two voltameters, one having a solution of silver salt and the other of a trivalent-metal salt, are connected in series and a current of 2 A is maintained for $1 \cdot 50$ hours. It is found that $1 \cdot 00$ g of the trivalent-metal is deposited. (a) What is the atomic weight of the trivalent metal ? (b) How much silver is deposited during this period ? Atomic weight of silver is $107 \cdot 9$ g/mole.

21. A brass plate having surface area 200 cm$^2$ on one side is electroplated with $0 \cdot 10$ mm thick silver layers on both sides using a 15 A current. Find the time taken to do the job. The specific gravity of silver is $10 \cdot 5$ and its atomic weight is $107 \cdot 9$ g/mol.

22. Figure (33-E2) shows an electrolyte of AgCl through which a current is passed. It is observed that $2 \cdot 68$ g of silver is deposited in 10 minutes on the cathode. Find the heat developed in the 20 $\Omega$ resistor during this period. Atomic weight of silver is $107 \cdot 9$ g/mole.
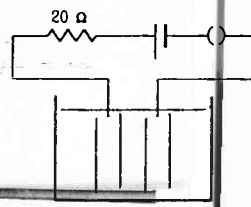


Figure 33-E2

23. The potential difference across the terminals of a battery of emf 12 V and internal resistance 2 $\Omega$ drops to 10 V when it is connected to a silver voltameter. Find the silver deposited at the cathode in half an hour. Atomic weight of silver is $107 \cdot 9$ g/mole.

24. A plate of area 10 cm$^2$ is to be electroplated with copper (density 9000 kg/m$^3$) to a thickness of 10 micrometres on both sides, using a cell of 12 V. Calculate the energy spent by the cell in the process of deposition. If this

energy is used to heat 100 g of water, calculate the rise in the temperature of the water. ECE of copper

$= 3 \times 10^{-7}$ kg/C and specific heat capacity of water $= 4200$ J/kg–K.

## ANSWERS

### OBJECTIVE I

1. (a)    2. (d)    3. (b)    4. (c)    5. (a)    6. (b)

7. (c)    8. (c)

### OBJECTIVE II

1. (a), (d)      2. all      3. (a), (b)

4. (c)      5. (a)

### EXERCISES

1. $6.0 \times 10^3$ J

2. 2.8 min

3. 125 Ω, 62.5 Ω

4. (a) 125 Ω    (b) 62.5 m    (c) ≈ 2500 turns

5. 8.4 mW

6. 40 W, 71 W

7. 98 V², 102 W

8. up to 270 V

9. 29 minutes

10. (a) 7 paisa (b) 9 paisa

11. 29%

12. (a) 2.9°C (b) 3.6°C

13. $-4.6 \times 10^{-8}$ V

14. $1.04 \times 10^{-5}$ V

15. 330°C, 659°C

16. (a) $1.6 \times 10^{-19}$ C (b) $3.2 \times 10^{-19}$ C

17. 2.01 g

18. 15 A

19. 29 minutes

20. (a) 26.8 g/mole (b) 12.1 g

21. 42 minutes

22. 190 kJ

23. 2 g

24. 7.2 kJ, 17 K

# CHAPTER 34
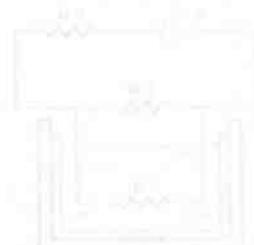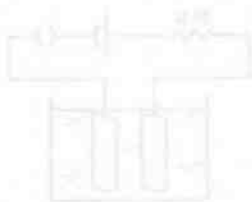
# MAGNETIC FIELD

## 34.1 INTRODUCTION

If a charge $q$ is placed at rest at a point $P$ near a metallic wire carrying a current $i$, it experiences almost no force. We conclude that there is no appreciable electric field at the point $P$. This is expected because in any volume of wire (which contains several thousand atoms) there are equal amounts of positive and negative charges. The wire is electrically neutral and does not produce an electric field.*
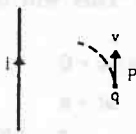


Figure 34.1

However, if the charge $q$ is projected from the point $P$ in the direction of the current (figure 34.1), it is deflected towards the wire ($q$ is assumed positive). There must be a field at $P$ which exerts a force on the charge when it is projected, but not when it is kept at rest. This field is different from the electric field which always exerts a force on a charged particle whether it is at rest or in motion. This new field is called *magnetic field* and is denoted by the symbol $\vec{B}$. The force exerted by a magnetic field is called *magnetic force*.

## 34.2 DEFINITION OF MAGNETIC FIELD $\vec{B}$

If a charged particle is projected in a magnetic field, in general, it experiences a magnetic force. By projecting the particle in different directions from the same point $P$ with different speeds, we can observe the following facts about the magnetic force:

(a) There is one line through the point $P$, such that, if the velocity of the particle is along this line, there is no magnetic force. We define the direction of the magnetic field to be along this line (the direction

is not uniquely defined yet, because there are two opposite directions along any line).

(b) If the speed of the particle is $v$ and it makes an angle $\theta$ with the line identified in (a), i.e., with the direction of the magnetic field, the magnitude of the magnetic force is proportional to $|\, v \sin\theta \,|$.

(c) The direction of the magnetic force is perpendicular to the direction of the magnetic field as well as to the direction of the velocity.

(d) The force is proportional to the magnitude of the charge $q$ and its direction is opposite for positive and negative charges.

All the above facts may be explained if we define the magnetic field by the equation

$$\vec{F} = q\vec{v} \times \vec{B}. \qquad \dots (34.1)$$

By measuring the magnetic force $\vec{F}$ acting on a charge $q$ moving at velocity $\vec{v}$, we can obtain $\vec{B}$. If $\vec{v} \parallel \vec{B}$, the force is zero. By taking magnitudes in equation (34.1), we see that the force is proportional to $|\, v \sin\theta \,|$. By the rules of vector product, the force is perpendicular to both $\vec{B}$ and $\vec{v}$. Also, the observation (d) follows from equation (34.1).

Equation (34.1) uniquely determines the direction of $\vec{B}$ from the rules of vector product. The SI unit of magnetic field is newton/ampere–metre. This is written as tesla and abbreviated as T. Another unit in common use is gauss. The relation between gauss and tesla is $1\,\text{T} = 10^4$ gauss.

The unit weber/m$^2$ is also used for magnetic field and is the same as tesla. Tesla is quite a large unit for many practical applications. We have a magnetic field of the order of $10^{-5}$ T near the earth's surface. Large superconducting magnets are needed to produce a field of the order of 10 T in laboratories.

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* In fact, there is a small charge density on the surface of the wire which does produce an electric field near the wire. This field is very small and we shall neglect it.

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in a direction perpendicular to the loop.

(c) The angular momentum of the electron is $l = mvr$. Its direction is opposite to that of the magnetic moment. Thus,

$$\frac{\mu}{l} = \frac{-evr}{2mvr} = \frac{-e}{2m}.$$

11. *An electron is released from the origin at a place where a uniform electric field E and a uniform magnetic field B exist along the negative Y-axis and the negative Z-axis respectively. Find the displacement of the electron along the Y-axis when its velocity becomes perpendicular to the electric field for the first time.*
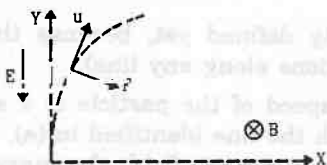


Figure 34-W8

*Solution* : Let us take axes as shown in figure (34-W8). According to the right-handed system, the Z-axis is upward in the figure and hence the magnetic field is shown downwards. At any time, the velocity of the electron may be written as

$$\vec{u} = u_x \vec{i} + u_y \vec{j}.$$

The electric and magnetic fields may be written as

$$\vec{E} = -E\vec{j}$$

and

$$\vec{B} = -B\vec{k}$$

respectively. The force on the electron is

$$\vec{F} = -e(\vec{E} + \vec{u} \times \vec{B})$$
$$= eE\vec{j} + eB(u_y \vec{i} - u_x \vec{j}).$$

Thus,

$$F_x = eu_y B$$

and

$$F_y = e(E - u_x B).$$

The components of the acceleration are

$$a_x = \frac{du_x}{dt} = \frac{eB}{m} u_y \qquad \ldots \text{(i)}$$

and

$$a_y = \frac{du_y}{dt} = \frac{e}{m}(E - u_x B). \qquad \ldots \text{(ii)}$$

We have,

$$\frac{d^2 u_y}{dt^2} = -\frac{eB}{m}\frac{du_x}{dt}$$
$$= -\frac{eB}{m} \cdot \frac{eB}{m} u_y$$
$$= -\omega^2 u_y$$

where

$$\omega = \frac{eB}{m}. \qquad \ldots \text{(iii)}$$

This equation is similar to that for a simple harmonic motion. Thus,

$$u_y = A \sin(\omega t + \delta) \qquad \ldots \text{(iv)}$$

and hence,

$$\frac{du_y}{dt} = A\omega\cos(\omega t + \delta). \qquad \ldots \text{(v)}$$

At $t = 0$, $u_y = 0$ and $\frac{du_y}{dt} = \frac{F}{m} = \frac{eE}{m}$.

Putting in (iv) and (v),

$$\delta = 0 \text{ and } A = \frac{eE}{m\omega} = \frac{E}{B}.$$

Thus,

$$u_y = \frac{E}{B}\sin \omega t.$$

The path of the electron will be perpendicular to the Y-axis when $u_y = 0$. This will be the case for the first time at $t$ where

$$\sin \omega t = 0$$

or,

$$\omega t = \pi$$

or,

$$t = \frac{\pi}{\omega} = \frac{\pi m}{eB}.$$

Also,

$$u_y = \frac{dy}{dt} = \frac{E}{B}\sin \omega t$$

or,

$$\int_0^y dy = \frac{E}{B}\int_0^t \sin \omega t\, dt$$

or,

$$y = \frac{E}{B\omega}(1 - \cos \omega t).$$

At

$$t = \frac{\pi}{\omega},$$

$$y = \frac{E}{B\omega}(1 - \cos \pi) = \frac{2E}{B\omega}.$$

Thus, the displacement along the Y-axis is

$$\frac{2E}{B\omega} = \frac{2Em}{BeB} = \frac{2Em}{eB^2}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. Suppose a charged particle moves with a velocity $v$ near a wire carrying an electric current. A magnetic force, therefore, acts on it. If the same particle is seen from a frame moving with velocity $v$ in the same direction, the charge will be found at rest. Will the magnetic force become zero in this frame ? Will the magnetic field become zero in this frame ?

2. Can a charged particle be accelerated by a magnetic field ? Can its speed be increased ?

3. Will a current loop placed in a magnetic field always experience a zero force ?

4. The free electrons in a conducting wire are in constant thermal motion. If such a wire, carrying no current, is placed in a magnetic field, is there a magnetic force on each free electron ? On the wire ?

5. Assume that the magnetic field is uniform in a cubical region and is zero outside. Can you project a charged particle from outside into the field so that the particle describes a complete circle in the field ?

6. An electron beam projected along the positive X-axis deflects along the positive Y-axis. If this deflection is caused by a magnetic field, what is the direction of the field ? Can we conclude that the field is parallel to the Z-axis ?

7. Is it possible for a current loop to stay without rotating in a uniform magnetic field ? If yes, what should be the orientation of the loop ?

8. The net charge in a current-carrying wire is zero. Then, why does a magnetic field exert a force on it ?

9. The torque on a current loop is zero if the angle between the positive normal and the magnetic field is either $\theta = 0$ or $\theta = 180°$. In which of the two orientations, the equilibrium is stable ?

10. Verify that the units weber and volt–second are the same.

## OBJECTIVE I

1. A positively charged particle projected towards east is deflected towards north by a magnetic field. The field may be
   (a) towards west        (b) towards south
   (c) upward              (d) downward.

2. A charged particle is whirled in a horizontal circle on a frictionless table by attaching it to a string fixed at one point. If a magnetic field is switched on in the vertical direction, the tension in the string
   (a) will increase        (b) will decrease
   (c) will remain the same  (d) may increase or decrease.

3. Which of the following particles will experience maximum magnetic force (magnitude) when projected with the same velocity perpendicular to a magnetic field ?
   (a) electron   (b) proton   (c) He$^{\cdot}$   (d) Li$^{++}$

4. Which of the following particles will describe the smallest circle when projected with the same velocity perpendicular to a magnetic field ?
   (a) electron   (b) proton   (c) He$^{\cdot}$   (d) Li$^{\cdot}$.

5. Which of the following particles will have minimum frequency of revolution when projected with the same velocity perpendicular to a magnetic field ?
   (a) electron   (b) proton   (c) He$^{\cdot}$   (d) Li$^{\cdot}$.

6. A circular loop of area 1 cm$^2$, carrying a current of 10 A, is placed in a magnetic field of 0·1 T perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is
   (a) zero   (b) 10$^{-4}$ N-m   (c) 10$^{-2}$ N-m   (d) 1 N-m.

7. A beam consisting of protons and electrons moving at the same speed goes through a thin region in which there is a magnetic field perpendicular to the beam. The protons and the electrons
   (a) will go undeviated
   (b) will be deviated by the same angle and will not separate
   (c) will be deviated by different angles and hence separate
   (d) will be deviated by the same angle but will separate.

8. A charged particle moves in a uniform magnetic field. The velocity of the particle at some instant makes an acute angle with the magnetic field. The path of the particle will be
   (a) a straight line        (b) a circle
   (c) a helix with uniform pitch
   (d) a helix with nonuniform pitch.

9. A particle moves in a region having a uniform magnetic field and a parallel, uniform electric field. At some instant, the velocity of the particle is perpendicular to the field direction. The path of the particle will be
   (a) a straight line        (b) a circle
   (c) a helix with uniform pitch
   (d) a helix with nonuniform pitch.

10. An electric current $i$ enters and leaves a uniform circular wire of radius $a$ through diametrically opposite points. A charged particle $q$ moving along the axis of the circular wire passes through its centre at speed $v$. The magnetic force acting on the particle when it passes through the centre has a magnitude
    (a) $qv \dfrac{\mu_0 i}{2a}$   (b) $qv \dfrac{\mu_0 i}{2\pi a}$   (c) $qv \dfrac{\mu_0 i}{a}$   (d) zero.

## OBJECTIVE II

1. If a charged particle at rest experiences no electromagnetic force,
   (a) the electric field must be zero
   (b) the magnetic field must be zero
   (c) the electric field may or may not be zero
   (d) the magnetic field may or may not be zero.

2. If a charged particle kept at rest experiences an electromagnetic force,
   (a) the electric field must not be zero
   (b) the magnetic field must not be zero
   (c) the electric field may or may not be zero
   (d) the magnetic field may or may not be zero.

3. If a charged particle projected in a gravity-free room deflects,
   (a) there must be an electric field
   (b) there must be a magnetic field
   (c) both fields cannot be zero
   (d) both fields can be nonzero.

4. A charged particle moves in a gravity-free space without change in velocity. Which of the following is/are possible ?
   (a) $E = 0, B = 0$        (b) $E = 0, B \neq 0$
   (c) $E \neq 0, B = 0$     (d) $E \neq 0, B \neq 0$.

5. A charged particle moves along a circle under the action of possible constant electric and magnetic fields. Which of the following are possible ?
   (a) $E = 0, B = 0$        (b) $E = 0, B \neq 0$
   (c) $E \neq 0, B = 0$     (d) $E \neq 0, B \neq 0$.

6. A charged particle goes undeflected in a region containing electric and magnetic field. It is possible that
   (a) $\vec{E} \parallel \vec{B}, \vec{v} \parallel \vec{E}$        (b) $\vec{E}$ is not parallel to $\vec{B}$
   (c) $\vec{v} \parallel \vec{B}$ but $\vec{E}$ is not parallel to $\vec{B}$
   (d) $\vec{E} \parallel \vec{B}$ but $\vec{v}$ is not parallel to $\vec{E}$.

7. If a charged particle goes unaccelerated in a region containing electric and magnetic fields,

8. (a) $\vec{E}$ must be perpendicular to $\vec{B}$
   (b) $\vec{v}$ must be perpendicular to $\vec{E}$
   (c) $\vec{v}$ must be perpendicular to $\vec{B}$
   (d) $E$ must be equal to $vB$.

8. Two ions have equal masses but one is singly-ionized and the other is doubly-ionized. They are projected from the same place in a uniform magnetic field with the same velocity perpendicular to the field.
   (a) Both ions will go along circles of equal radii.
   (b) The circle described by the singly-ionized charge will have a radius double that of the other circle.
   (c) The two circles do not touch each other.
   (d) The two circles touch each other.

9. An electron is moving along the positive X-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative X-axis. This can be done by applying the magnetic field along
   (a) Y-axis   (b) Z-axis   (c) Y-axis only   (d) Z-axis only.

10. Let $\vec{E}$ and $\vec{B}$ denote electric and magnetic fields in a frame $S$ and $\vec{E}'$ and $\vec{B}'$ in another frame $S'$ moving with respect to $S$ at a velocity $\vec{v}$. Two of the following equations are wrong. Identify them.
    (a) $B_y' = B_y + \dfrac{vE_z}{c^2}$        (b) $E_y' = E_y - \dfrac{vB_z}{c^2}$
    (c) $B_y' = B_y + vE_z$                     (d) $E_y' = E_y + vB_z$.

## EXERCISES

1. An alpha particle is projected vertically upward with a speed of $3.0 \times 10^4$ km/s in a region where a magnetic field of magnitude $1.0$ T exists in the direction south to north. Find the magnetic force that acts on the $\alpha$-particle.

2. An electron is projected horizontally with a kinetic energy of 10 keV. A magnetic field of strength $1.0 \times 10^{-7}$ T exists in the vertically upward direction. (a) Will the electron deflect towards right or towards left of its motion ? (b) Calculate the sideways deflection of the electron in travelling through 1 m. Make appropriate approximations.

3. A magnetic field of $(4.0 \times 10^{-3} \vec{k})$ T exerts a force of $(4.0 \vec{i} + 3.0 \vec{j}) \times 10^{-10}$ N on a particle having a charge of $1.0 \times 10^{-9}$ C and going in the X–Y plane. Find the velocity of the particle.

4. An experimenter's diary reads as follows: "a charged particle is projected in a magnetic field of $(7.0 \vec{i} - 3.0 \vec{j}) \times 10^{-3}$ T. The acceleration of the particle is found to be $(\square \vec{i} + 7.0 \vec{j}) \times 10^{-10}$ m/s$^2$. The number to the left of $\vec{i}$ in the last expression was not readable. What can this number be ?

5. A 10 g bullet having a charge of $4.00$ μC is fired at a speed of 270 m/s in a horizontal direction. A vertical magnetic field of 500 μT exists in the space. Find the deflection of the bullet due to the magnetic field as it travels through 100 m. Make appropriate approximations.

6. When a proton is released from rest in a room, it starts with an initial acceleration $a_0$ towards west. When it is projected towards north with a speed $v_0$, it moves with an initial acceleration $3a_0$ towards west. Find the electric field and the maximum possible magnetic field in the room.

7. Consider a 10 cm long portion of a straight wire carrying a current of 10 A placed in a magnetic field of $0.1$ T making an angle of 53° with the wire. What magnetic force does the wire experience ?

8. A current of 2 A enters at the corner $d$ of a square frame $abcd$ of side 20 cm and leaves at the opposite corner $b$. A magnetic field $B = 0.1$ T exists in the space in a direction perpendicular to the plane of the frame as shown in figure
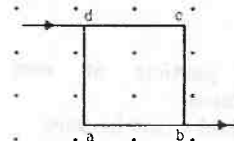


Figure 34-E1

(34-E1). Find the magnitude and direction of the magnetic forces on the four sides of the frame.

9. A magnetic field of strength 1·0 T is produced by a strong electromagnet in a cylindrical region of radius 4·0 cm as shown in figure (34-E2). A wire, carrying a current of 2·0 A, is placed perpendicular to and intersecting the axis of the cylindrical region. Find the magnitude of the force acting on the wire.
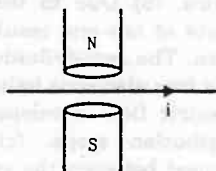


Figure 34-E2

10. A wire of length $l$ carries a current $i$ along the $X$-axis. A magnetic field exists which is given as $\vec{B} = B_0(\vec{i} + \vec{j} + \vec{k})$ T. Find the magnitude of the magnetic force acting on the wire.

11. A current of 5·0 A exists in the circuit shown in figure (34-E3). The wire $PQ$ has a length of 50 cm and the magnetic field in which it is immersed has a magnitude of 0·20 T. Find the magnetic force acting on the wire $PQ$.
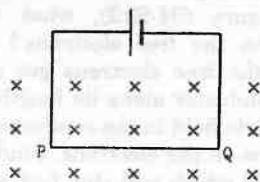


Figure 34-E3

12. A circular loop of radius $a$, carrying a current $i$, is placed in a two-dimensional magnetic field. The centre of the loop coincides with the centre of the field (figure 34-E4). The strength of the magnetic field at the periphery of the loop is $B$. Find the magnetic force on the wire.
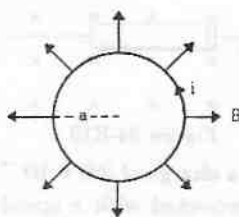


Figure 34-E4

13. A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0 \vec{e}_r$, where $\vec{e}_r$ denotes the unit vector along the radial direction. A circular loop of radius $a$, carrying a current $i$, is placed with its plane parallel to the $X-Y$ plane and the centre at $(0, 0, d)$. Find the magnitude of the magnetic force acting on the loop.

14. A rectangular wire-loop of width $a$ is suspended from the insulated pan of a spring balance as shown in figure (34-E5). A current $i$ exists in the anticlockwise direction in the loop. A magnetic field $B$ exists in the lower region. Find the change in the tension of the spring if the current in the loop is reversed.
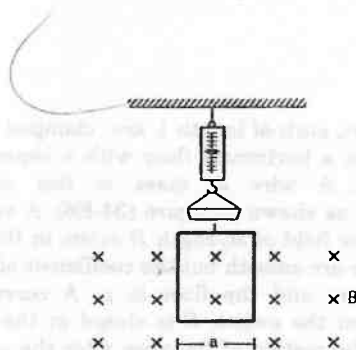


figure 34-E5

15. A current loop of arbitrary shape lies in a uniform magnetic field $B$. Show that the net magnetic force acting on the loop is zero.

16. Prove that the force acting on a current-carrying wire, joining two fixed points $a$ and $b$ in a uniform magnetic field, is independent of the shape of the wire.

17. A semicircular wire of radius 5·0 cm carries a current of 5·0 A. A magnetic field $B$ of magnitude 0·50 T exists along the perpendicular to the plane of the wire. Find the magnitude of the magnetic force acting on the wire.

18. A wire, carrying a current $i$, is kept in the $X-Y$ plane along the curve $y = A \sin\left(\dfrac{2\pi}{\lambda} x\right)$. A magnetic field $B$ exists in the $z$-direction. Find the magnitude of the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda$.

19. A rigid wire consists of a semicircular portion of radius $R$ and two straight sections (figure 34-E6). The wire is partially immersed in a perpendicular magnetic field $B$ as shown in the figure. Find the magnetic force on the wire if it carries a current $i$.
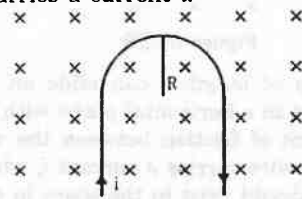


Figure 34-E6

20. A straight, horizontal wire of mass 10 mg and length 1·0 m carries a current of 2·0 A. What minimum magnetic field $B$ should be applied in the region so that the magnetic force on the wire may balance its weight ?

21. Figure (34-E7) shows a rod $PQ$ of length 20·0 cm and mass 200 g suspended through a fixed point $O$ by two threads of lengths 20·0 cm each. A magnetic field of strength 0·500 T exists in the vicinity of the wire $PQ$ as shown in the figure. The wires connecting $PQ$ with the battery are loose and exert no force on $PQ$. (a) Find the tension in the threads when the switch $S$ is open. (b) A current of 2·0 A is established when the switch $S$ is closed. Find the tension in the threads now.
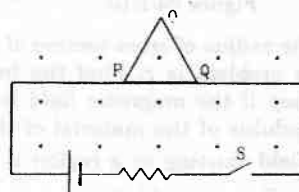


Figure 34-E7

22. Two metal strips, each of length $l$, are clamped parallel to each other on a horizontal floor with a separation $b$ between them. A wire of mass $m$ lies on them perpendicularly as shown in figure (34-E8). A vertically upward magnetic field of strength $B$ exists in the space. The metal strips are smooth but the coefficient of friction between the wire and the floor is $\mu$. A current $i$ is established when the switch $S$ is closed at the instant $t = 0$. Discuss the motion of the wire after the switch is closed. How far away from the strips will the wire reach ?
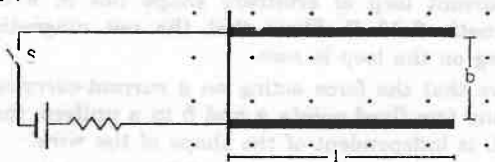


Figure 34-E8

23. A metal wire $PQ$ of mass 10 g lies at rest on two horizontal metal rails separated by 4·90 cm (figure 34·E9). A vertically downward magnetic field of magnitude 0·800 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20·0 $\Omega$, the wire $PQ$ starts sliding on the rails. Find the coefficient of friction.
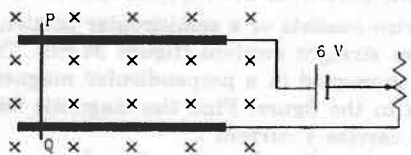


Figure 34-E9

24. A straight wire of length $l$ can slide on two parallel plastic rails kept in a horizontal plane with a separation $d$. The coefficient of friction between the wire and the rails is $\mu$. If the wire carries a current $i$, what minimum magnetic field should exist in the space in order to slide the wire on the rails.

25. Figure (34-E10) shows a circular wire-loop of radius $a$, carrying a current $i$, placed in a perpendicular magnetic field $B$. (a) Consider a small part $dl$ of the wire. Find the force on this part of the wire exerted by the magnetic field. (b) Find the force of compression in the wire.
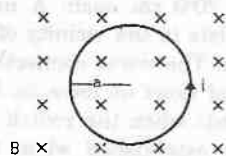


Figure 34-E10

26. Suppose that the radius of cross-section of the wire used in the previous problem is $r$. Find the increase in the radius of the loop if the magnetic field is switched off. The Young's modulus of the material of the wire is $Y$.

27. The magnetic field existing in a region is given by

$$\vec{B} = B_o\left(1 + \frac{x}{l}\right)\vec{k}.$$

A square loop of edge $l$ and carrying a current $i$, is placed with its edges parallel to the $X$-$Y$ axes. Find the magnitude of the net magnetic force experienced by the loop.

28. A conducting wire of length $l$, lying normal to a magnetic field $B$, moves with a velocity $v$ as shown in figure (34-E11). (a) Find the average magnetic force on a free electron of the wire. (b) Due to this magnetic force, electrons concentrate at one end resulting in an electric field inside the wire. The redistribution stops when the electric force on the free electrons balances the magnetic force. Find the electric field developed inside the wire when the redistribution stops. (c) What potential difference is developed between the ends of the wire ?
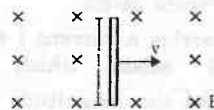


Figure 34-E11

29. A current $i$ is passed through a silver strip of width $d$ and area of cross-section $A$. The number of free electrons per unit volume is $n$. (a) Find the drift velocity $v$ of the electrons. (b) If a magnetic field $B$ exists in the region as shown in figure (34-E12), what is the average magnetic force on the free electrons ? (c) Due to the magnetic force, the free electrons get accumulated on one side of the conductor along its length. This produces a transverse electric field in the conductor which opposes the magnetic force on the electrons. Find the magnitude of the electric field which will stop further accumulation of electrons. (d) What will be the potential difference developed across the width of the conductor due to the electron-accumulation ? The appearance of a transverse emf, when a current-carrying wire is placed in a magnetic field, is called *Hall effect*.
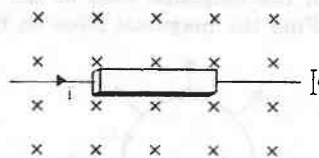


Figure 34-E12

30. A particle having a charge of $2·0 \times 10^{-8}$ C and a mass of $2·0 \times 10^{-10}$ g is projected with a speed of $2·0 \times 10^3$ m/s in a region having a uniform magnetic field of 0·10 T. The velocity is perpendicular to the field. Find the radius of the circle formed by the particle and also the time period.

31. A proton describes a circle of radius 1 cm in a magnetic field of strength 0·10 T. What would be the radius of the circle described by an $\alpha$-particle moving with the same speed in the same magnetic field ?

32. An electron having a kinetic energy of 100 eV circulates in a path of radius 10 cm in a magnetic field. Find the magnetic field and the number of revolutions per second made by the electron.

33. Protons having kinetic energy $K$ emerge from an accelerator as a narrow beam. The beam is bent by a perpendicular magnetic field so that it just misses a

plane target kept at a distance $l$ in front of the accelerator. Find the magnetic field.

34. A charged particle is accelerated through a potential difference of 12 kV and acquires a speed of $1.0 \times 10^{6}$ m/s. It is then injected perpendicularly into a magnetic field of strength 0.2 T. Find the radius of the circle described by it.

35. Doubly-ionized helium ions are projected with a speed of 10 km/s in a direction perpendicular to a uniform magnetic field of magnitude 1.0 T. Find (a) the force acting on an ion, (b) the radius of the circle in which it circulates and (c) the time taken by an ion to complete the circle.

36. A proton is projected with a velocity of $3 \times 10^{6}$ m/s perpendicular to a uniform magnetic field of 0.6 T. Find the acceleration of the proton.

37. (a) An electron moves along a circle of radius 1 m in a perpendicular magnetic field of strength 0.50 T. What would be its speed? Is it reasonable? (b) If a proton moves along a circle of the same radius in the same magnetic field, what would be its speed?

38. A particle of mass $m$ and positive charge $q$, moving with a uniform velocity $v$, enters a magnetic field $B$ as shown in figure (34-E13). (a) Find the radius of the circular arc it describes in the magnetic field. (b) Find the angle subtended by the arc at the centre. (c) How long does the particle stay inside the magnetic field? (d) Solve the three parts of the above problem if the charge $q$ on the particle is negative.
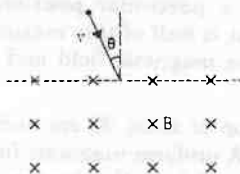


Figure 34-E13

39. A particle of mass $m$ and charge $q$ is projected into a region having a perpendicular magnetic field $B$. Find the angle of deviation (figure 34-E14) of the particle as it comes out of the magnetic field if the width $d$ of the region is very slightly smaller than

(a) $\dfrac{mv}{qB}$     (b) $\dfrac{mv}{2qB}$     (c) $\dfrac{2mv}{qB}$.
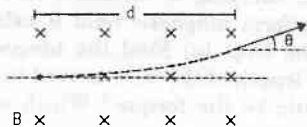


Figure 34-E14

40. A narrow beam of singly-charged carbon ions, moving at a constant velocity of $6.0 \times 10^{4}$ m/s, is sent perpendicularly in a rectangular region having uniform magnetic field $B = 0.5$ T (figure 34-E15). It is found that two beams emerge from the field in the backward direction, the separations from the incident beam being 3.0 cm and 3.5 cm. Identify the isotopes present in the ion beam. Take the mass of an ion = $A(1.6 \times 10^{-27})$ kg, where $A$ is the mass number.
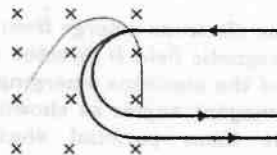


Figure 34-E15

41. $Fe^{+}$ ions are accelerated through a potential difference of 500 V and are injected normally into a homogeneous magnetic field $B$ of strength 20.0 mT. Find the radius of the circular paths followed by the isotopes with mass numbers 57 and 58. Take the mass of an ion = $A (1.6 \times 10^{-27})$ kg where $A$ is the mass number.

42. A narrow beam of singly-charged potassium ions of kinetic energy 32 keV is injected into a region of width 1.00 cm having a magnetic field of strength 0.500 T as shown in figure (34-E16). The ions are collected at a screen 95.5 cm away from the field region. If the beam contains isotopes of atomic weights 39 and 41, find the separation between the points where these isotopes strike the screen. Take the mass of a potassium ion = $A (1.6 \times 10^{-27})$ kg where $A$ is the mass number.
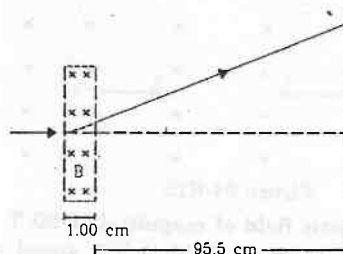


Figure 34-E16

43. Figure (34-E17) shows a convex lens of focal length 12 cm lying in a uniform magnetic field $B$ of magnitude 1.2 T parallel to its principal axis. A particle having a charge $2.0 \times 10^{-3}$ C and mass $2.0 \times 10^{-5}$ kg is projected perpendicular to the plane of the diagram with a speed of 4.8 m/s. The particle moves along a circle with its centre on the principal axis at a distance of 18 cm from the lens. Show that the image of the particle goes along a circle and find the radius of that circle.
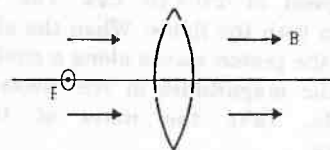


Figure 34-E17

44. Electrons emitted with negligible speed from an electron gun are accelerated through a potential difference $V$
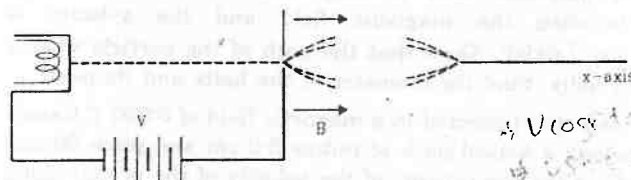


Figure 34-E18

along the $X$-axis. These electrons emerge from a narrow hole into a uniform magnetic field $B$ directed along this axis. However, some of the electrons emerging from the hole make slightly divergent angles as shown in figure (34-E18). Show that these paraxial electrons are refocused on the $X$-axis at a distance

$$\sqrt{\frac{8\pi^2 mV}{eB^2}}$$

45. Two particles, each having a mass $m$ are placed at a separation $d$ in a uniform magnetic field $B$ as shown in figure (34-E19). They have opposite charges of equal magnitude $q$. At time $t = 0$, the particles are projected towards each other, each with a speed $v$. Suppose the Coulomb force between the charges is switched off. (a) Find the maximum value $v_m$ of the projection speed so that the two particles do not collide. (b) What would be the minimum and maximum separation between the particles if $v = v_m/2$ ? (c) At what instant will a collision occur between the particles if $v = 2v_m$ ? (d) Suppose $v = 2v_m$ and the collision between the particles is completely inelastic. Describe the motion after the collision.
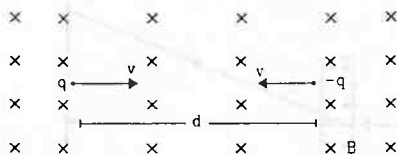


Figure 34-E19

46. A uniform magnetic field of magnitude 0·20 T exists in space from east to west. With what speed should a particle of mass 0·010 g and having a charge $1·0 \times 10^{-5}$ C be projected from south to north so that it moves with a uniform velocity ?

47. A particle moves in a circle of diameter 1·0 cm under the action of a magnetic field of 0·40 T. An electric field of 200 V/m makes the path straight. Find the charge/mass ratio of the particle.

48. A proton goes undeflected in a crossed electric and magnetic field (the fields are perpendicular to each other) at a speed of $2·0 \times 10^5$ m/s. The velocity is perpendicular to both the fields. When the electric field is switched off, the proton moves along a circle of radius 4·0 cm. Find the magnitudes of the electric and the magnetic fields. Take the mass of the proton $= 1·6 \times 10^{-27}$ kg.

49. A particle having a charge of 5·0 µC and a mass of $5·0 \times 10^{-12}$ kg is projected with a speed of 1·0 km/s in a magnetic field of magnitude 5·0 mT. The angle between the magnetic field and the velocity is $\sin^{-1}(0·90)$. Show that the path of the particle will be a helix. Find the diameter of the helix and its pitch.

50. A proton projected in a magnetic field of 0·020 T travels along a helical path of radius 5·0 cm and pitch 20 cm. Find the components of the velocity of the proton along and perpendicular to the magnetic field. Take the mass of the proton $= 1·6 \times 10^{-27}$ kg.

51. A particle having mass $m$ and charge $q$ is released from the origin in a region in which electric field and magnetic field are given by

$$\vec{B} = -B_0\,\vec{j} \quad \text{and} \quad \vec{E} = E_0\,\vec{k}.$$

Find the speed of the particle as a function of its $z$-coordinate.

52. An electron is emitted with negligible speed from the negative plate of a parallel plate capacitor charged to a potential difference $V$. The separation between the plates is $d$ and a magnetic field $B$ exists in the space as shown in figure (34-E20). Show that the electron will fail to strike the upper plate if

$$d > \left(\frac{2m_e V}{eB_0^2}\right)^{\frac{1}{2}}.$$
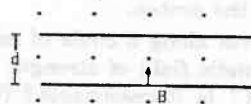


Figure 34-E20

53. A rectangular coil of 100 turns has length 5 cm and width 4 cm. It is placed with its plane parallel to a uniform magnetic field and a current of 2 A is sent through the coil. Find the magnitude of the magnetic field $B$, if the torque acting on the coil is 0·2 N-m.

54. A 50-turn circular coil of radius 2·0 cm carrying a current of 5·0 A is rotated in a magnetic field of strength 0·20 T. (a) What is the maximum torque that acts on the coil ? (b) In a particular position of the coil, the torque acting on it is half of this maximum. What is the angle between the magnetic field and the plane of the coil ?

55. A rectangular loop of sides 20 cm and 10 cm carries a current of 5·0 A. A uniform magnetic field of magnitude 0·20 T exists parallel to the longer side of the loop. (a) What is the force acting on the loop ? (b) What is the torque acting on the loop ?

56. A circular coil of radius 2·0 cm has 500 turns in it and carries a current of 1·0 A. Its axis makes an angle of 30° with the uniform magnetic field of magnitude 0·40 T that exists in the space. Find the torque acting on the coil.

57. A circular loop carrying a current $i$ has wire of total length $L$. A uniform magnetic field $B$ exists parallel to the plane of the loop. (a) Find the torque on the loop. (b) If the same length of the wire is used to form a square loop, what would be the torque ? Which is larger ?

58. A square coil of edge $l$ having $n$ turns carries a current $i$. It is kept on a smooth horizontal plate. A uniform magnetic field $B$ exists in a direction parallel to an edge. The total mass of the coil is $M$. What should be the minimum value of $B$ for which the coil will start tipping over ?

59. Consider a nonconducting ring of radius $r$ and mass $m$ which has a total charge $q$ distributed uniformly on it. The ring is rotated about its axis with an angular speed $\omega$. (a) Find the equivalent electric current in the ring. (b) Find the magnetic moment $\mu$ of the ring. (c) Show

that $\mu = \dfrac{q}{2m} l$ where $l$ is the angular momentum of the ring about its axis of rotation.

60. Consider a nonconducting plate of radius $r$ and mass $m$ which has a charge $q$ distributed uniformly over it. The plate is rotated about its axis with an angular speed $\omega$. Show that the magnetic moment $\mu$ and the angular momentum $l$ of the plate are related as $\mu = \dfrac{q}{2m} l$.

61. Consider a solid sphere of radius $r$ and mass $m$ which has a charge $q$ distributed uniformly over its volume. The sphere is rotated about a diameter with an angular speed $\omega$. Show that the magnetic moment $\mu$ and the angular momentum $l$ of the sphere are related as $\mu = \dfrac{q}{2m} l$.

□

## ANSWERS

### OBJECTIVE I

1. (d)  2. (d)  3. (d)  4. (a)  5. (d)  6. (a)
7. (c)  8. (c)  9. (d)  10. (d)

### OBJECTIVE II

1. (a), (d)  2. (a), (d)  3. (c), (d)
4. (a), (b), (d)  5. (b)  6. (a), (b)
7. (a), (b)  8. (b), (d)  9. (a), (b)
10. (b), (c)

### EXERCISES

1. $9.6 \times 10^{-14}$ N towards west
2. (a) left (b) $\approx 1.5$ cm
3. $(-75\,\vec{i} + 100\,\vec{j})$ m/s
4. $3.0$
5. $3.7 \times 10^{-6}$ m
6. $\dfrac{m a_0}{e}$ towards west, $\dfrac{2 m a_t}{e v_0}$ downward
7. $0.08$ N perpendicular to both the wire and the field
8. $0.02$ N on each wire, on $da$ and $cb$ towards left and on $dc$ and $ab$ downward
9. $0.16$ N
10. $\sqrt{2}\,B_0 il$
11. $0.50$ N towards the inside of the circuit
12. $2\pi a i B$, perpendicular to the plane of the figure going into it
13. $\dfrac{2\pi a^2 i B_0}{\sqrt{a^2 + d^2}}$
14. $2iBa$
17. $0.25$ N
18. $i\lambda B$
19. $2iRB$, upward in the figure
20. $4.9 \times 10^{-5}$ T
21. (a) $1.13$ N (b) $1.25$ N

22. $\dfrac{ilbB}{\mu mg}$
23. $0.12$
24. $\dfrac{\mu mg}{il}$
25. (a) $idlB$ towards the centre  (b) $iaB$
26. $\dfrac{ia^2 B}{\pi r^2 Y}$
27. $iB_0 l$
28. (a) $evB$  (b) $vB$  (c) $lBv$
29. (a) $\dfrac{i}{Ane}$  (b) $\dfrac{iB}{An}$ upwards in the figure
    (c) $\dfrac{iB}{Ane}$  (d) $\dfrac{iBd}{Ane}$
30. $20$ cm, $6.3 \times 10^{-4}$ s
31. $2$ cm
32. $3.4 \times 10^{-4}$ T, $9.4 \times 10^{6}$
33. $\dfrac{\sqrt{2 m_p K}}{\epsilon l}$ where $m_p$ = mass of a proton
34. $12$ cm
35. (a) $3.2 \times 10^{-15}$ N (b) $2.1 \times 10^{-4}$ m (c) $1.31 \times 10^{-7}$ s
36. $1.72 \times 10^{14}$ m/s$^2$
37. (a) $8.8 \times 10^{10}$ m/s (b) $4.8 \times 10^{7}$ m/s
38. (a) $\dfrac{mv}{qB}$  (b) $\pi - 2\theta$
    (c) $\dfrac{m}{qB}(\pi - 2\theta)$  (d) $\dfrac{mv}{qB}$, $\pi + 2\theta$, $\dfrac{m}{qB}(\pi + 2\theta)$
39. (a) $\pi/2$  (b) $\pi/6$  (c) $\pi$
40. $^{12}$C and $^{14}$C
41. $119$ cm and $120$ cm
42. $0.75$ mm
43. $8$ cm
45. (a) $\dfrac{qBd}{2m}$ (b) $\dfrac{d}{2}$, $\dfrac{3d}{2}$ (c) $\dfrac{\pi m}{6qB}$ (d) the particles stick together and the combined mass moves with constant speed $v_m$ along the straight line drawn upward in the plane of figure through the point of collision

46. 49 m/s

47. $2.5 \times 10^5$ C/kg

48. $1.0 \times 10^4$ N/C, $0.05$ T

49. 36 cm, 55 cm

50. $6.4 \times 10^4$ m/s, $1.0 \times 10^5$

51. $\pm\sqrt{\dfrac{2qE_0 z}{m}}$

53. $0.5$ T

54. (a) $6.3 \times 10^{-2}$ N–m    (b) $60°$

55. (a) zero (b) $0.02$ N–m parallel to the shorter side.

56. $0.13$ N–m

57. (a) $\dfrac{iL^2 B}{4\pi}$    (b) $\dfrac{iL^2 B}{16}$

58. $\dfrac{Mg}{2nil}$

59. (a) $\dfrac{q\omega}{2\pi}$    (b) $\dfrac{q\omega r^2}{2}$

□

# QUESTIONS FOR SHORT ANSWER

1. An electric current flows in a wire from north to south. What will be the direction of the magnetic field due to this wire at a point east of the wire ? West of the wire ? Vertically above the wire ? Vertically below the wire ?

2. The magnetic field due to a long straight wire has been derived in terms of $\mu_0$, $i$ and $d$. Express this in terms of $\varepsilon_0$, $c$, $i$ and $d$.

3. You are facing a circular wire carrying an electric current. The current is clockwise as seen by you. Is the field at the centre coming towards you or going away from you ?

4. In Ampere's law $\oint \vec{B} \cdot \vec{dl} = \mu_0 i$, the current outside the curve is not included on the right hand side. Does it mean that the magnetic field $B$ calculated by using Ampere's law, gives the contribution of only the currents crossing the area bounded by the curve ?

5. The magnetic field inside a tightly wound, long solenoid is $B = \mu_0 ni$. It suggests that the field does not depend on the total length of the solenoid, and hence if we add more loops at the ends of a solenoid the field should not increase. Explain qualitatively why the extra-added loops do not have a considerable effect on the field inside the solenoid.

6. A long, straight wire carries a current. Is Ampere's law valid for a loop that does not enclose the wire ? That encloses the wire but is not circular ?

7. A straight wire carrying an electric current is placed along the axis of a uniformly charged ring. Will there be a magnetic force on the wire if the ring starts rotating about the wire ? If yes, in which direction ?

8. Two wires carrying equal currents $i$ each, are placed perpendicular to each other, just avoiding a contact. If one wire is held fixed and the other is free to move under magnetic forces, what kind of motion will result ?

9. Two proton beams going in the same direction repel each other whereas two wires carrying currents in the same direction attract each other. Explain.

10. In order to have a current in a long wire, it should be connected to a battery or some such device. Can we obtain the magnetic field due to a straight, long wire by using Ampere's law without mentioning this other part of the circuit ?

11. Quite often, connecting wires carrying currents in opposite directions are twisted together in using electrical appliances. Explain how it avoids unwanted magnetic fields.

12. Two current-carrying wires may attract each other. In absence of other forces, the wires will move towards each other increasing the kinetic energy. Does it contradict the fact that the magnetic force cannot do any work and hence cannot increase the kinetic energy ?

# OBJECTIVE I

1. A vertical wire carries a current in upward direction. An electron beam sent horizontally towards the wire will be deflected
   (a) towards right          (b) towards left
   (c) upwards                (d) downwards.

2. A current-carrying, straight wire is kept along the axis of a circular loop carrying a current. The straight wire
   (a) will exert an inward force on the circular loop
   (b) will exert an outward force on the circular loop
   (c) will not exert any force on the circular loop
   (d) will exert a force on the circular loop parallel to itself.

3. A proton beam is going from north to south and an electron beam is going from south to north. Neglecting the earth's magnetic field, the electron beam will be deflected
   (a) towards the proton beam
   (b) away from the proton beam
   (c) upwards               (d) downwards.

4. A circular loop is kept in that vertical plane which contains the north-south direction. It carries a current that is towards north at the topmost point. Let $A$ be a point on the axis of the circle to the east of it and $B$ a point on this axis to the west of it. The magnetic field due to the loop

(a) is towards east at $A$ and towards west at $B$
(b) is towards west at $A$ and towards east at $B$
(c) is towards east at both $A$ and $B$
(d) is towards west at both $A$ and $B$.

5. Consider the situation shown in figure (35-Q1). The straight wire is fixed but the loop can move under magnetic force. The loop will
   (a) remain stationary
   (b) move towards the wire
   (c) move away from the wire
   (d) rotate about the wire.
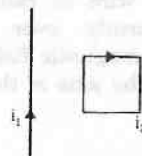


Figure 35-Q1

6. A charged particle is moved along a magnetic field line. The magnetic force on the particle is
   (a) along its velocity        (b) opposite to its velocity
   (c) perpendicular to its velocity   (d) zero.

7. A moving charge produces
   (a) electric field only       (b) magnetic field only
   (c) both of them             (d) none of them.

8. A particle is projected in a plane perpendicular to a uniform magnetic field. The area bounded by the path described by the particle is proportional to
(a) the velocity  (b) the momentum
(c) the kinetic energy  (d) none of these.

9. Two particles $X$ and $Y$ having equal charge, after being accelerated through the same potential difference enter a region of uniform magnetic field and describe circular paths of radii $R_1$ and $R_2$ respectively. The ratio of the mass of $X$ to that of $Y$ is
(a) $(R_1/R_2)^{1/2}$  (b) $R_1/R_2$  (c) $(R_1/R_2)^2$  (d) $R_1R_2$.

10. Two parallel wires carry currents of 20 A and 40 A in opposite directions. Another wire carrying a current antiparallel to 20 A is placed midway between the two wires. The magnetic force on it will be
(a) towards 20 A  (b) towards 40 A  (c) zero
(d) perpendicular to the plane of the currents.

11. Two parallel, long wires carry currents $i_1$ and $i_2$ with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of $i_2$ is reversed, the field becomes 30 μT. The ratio $i_1/i_2$ is
(a) 4  (b) 3  (c) 2  (d) 1.

12. Consider a long, straight wire of cross-sectional area $A$ carrying a current $i$. Let there be $n$ free electrons per unit volume. An observer places himself on a trolley moving in the direction opposite to the current with a speed $v = \dfrac{i}{nAe}$ and separated from the wire by a distance $r$. The magnetic field seen by the observer is very nearly
(a) $\dfrac{\mu_0 i}{2\pi r}$  (b) zero  (c) $\dfrac{\mu_0 i}{\pi r}$  (d) $\dfrac{2\mu_0 i}{\pi r}$.

## OBJECTIVE II

1. The magnetic field at the origin due to a current element $i\,d\vec{l}$ placed at a position $\vec{r}$ is
(a) $\dfrac{\mu_0 i}{4\pi}\dfrac{d\vec{l} \times \vec{r}}{r^3}$  (b) $-\dfrac{\mu_0 i}{4\pi}\dfrac{\vec{r} \times d\vec{l}}{r^3}$
(c) $\dfrac{\mu_0 i}{4\pi}\dfrac{\vec{r} \times d\vec{l}}{r^3}$  (d) $-\dfrac{\mu_0 i}{4\pi}\dfrac{d\vec{l} \times \vec{r}}{r^3}$.

2. Consider three quantities $x = E/B$, $y = \sqrt{1/\mu_0\varepsilon_0}$ and $z = \dfrac{l}{CR}$. Here, $l$ is the length of a wire, $C$ is a capacitance and $R$ is a resistance. All other symbols have standard meanings.
(a) $x, y$ have the same dimensions.
(b) $y, z$ have the same dimensions.
(c) $z, x$ have the same dimensions.
(d) None of the three pairs have the same dimensions.

3. A long, straight wire carries a current along the $Z$-axis. One can find two points in the $X$–$Y$ plane such that
(a) the magnetic fields are equal
(b) the directions of the magnetic fields are the same
(c) the magnitudes of the magnetic fields are equal
(d) the field at one point is opposite to that at the other point.

4. A long, straight wire of radius $R$ carries a current distributed uniformly over its cross-section. The magnitude of the magnetic field is
(a) maximum at the axis of the wire

(b) minimum at the axis of the wire
(c) maximum at the surface of the wire
(d) minimum at the surface of the wire.

5. A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field
(a) increases linearly from the axis to the surface
(b) is constant inside the tube
(c) is zero at the axis
(d) is zero just outside the tube.

6. In a coaxial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero
(a) outside the cable
(b) inside the inner conductor
(c) inside the outer conductor
(d) in between the two conductors.

7. A steady electric current is flowing through a cylindrical conductor.
(a) The electric field at the axis of the conductor is zero.
(b) The magnetic field at the axis of the conductor is zero.
(c) The electric field in the vicinity of the conductor is zero.
(d) The magnetic field in the vicinity of the conductor is zero.

## EXERCISES

1. Using the formulae $\vec{F} = q\vec{v} \times \vec{B}$ and $B = \dfrac{\mu_0 i}{2\pi r}$, show that the SI units of the magnetic field B and the permeability constant $\mu_0$ may be written as N/A–m and N/A$^2$ respectively.

2. A current of 10 A is established in a long wire along the positive $Z$-axis. Find the magnetic field $\vec{B}$ at the point (1 m, 0, 0).

3. A copper wire of diameter 1.6 mm carries a current of 20 A. Find the maximum magnitude of the magnetic field $\vec{B}$ due to this current.

4. A transmission wire carries a current of 100 A. What would be the magnetic field $B$ at a point on the road if the wire is 8 m above the road?

5. A long, straight wire carrying a current of 1.0 A is placed horizontally in a uniform magnetic field $B = 1.0 \times 10^{-5}$ T pointing vertically upward (figure 35-E1). Find the magnitude of the resultant magnetic field at the points $P$ and $Q$, both situated at a distance of 2.0 cm from the wire in the same horizontal plane.
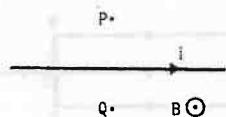


**Figure 35-E1**

6. A long, straight wire of radius $r$ carries a current $i$ and is placed horizontally in a uniform magnetic field $B$ pointing vertically upward. The current is uniformly distributed over its cross-section. (a) At what points will the resultant magnetic field have maximum magnitude? What will be the maximum magnitude? (b) What will be the minimum magnitude of the resultant magnetic field?

7. A long, straight wire carrying a current of 30 A is placed in an external, uniform magnetic field of $4.0 \times 10^{-4}$ T parallel to the current. Find the magnitude of the resultant magnetic field at a point 2.0 cm away from the wire.

8. A long, vertical wire carrying a current of 10 A in the upward direction is placed in a region where a horizontal magnetic field of magnitude $2.0 \times 10^{-3}$ T exists from south to north. Find the point where the resultant magnetic field is zero.

9. Figure (35-E2) shows two parallel wires separated by a distance of 4.0 cm and carrying equal currents of 10 A along opposite directions. Find the magnitude of the magnetic field $B$ at the points $A_1$, $A_2$, $A_3$ and $A_4$.
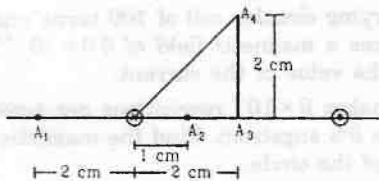


**Figure 35-E2**

10. Two parallel wires carry equal currents of 10 A along the same direction and are separated by a distance of 2.0 cm. Find the magnetic field at a point which is 2.0 cm away from each of these wires.

11. Two long, straight wires, each carrying a current of 5 A, are placed along the X- and Y-axes respectively. The currents point along the positive directions of the axes. Find the magnetic fields at the points (a) (1 m, 1 m), (b) (– 1 m, 1 m), (c) (– 1 m, – 1 m) and (d) ( 1 m, – 1 m).

12. Four long, straight wires, each carrying a current of 5.0 A, are placed in a plane as shown in figure (35-E3).

The points of intersection form a square of side 5.0 cm. (a) Find the magnetic field at the centre $P$ of the square. (b) $Q_1$, $Q_2$, $Q_3$ and $Q_4$ are points situated on the diagonals of the square and at a distance from $P$ that is equal to the length of the diagonal of the square. Find the magnetic fields at these points.
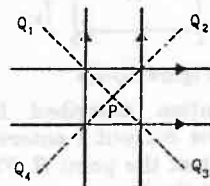


**Figure 35-E3**

13. Figure (35-E4) shows a long wire bent at the middle to form a right angle. Show that the magnitudes of the magnetic fields at the points $P$, $Q$, $R$ and $S$ are equal and find this magnitude.
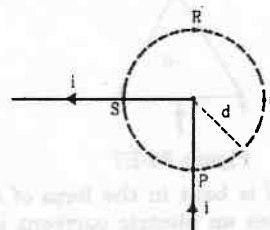


**Figure 35-E4**

14. Consider a straight piece of length $x$ of a wire carrying a current $i$. Let $P$ be a point on the perpendicular bisector of the piece, situated at a distance $d$ from its middle point. Show that for $d \gg x$, the magnetic field at $P$ varies as $1/d^2$ whereas for $d \ll x$, it varies as $1/d$.

15. Consider a 10 cm long piece of a wire which carries a current of 10 A. Find the magnitude of the magnetic field due to the piece at a point which makes an equilateral triangle with the ends of the piece.

16. A long, straight wire carries a current $i$. Let $B_1$ be the magnetic field at a point $P$ at a distance $d$ from the wire. Consider a section of length $l$ of this wire such that the point $P$ lies on a perpendicular bisector of the section. Let $B_2$ be the magnetic field at this point due to this section only. Find the value of $d/l$ so that $B_2$ differs from $B_1$ by 1%.

17. Figure (35-E5) shows a square loop $ABCD$ with edge-length $a$. The resistance of the wire $ABC$ is $r$ and that of $ADC$ is $2r$. Find the magnetic field $B$ at the centre of the loop assuming uniform wires.
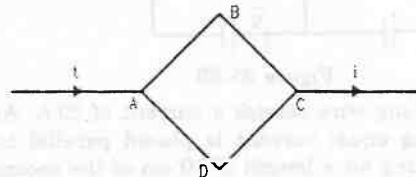


**Figure 35-E5**

18. Figure (35-E6) shows a square loop of edge $a$ made of a uniform wire. A current $i$ enters the loop at the point $A$ and leaves it at the point $C$. Find the magnetic field at

the point $P$ which is on the perpendicular bisector of $AB$ at a distance $a/4$ from it.
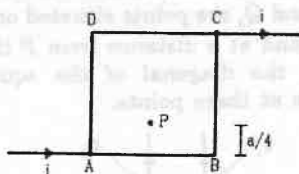


Figure 35-E6

19. Consider the situation described in the previous problem. Suppose the current $i$ enters the loop at the point $A$ and leaves it at the point $B$. Find the magnetic field at the centre of the loop.

20. The wire $ABC$ shown in figure (35-E7) forms an equilateral triangle. Find the magnetic field $B$ at the centre $O$ of the triangle assuming the wire to be uniform.
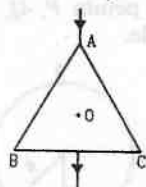


Figure 35-E7

21. A wire of length $l$ is bent in the form of an equilateral triangle and carries an electric current $i$. (a) Find the magnetic field $B$ at the centre. (b) If the wire is bent in the form of a square, what would be the value of $B$ at the centre?

22. A long wire carrying a current $i$ is bent to form a plane angle $\alpha$. Find the magnetic field $B$ at a point on the bisector of this angle situated at a distance $x$ from the vertex.

23. Find the magnetic field $B$ at the centre of a rectangular loop of length $l$ and width $b$, carrying a current $i$.

24. A regular polygon of $n$ sides is formed by bending a wire of total length $2\pi r$ which carries a current $i$. (a) Find the magnetic field $B$ at the centre of the polygon. (b) By letting $n \to \infty$, deduce the expression for the magnetic field at the centre of a circular current.

25. Each of the batteries shown in figure (35-E8) has an emf equal to 5 V. Show that the magnetic field $B$ at the point $P$ is zero for any set of values of the resistances.
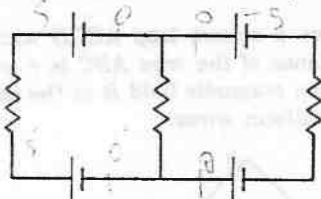


Figure 35-E8

26. A straight, long wire carries a current of 20 A. Another wire carrying equal current is placed parallel to it. If the force acting on a length of 10 cm of the second wire is $2.0 \times 10^{-5}$ N, what is the separation between them?

27. Three coplanar parallel wires, each carrying a current of 10 A along the same direction, are placed with a separation 5.0 cm between the consecutive ones. Find

the magnitude of the magnetic force per unit length acting on the wires.

28. Two parallel wires separated by a distance of 10 cm carry currents of 10 A and 40 A along the same direction. Where should a third current be placed so that it experiences no magnetic force?

29. Figure (35-E9) shows a part of an electric circuit. The wires $AB$, $CD$ and $EF$ are long and have identical resistances. The separation between the neighbouring wires is 1.0 cm. The wires $AE$ and $BF$ have negligible resistance and the ammeter reads 30 A. Calculate the magnetic force per unit length of $AB$ and $CD$.
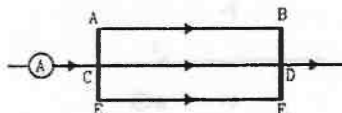


Figure 35-E9

30. A long, straight wire is fixed horizontally and carries a current of 50.0 A. A second wire having linear mass density $1.0 \times 10^{-4}$ kg/m is placed parallel to and directly above this wire at a separation of 5.0 mm. What current should this second wire carry such that the magnetic repulsion can balance its weight?

31. A square loop $PQRS$ carrying a current of 6.0 A is placed near a long wire carrying 10 A as shown in figure (35-E10). (a) Show that the magnetic force acting on the part $PQ$ is equal and opposite to that on the part $RS$. (b) Find the magnetic force on the square loop.
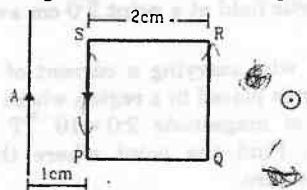


Figure 35-E10

32. A circular loop of one turn carries a current of 5.00 A. If the magnetic field $B$ at the centre is 0.200 mT, find the radius of the loop.

33. A current-carrying circular coil of 100 turns and radius 5.0 cm produces a magnetic field of $6.0 \times 10^{-5}$ T at its centre. Find the value of the current.

34. An electron makes $3 \times 10^5$ revolutions per second in a circle of radius 0.5 angstrom. Find the magnetic field $B$ at the centre of the circle.

35. A conducting circular loop of radius $a$ is connected to two long, straight wires. The straight wires carry a current $i$ as shown in figure (35-E11). Find the magnetic field $B$ at the centre of the loop.
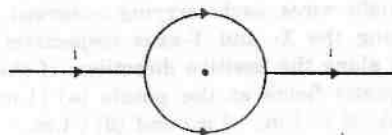


Figure 35-E11

36. Two circular coils of radii 5·0 cm and 10 cm carry equal currents of 2·0 A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as the centres coincide. Find the magnitude of the magnetic field $B$ at the common centre of the coils if the currents in the coils are (a) in the same sense (b) in the opposite sense.

37. If the outer coil of the previous problem is rotated through 90° about a diameter, what would be the magnitude of the magnetic field $B$ at the centre ?

38. A circular loop of radius 20 cm carries a current of 10 A. An electron crosses the plane of the loop with a speed of $2·0 \times 10^{6}$ m/s. The direction of motion makes an angle of 30° with the axis of the circle and passes through its centre. Find the magnitude of the magnetic force on the electron at the instant it crosses the plane.

39. A circular loop of radius $R$ carries a current $I$. Another circular loop of radius $r (<< R)$ carries a current $i$ and is placed at the centre of the larger loop. The planes of the two circles are at right angle to each other. Find the torque acting on the smaller loop.

40. A circular loop of radius $r$ carrying a current $i$ is held at the centre of another circular loop of radius $R(>> r)$ carrying a current $I$. The plane of the smaller loop makes an angle of 30° with that of the larger loop. If the smaller loop is held fixed in this position by applying a single force at a point on its periphery, what would be the minimum magnitude of this force ?

41. Find the magnetic field $B$ due to a semicircular wire of radius 10·0 cm carrying a current of 5·0 A at its centre of curvature.

42. A piece of wire carrying a current of 6·00 A is bent in the form of a circular arc of radius 10·0 cm, and it subtends an angle of 120° at the centre. Find the magnetic field $B$ due to this piece of wire at the centre.

43. A circular loop of radius $r$ carries a current $i$. How should a long, straight wire carrying a current $4i$ be placed in the plane of the circle so that the magnetic field at the centre becomes zero ?

44. A circular coil of 200 turns has a radius of 10 cm and carries a current of 2·0 A. (a) Find the magnitude of the magnetic field $\vec{B}$ at the centre of the coil. (b) At what distance from the centre along the axis of the coil will the field $B$ drop to half its value at the centre ? $(\sqrt[3]{4} = 1·5874 ...)$

45. A circular loop of radius 4·0 cm is placed in a horizontal plane and carries an electric current of 5·0 A in the clockwise direction as seen from above. Find the magnetic field (a) at a point 3·0 cm above the centre of the loop (b) at a point 3·0 cm below the centre of the loop.

46. A charge of $3·14 \times 10^{-6}$ C is distributed uniformly over a circular ring of radius 20·0 cm. The ring rotates about its axis with an angular velocity of 60·0 rad/s. Find the ratio of the electric field to the magnetic field at a point on the axis at a distance of 5·00 cm from the centre.

47. A thin but long, hollow, cylindrical tube of radius $r$ carries a current $i$ along its length. Find the magnitude of the magnetic field at a distance $r/2$ from the surface (a) inside the tube (b) outside the tube.

48. A long, cylindrical tube of inner and outer radii $a$ and $b$ carries a current $i$ distributed uniformly over its cross-section. Find the magnitude of the magnetic field at a point (a) just inside the tube (b) just outside the tube.

49. A long, cylindrical wire of radius $b$ carries a current $i$ distributed uniformly over its cross-section. Find the magnitude of the magnetic field at a point inside the wire at a distance $a$ from the axis.

50. A solid wire of radius 10 cm carries a current of 5·0 A distributed uniformly over its cross-section. Find the magnetic field $B$ at a point at a distance (a) 2 cm (b) 10 cm and (c) 20 cm away from the axis. Sketch a graph of $B$ versus $x$ for $0 < x < 20$ cm.

51. Sometimes we show an idealised magnetic field which is uniform in a given region and falls to zero abruptly. One such field is represented in figure (35-E12). Using Ampere's law over the path $PQRS$, show that such a field is not possible.
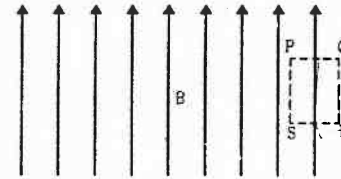


Figure 35-E12

52. Two large metal sheets carry surface currents as shown in figure (35-E13). The current through a strip of width $dl$ is $Kdl$ where $K$ is a constant. Find the magnetic field at the points $P$, $Q$ and $R$.
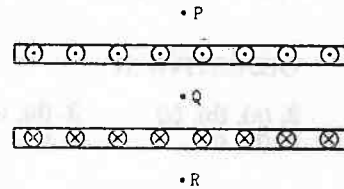


Figure 35-E13

53. Consider the situation of the previous problem. A particle having charge $q$ and mass $m$ is projected from the point $Q$ in a direction going into the plane of the diagram. It is found to describe a circle of radius $r$ between the two plates. Find the speed of the charged particle.

54. The magnetic field $B$ inside a long solenoid, carrying a current of 5·00 A, is $3·14 \times 10^{-2}$ T. Find the number of turns per unit length of the solenoid.

55. A long solenoid is fabricated by closely winding a wire of radius 0·5 mm over a cylindrical nonmagnetic frame so that the successive turns nearly touch each other. What would be the magnetic field $B$ at the centre of the solenoid if it carries a current of 5 A ?

56. A copper wire having resistance 0·01 ohm in each metre is used to wind a 400-turn solenoid of radius 1·0 cm and length 20 cm. Find the emf of a battery which when

connected across the solenoid will cause a magnetic field of $1.0 \times 10^{-2}$ T near the centre of the solenoid.

57. A tightly-wound solenoid of radius $a$ and length $l$ has $n$ turns per unit length. It carries an electric current $i$. Consider a length $dx$ of the solenoid at a distance $x$ from one end. This contains $n\,dx$ turns and may be approximated as a circular current $i\,n\,dx$. (a) Write the magnetic field at the centre of the solenoid due to this circular current. Integrate this expression under proper limits to find the magnetic field at the centre of the solenoid. (b) Verify that if $l \gg a$, the field tends to $B = \mu_0 ni$ and if $a \gg l$, the field tends to $B = \dfrac{\mu_0 nil}{2a}$. Interpret these results.

58. A tightly-wound, long solenoid carries a current of $2.00$ A. An electron is found to execute a uniform circular motion inside the solenoid with a frequency of $1.00 \times 10^8$ rev/s. Find the number of turns per metre in the solenoid.

59. A tightly-wound, long solenoid has $n$ turns per unit length, a radius $r$ and carries a current $i$. A particle having charge $q$ and mass $m$ is projected from a point

on the axis in a direction perpendicular to the axis. What can be the maximum speed for which the particle does not strike the solenoid?

60. A tightly-wound, long solenoid is kept with its axis parallel to a large metal sheet carrying a surface current. The surface current through a width $dl$ of the sheet is $Kdl$ and the number of turns per unit length of the solenoid is $n$. The magnetic field near the centre of the solenoid is found to be zero. (a) Find the current in the solenoid. (b) If the solenoid is rotated to make its axis perpendicular to the metal sheet, what would be the magnitude of the magnetic field near its centre?

61. A capacitor of capacitance $100\,\mu$F is connected to a battery of 20 volts for a long time and then disconnected from it. It is now connected across a long solenoid having 4000 turns per metre. It is found that the potential difference across the capacitor drops to 90% of its maximum value in $2.0$ seconds. Estimate the average magnetic field produced at the centre of the solenoid during this period.

□

# ANSWERS

## OBJECTIVE I

| 1. (c) | 2. (c) | 3. (a) | 4. (d) | 5. (b) | 6. (d) |
| 7. (c) | 8. (c) | 9. (c) | 10. (b) | 11. (c) | 12. (a) |

## OBJECTIVE II

1. (c), (d)  2. (a), (b), (c)  3. (b), (c), (d)
4. (b), (c)  5. (b), (c)  6. (a)
7. (b), (c)

## EXERCISES

2. $2\,\mu$T along the positive $Y$-axis
3. $5.0$ mT
4. $2.5\,\mu$T
5. $20\,\mu$T, zero
6. (a) looking along the current, at the leftmost points on the wire's surface, $B = \dfrac{\mu_0 i}{2\pi r}$

   (b) zero if $r \leq \dfrac{\mu_0 i}{2\pi B}$, $B = \dfrac{\mu_0 i}{2\pi r}$ if $r > \dfrac{\mu_0 i}{2\pi B}$
7. $5 \times 10^{-4}$ T
8. $1.0$ mm west to the wire
9. (a) $0.67 \times 10^{-4}$ T (b) $2.7 \times 10^{-4}$ T
   (c) $2.0 \times 10^{-4}$ T (d) $1.0 \times 10^{-4}$ T

10. $1.7 \times 10^{-?}$ T in a direction parallel to the plane of the wires and perpendicular to the wires

11. (a) zero  (b) $2\,\mu$T along the $Z$-axis
    (c) zero and (d) $2\,\mu$T along the negative $Z$-axis

12. (a) zero (b) $Q_1 : 1.1 \times 10^{-4}$ T, $\odot$ , $Q_2$ : zero,
    $Q_3 : 1.1 \times 10^{-4}$ T, $\otimes$, and $Q_4$ : zero

13. $\dfrac{\mu_0 i}{4\pi d}$

15. $11.5\,\mu$T

16. $0.07$

17. $\dfrac{\sqrt{2}\,\mu_0 i}{3\pi a}$ , $\otimes$

18. $\dfrac{2\mu_0 i}{\pi a}\left(\dfrac{1}{\sqrt{5}} - \dfrac{1}{3\sqrt{13}}\right)$, $\odot$

19. zero

20. zero

21. (a) $\dfrac{27\mu_0 i}{2\pi l}$ (b) $\dfrac{8\sqrt{2}\mu_0 i}{\pi l}$

22. $\dfrac{\mu_0 i}{2\pi x}\cot\dfrac{\alpha}{4}$

23. $\dfrac{2\mu_0 i\sqrt{l^2 + b^2}}{\pi lb}$

24. (a) $\dfrac{\mu_0 in^2 \sin\dfrac{\pi}{n}\tan\dfrac{\pi}{n}}{2\pi^2 r}$

26. 40 cm

27. zero on the middle wire and $6.0 \times 10^{-4}$ N towards the middle wire on each of the rest two

28. 2 cm from the 10 A current and 8 cm from the other

29. $3 \times 10^{-3}$ N/m, downward          zero

30. 0.49 A in opposite directiom

31. (b) $1.6 \times 10^{-7}$ N towards right

32. 1.57 cm

33. 48 mA

34. $6 \times 10^{-10}$ T

35. zero

36. (a) $8\pi \times 10^{-4}$ T (b) zero

37. 1.8 mT

38. $16\pi \times 10^{-11}$ N

39. $\dfrac{\mu_0 \pi i I r^2}{2R}$

40. $\dfrac{\mu_0 \pi i I r}{4R}$

41. $1.6 \times 10^{-7}$ T

42. $1.26 \times 10^{-5}$ T

43. at a distance of $4r/\pi$ from the centre in such a way that the direction of the current in it is opposite to that in the nearest part of the circular wire

44. (a) 2.51 mT     (b) 7.66 cm

45. $4.0 \times 10^{-5}$ T, downwards in both the cases

46. $1.88 \times 10^{15}$ m/s

47. (a) zero (b) $\dfrac{\mu_0 i}{3\pi r}$

48. (a) zero (b) $\dfrac{\mu_0 i}{2\pi b}$

49. $\dfrac{\mu_0 i a}{2\pi b^2}$

50. (a) 2.0 μT (b) 10 μT (c) 5.0 μT

52. 0, $\mu_0 K$ towards right in the figure, 0

53. $\dfrac{\mu_0 K q r}{m}$

54. 5000 turns/m

55. $2\pi \times 10^{-3}$ T

56. 1 V

57. (a) $\dfrac{\mu_0 n i}{\sqrt{1 + \left(\dfrac{2a}{l}\right)^2}}$

58. 1420 turns/m

59. $\dfrac{\mu_0 q r n i}{2m}$

60. (a) $\dfrac{K}{2n}$ (b) $\dfrac{\mu_0 K}{\sqrt{2}}$

61. $16\pi \times 10^{-8}$ T

□

# CHAPTER 36

# PERMANENT MAGNETS

## 36.1 MAGNETIC POLES AND BAR MAGNETS

We have seen that a small current-loop carrying a current $i$, produces a magnetic field

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{d^3} \qquad \ldots \text{(i)}$$

at an axial point. Here $\vec{\mu} = i\vec{A}$ is the magnetic dipole moment of the current loop. The vector $\vec{A}$ represents the area-vector of the current loop. Also, a current loop placed in a magnetic field $\vec{B}$ experiences a torque

$$\vec{\Gamma} = \vec{\mu} \times \vec{B}. \qquad \ldots \text{(ii)}$$

We also know that an electric dipole produces an electric field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{d^3} \qquad \ldots \text{(iii)}$$

at an axial point and it experiences a torque

$$\vec{\Gamma} = \vec{p} \times \vec{E} \qquad \ldots \text{(iv)}$$

when placed in an electric field. Equations (i) and (ii) for a current loop are similar in structure to the equations (iii) and (iv) for an electric dipole with $\vec{\mu}$ taking the role of $\vec{p}$ and $\frac{\mu_0}{4\pi}$ taking the role of $\frac{1}{4\pi\varepsilon_0}$. The similarity suggests that the behaviour of a current loop can be described by the following hypothetical model:

(a) There are two types of magnetic charges, positive magnetic charge and negative magnetic charge. A magnetic charge $m$ placed in a magnetic field $\vec{B}$ experiences a force

$$\vec{F} = m\vec{B}. \qquad \ldots \text{(36.1)}$$

The force on a positive magnetic charge is along the field and the force on a negative magnetic charge is opposite to the field.

(b) A magnetic charge $m$ produces a magnetic field

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^2} \qquad \ldots \text{(36.2)}$$

at a distance $r$ from it. The field is radially outward if the magnetic charge is positive and is inward if it is negative.

(c) A magnetic dipole is formed when a negative magnetic charge $-m$ and a positive magnetic charge $+m$ are placed at a small separation $d$. The magnetic dipole moment is $\mu = md$ and its direction is from $-m$ to $+m$. The line joining $-m$ and $+m$ is called the *axis* of the dipole.

(d) A current loop of area $A$ carrying a current $i$ may be replaced by a magnetic dipole of dipole moment $\mu = md = iA$ placed along the axis of the loop. The area-vector $\vec{A}$ points in the direction $-m$ to $+m$.
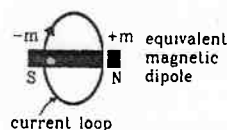


Figure 36.1

The model is very useful in studying magnetic effects and is widely used. It is customary to call a positive magnetic charge a *north pole* and a negative magnetic charge a *south pole*. They are represented by the letters $N$ and $S$ respectively. The quantity $m$ is called *pole strength*. From the equation $md = iA$ or $F = mB$, we can easily see that the unit of pole strength is A–m. We can find the magnetic field due to a magnetic dipole at any point $P$ using equation (36.2) for both the poles.

A solenoid very closely resembles a combination of circular loops placed side by side. If $i$ be the current through it and $A$ be the area of cross-section, the dipole moment of each turn is $\mu = iA$. In our model, each turn may be replaced by a small dipole placed at the centre
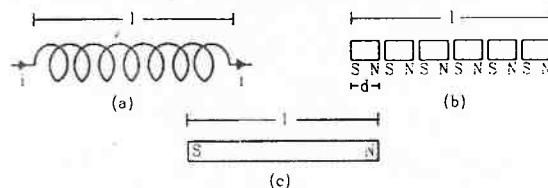


Figure 36.2

$$v' = \frac{1}{2\pi} \sqrt{\frac{M(B_H + B)}{I}}$$

or, $\dfrac{v'^2}{v^2} = \dfrac{B_H + B}{B_H}$

or, $\left(\dfrac{60}{40}\right)^2 = 1 + \dfrac{B}{B_H}$

or, $\dfrac{B}{B_H} = 1.25$

or, $B = 1.25 \times 24\ \mu T = 30 \times 10^{-6}\ T.$

The oscillating magnet is in end-on position of the short magnet. Thus, the field $B$ can be written as

$$B = \frac{\mu_0}{4\pi} \frac{2M'}{d^3}$$

or, $M' = \dfrac{2\pi}{\mu_0} B d^3$

$= 0.5 \times 10^7 \dfrac{A}{T\text{-}m} \times (30 \times 10^{-6}\ T) \times (20 \times 10^{-2}\ m)^3$

$= 1.2\ A\text{-}m^2.$

19. *A bar magnet of mass 100 g, length 7·0 cm, width 1·0 cm and height 0·50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25 $\mu$T. (a) Find the magnetic moment of the magnet. (b) If the magnet is put in the magnetometer with its 0·50 cm edge horizontal, what would be the time period?*

**Solution :** (a) The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2)$$

$= \dfrac{100 \times 10^{-3}}{12}[(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2]\ kg\text{-}m^2$

$= \dfrac{25}{6} \times 10^{-5}\ kg\text{-}m^2.$

We have,

$$T = 2\pi \sqrt{\frac{I}{MB}} \qquad \ldots\text{(i)}$$

or, $M = \dfrac{4\pi^2 I}{BT^2} = \dfrac{4\pi^2 \times 25 \times 10^{-5}\ kg/m^2}{6 \times (25 \times 10^{-6}\ T) \times \dfrac{\pi^2}{4}\ s^2}$

$= 27\ A\text{-}m^2.$

(b) In this case the moment of inertia becomes

$I' = \dfrac{m'}{12}(L^2 + b'^2)$ where $b' = 0.5$ cm.

The time period would be

$$T' = \sqrt{\frac{I'}{MB}} \qquad \ldots\text{(ii)}$$

Dividing by equation (i),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}}$$

$= \dfrac{\sqrt{\dfrac{m'}{12}(L^2 + b'^2)}}{\sqrt{\dfrac{m'}{12}(L^2 + b^2)}} = \dfrac{\sqrt{(7\ cm)^2 + (0.5\ cm)^2}}{\sqrt{(7\ cm)^2 + (1.0\ cm)^2}}$

$= 0.992$

or, $T' = \dfrac{0.992 \times \pi}{2}\ s = 0.496\pi\ s.$

□

## QUESTIONS FOR SHORT ANSWER

1. Can we have a single north pole? A single south pole?

2. Do two distinct poles actually exist at two nearby points in a magnetic dipole?

3. An iron needle is attracted to the ends of a bar magnet but not to the middle region of the magnet. Is the material making up the ends of a bar magnet different from that of the middle region?

4. Compare the direction of the magnetic field inside a solenoid with that of the field there if the solenoid is replaced by its equivalent combination of north pole and south pole.

5. Sketch the magnetic field lines for a current-carrying circular loop near its centre. Replace the loop by an equivalent magnetic dipole and sketch the magnetic field lines near the centre of the dipole. Identify the difference.

6. The force on a north pole, $\vec{F} = m\vec{B}$, is parallel to the field $\vec{B}$. Does it contradict our earlier knowledge that a magnetic field can exert forces only perpendicular to itself?

7. Two bar magnets are placed close to each other with their opposite poles facing each other. In absence of other forces, the magnets are pulled towards each other and their kinetic energy increases. Does it contradict our earlier knowledge that magnetic forces cannot do any work and hence cannot increase kinetic energy of a system?

8. Magnetic scalar potential is defined as

$$U(\vec{r_2}) - U(\vec{r_1}) = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{B} \cdot d\vec{l}.$$

Apply this equation to a closed curve enclosing a long

straight wire. The RHS of the above equation is then $-\mu_0 i$ by Ampere's law. We see that $U(\vec{r_2}) \neq U(\vec{r_1})$ even when $\vec{r_2} = \vec{r_1}$. Can we have a magnetic scalar potential in this case?

9. Can the earth's magnetic field be vertical at a place? What will happen to a freely suspended magnet at such a place? What is the value of dip here?

10. Can the dip at a place be zero? 90°?

11. The reduction factor $K$ of a tangent galvanometer is written on the instrument. The manual says that the current is obtained by multiplying this factor to $\tan\theta$. The procedure works well at Bhuwaneshwar. Will the procedure work if the instrument is taken to Nepal? If there is some error. can it be corrected by correcting the manual or the instrument will have to be taken back to the factory?

## OBJECTIVE I

1. A circular loop carrying a current is replaced by an equivalent magnetic dipole. A point on the axis of the loop is in
   (a) end-on position    (b) broadside-on position
   (c) both               (d) none.

2. A circular loop carrying a current is replaced by an equivalent magnetic dipole. A point on the loop is in
   (a) end-on position    (b) broadside-on position
   (c) both               (d) none.

3. When a current in a circular loop is equivalently replaced by a magnetic dipole,
   (a) the pole strength $m$ of each pole is fixed
   (b) the distance $d$ between the poles is fixed
   (c) the product $md$ is fixed
   (d) none of the above.

4. Let $r$ be the distance of a point on the axis of a bar magnet from its centre. The magnetic field at such a point is proportional to
   (a) $\dfrac{1}{r}$   (b) $\dfrac{1}{r^2}$   (c) $\dfrac{1}{r^3}$   (d) none of these.

5. Let $r$ be the distance of a point on the axis of a magnetic dipole from its centre. The magnetic field at such a point is proportional to
   (a) $\dfrac{1}{r}$   (b) $\dfrac{1}{r^2}$   (c) $\dfrac{1}{r^3}$   (d) none of these.

6. Two short magnets of equal dipole moments $M$ are fastened perpendicularly at their centres (figure 36-Q1). The magnitude of the magnetic field at a distance $d$ from the centre on the bisector of the right angle is
   (a) $\dfrac{\mu_0}{4\pi}\dfrac{M}{d^3}$   (b) $\dfrac{\mu_0}{4\pi}\dfrac{\sqrt{2}M}{d^3}$   (c) $\dfrac{\mu_0}{4\pi}\dfrac{2\sqrt{2}M}{d^3}$   (d) $\dfrac{\mu_0}{4\pi}\dfrac{2M}{d^3}$.
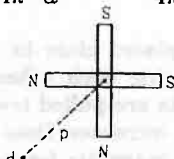
Figure 36-Q1

7. Magnetic meridian is
   (a) a point              (b) a line along north-south
   (c) a horizontal plane   (d) a vertical plane.

8. A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It
   (a) will stay in north-south direction only
   (b) will stay in east-west direction only
   (c) will become rigid showing no movement
   (d) will stay in any position.

9. A dip circle is taken to geomagnetic equator. The needle is allowed to move in a vertical plane perpendicular to the magnetic meridian. The needle will stay
   (a) in horizontal direction only
   (b) in vertical direction only
   (c) in any direction except vertical and horizontal
   (d) in any direction it is released.

10. Which of the following four graphs may best represent the current–deflection relation in a tangent galvanometer?
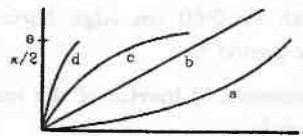
Figure 36-Q2

11. A tangent galvanometer is connected directly to an ideal battery. If the number of turns in the coil is doubled, the deflection will
    (a) increase            (b) decrease
    (c) remain unchanged    (d) either increase or decrease.

12. If the current is doubled, the deflection is also doubled in
    (a) a tangent galvanometer
    (b) a moving-coil galvanometer
    (c) both                (d) none.

13. A very long bar magnet is placed with its north pole coinciding with the centre of a circular loop carrying an electric current $i$. The magnetic field due to the magnet at a point on the periphery of the wire is $B$. The radius of the loop is $a$. The force on the wire is
    (a) very nearly $2\pi aiB$ perpendicular to the plane of the wire
    (b) $2\pi aiB$ in the plane of the wire
    (c) $\pi aiB$ along the magnet    (d) zero.

## OBJECTIVE II

1. Pick the correct options.
   (a) Magnetic field is produced by electric charges only.
   (b) Magnetic poles are only mathematical assumptions having no real existence.
   (c) A north pole is equivalent to a clockwise current and a south pole is equivalent to an anticlockwise current.
   (d) A bar magnet is equivalent to a long, straight current.

2. A horizontal circular loop carries a current that looks clockwise when viewed from above. It is replaced by an equivalent magnetic dipole consisting of a south pole $S$ and a north pole $N$.
   (a) The line $SN$ should be along a diameter of the loop.
   (b) The line $SN$ should be perpendicular to the plane of the loop.
   (c) The south pole should be below the loop.
   (d) The north pole should be below the loop.

3. Consider a magnetic dipole kept in the north-south direction. Let $P_1$, $P_2$, $Q_1$, $Q_2$ be four points at the same distance from the dipole towards north, south, east and west of the dipole respectively. The directions of the magnetic field due to the dipole are the same at
   (a) $P_1$ and $P_2$
   (b) $Q_1$ and $Q_2$
   (c) $P_1$ and $Q_1$
   (d) $P_2$ and $Q_2$.

4. Consider the situation of the previous problem. The directions of the magnetic field due to the dipole are opposite at
   (a) $P_1$ and $P_2$
   (b) $Q_1$ and $Q_2$
   (c) $P_1$ and $Q_1$
   (d) $P_2$ and $Q_2$

5. To measure the magnetic moment of a bar magnet, one may use
   (a) a tangent galvanometer
   (b) a deflection galvanometer if the earth's horizontal field is known
   (c) an oscillation magnetometer if the earth's horizontal field is known
   (d) both deflection and oscillation magnetometer if the earth's horizontal field is not known.

## EXERCISES

1. A long bar magnet has a pole strength of 10 A–m. Find the magnetic field at a point on the axis of the magnet at a distance of 5 cm from the north pole of the magnet.

2. Two long bar magnets are placed with their axes coinciding in such a way that the north pole of the first magnet is 2.0 cm from the south pole of the second. If both the magnets have a pole strength of 10 A–m, find the force exerted by one magnet on the other.

3. A uniform magnetic field of $0.20 \times 10^{-3}$ T exists in the space. Find the change in the magnetic scalar potential as one moves through 50 cm along the field.

4. Figure (36-E1) shows some of the equipotential surfaces of the magnetic scalar potential. Find the magnetic field $B$ at a point in the region.



Figure 36-E1

5. The magnetic field at a point, 10 cm away from a magnetic dipole, is found to be $2.0 \times 10^{-4}$ T. Find the magnetic moment of the dipole if the point is (a) in end-on position of the dipole and (b) in broadside-on position of the dipole.

6. Show that the magnetic field at a point due to a magnetic dipole is perpendicular to the magnetic axis if the line joining the point with the centre of the dipole makes an angle of $\tan^{-1}(\sqrt{2})$ with the magnetic axis.

7. A bar magnet has a length of 8 cm. The magnetic field at a point at a distance 3 cm from the centre in the broadside-on position is found to be $4 \times 10^{-6}$ T. Find the pole strength of the magnet.

8. A magnetic dipole of magnetic moment 1.44 A–m² is placed horizontally with the north pole pointing towards north. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 µT.

9. A magnetic dipole of magnetic moment 0.72 A–m² is placed horizontally with the north pole pointing towards south. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 µT.

10. A magnetic dipole of magnetic moment $0.72\sqrt{2}$ A–m² is placed horizontally with the north pole pointing towards east. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 µT.

11. The magnetic moment of the assumed dipole at the earth's centre is $8.0 \times 10^{22}$ A–m². Calculate the magnetic field $B$ at the geomagnetic poles of the earth. Radius of the earth is 6400 km.

12. If the earth's magnetic field has a magnitude $3.4 \times 10^{-5}$ T at the magnetic equator of the earth, what would be its value at the earth's geomagnetic poles ?

13. The magnetic field due to the earth has a horizontal component of 26 µT at a place where the dip is 60°. Find the vertical component and the magnitude of the field.

14. A magnetic needle is free to rotate in a vertical plane which makes an angle of 60° with the magnetic meridian. If the needle stays in a direction making an angle of $\tan^{-1}(2/\sqrt{3})$ with the horizontal, what would be the dip at that place ?

15. The needle of a dip circle shows an apparent dip of 45° in a particular position and 53° when the circle is rotated through 90°. Find the true dip.

16. A tangent galvanometer shows a deflection of 45° when 10 mA of current is passed through it. If the horizontal component of the earth's magnetic field is $B_H = 3.6 \times 10^{-5}$ T and radius of the coil is 10 cm, find the number of turns in the coil.

17. A moving-coil galvanometer has a 50-turn coil of size 2 cm × 2 cm. It is suspended between the magnetic poles producing a magnetic field of 0·5 T. Find the torque on the coil due to the magnetic field when a current of 20 mA passes through it.

18. A short magnet produces a deflection of 37° in a deflection magnetometer in Tan-A position when placed at a separation of 10 cm from the needle. Find the ratio of the magnetic moment of the magnet to the earth's horizontal magnetic field.

19. The magnetometer of the previous problem is used with the same magnet in Tan-B position. Where should the magnet be placed to produce a 37° deflection of the needle ?

20. A deflection magnetometer is placed with its arms in north-south direction. How and where should a short magnet having $M/B_H = 40$ A-m$^2$/T be placed so that the needle can stay in any position ?

21. A bar magnet takes $\pi/10$ second to complete one oscillation in an oscillation magnetometer. The moment of inertia of the magnet about the axis of rotation is $1.2 \times 10^{-4}$ kg-m$^2$ and the earth's horizontal magnetic field is 30 μT. Find the magnetic moment of the magnet.

22. The combination of two bar magnets makes 10 oscillations per second in an oscillation magnetometer when like poles are tied together and 2 oscillations per second when unlike poles are tied together. Find the ratio of the magnetic moments of the magnets. Neglect any induced magnetism.

23. A short magnet oscillates in an oscillation magnetometer with a time period of 0·10 s where the earth's horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire placed 20 cm east of the magnet. Find the new time period.

24. A bar magnet makes 40 oscillations per minute in an oscillation magnetometer. An identical magnet is demagnetized completely and is placed over the magnet in the magnetometer. Find the time taken for 40 oscillations by this combination. Neglect any induced magnetism.

25. A short magnet makes 40 oscillations per minute when used in an oscillation magnetometer at a place where the earth's horizontal magnetic field is 25 μT. Another short magnet of magnetic moment 1·6 A-m$^2$ is placed 20 cm east of the oscillating magnet. Find the new frequency of oscillation if the magnet has its north pole (a) towards north and (b) towards south.

□

# ANSWERS

## OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (a) | 2. (b) | 3. (c) | 4. (d) | 5. (c) | 6. (c) |
| 7. (d) | 8. (d) | 9. (d) | 10. (c) | 11. (c) | 12. (b) |
| 13. (a) | | | | | |

## OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (b) | 2. (b), (d) | 3. (a), (b) |
| 4. (c), (d) | 5. (b), (c), (d) | |

## EXERCISES

1. $4 \times 10^{-4}$ T

2. $2.5 \times 10^{-2}$ N

3. decreases by $0.10 \times 10^{-3}$ T–m

4. $2.0 \times 10^{-4}$ T

5. (a) $1.0$ A-m$^2$ and (b) $2.0$ A-m$^2$

7. $6 \times 10^{-5}$ A-m

8. at a distance of 20 cm in the plane bisecting the dipole

9. 20 cm south of the dipole

10. 20 cm from the dipole, $\tan^{-1}\sqrt{2}$ south of east

11. 60 μT

12. $6.8 \times 10^{-5}$ T

13. 45 μT, 52 μT

14. 30°

15. 39°

16. 570

17. $2 \times 10^{-4}$ N–m

18. $3.75 \times 10^3 \dfrac{\text{A–m}^2}{\text{T}}$

19. 7·9 cm from the centre

20. 2·0 cm from the needle, north pole pointing towards south

21. 1600 A-m$^2$

22. 13 : 12

23. 0·076 s

24. $\sqrt{2}$ minutes

25. (a) 18 oscillations/minute (b) 54 oscillations/minute

□

magnetic intensity is

$$H = H_n + H_s$$

$$= 159 \cdot 2 \text{ A/m towards the south pole.}$$

(c) The magnetic field $\vec{B}$ at the centre is

$$\vec{B} = \mu_0(\vec{H} + \vec{I})$$

or,

$$B = \left(4\pi \times 10^{-7} \frac{\text{T-m}}{\text{A}}\right)(4 \times 10^{1} - 159 \cdot 2) \frac{\text{A}}{\text{m}}$$

$$= 5 \cdot 0 \times 10^{-4} \text{ T}.$$

The field is towards the north pole.

**5.** *The maximum value of the permeability of $\mu$-metal (77% Ni, 16% Fe, 5% Cu, 2% Cr) is $0 \cdot 126$ T-m/A. Find the maximum relative permeability and susceptibility.*

**Solution :** Relative permeability is

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0 \cdot 126 \text{ T-m/A}}{4\pi \times 10^{-7} \text{ T-m/A}}$$

$$= 1 \cdot 00 \times 10^{5}.$$

Susceptibility $\chi = \mu_r - 1 \approx 1 \cdot 00 \times 10^{5}$.

**6.** *A toroid has a mean radius $R$ equal to $20/\pi$ cm, and a total of 400 turns of wire carrying a current of $2 \cdot 0$ A. An aluminium ring at temperature 280 K inside the toroid provides the core. (a) If the magnetization $I$ is $4 \cdot 8 \times 10^{-2}$ A/m, find the susceptibility of aluminium at* 280 K. *(b) If the temperature of the aluminium ring is raised to 320 K, what will be the magnetization ?*

**Solution :** (a) The number of turns per unit length of the toroid is

$$n = \frac{400}{2\pi R}.$$

The magnetic intensity $H$ in the core is

$$H = ni$$

$$= \frac{400 \times 2 \cdot 0 \text{ A}}{2\pi \times \frac{20}{\pi} \times 10^{-2} \text{ m}} = 2000 \text{ A/m}.$$

The susceptibility is

$$\chi = I/H$$

$$= \frac{4 \cdot 8 \times 10^{-2} \text{ A/m}}{2000 \text{ A/m}} = 2 \cdot 4 \times 10^{-5}.$$

(b) The susceptibility $\chi$ of a paramagnetic substance varies with absolute temperature as $\chi = c/T$. Thus, $\chi_2/\chi_1 = T_1/T_2$.

The susceptibility of aluminium at temperature 320 K is, therefore,

$$\chi = \frac{280}{320} \times 2 \cdot 4 \times 10^{-5} = 2 \cdot 1 \times 10^{-5}.$$

Thus, the magnetization at 320 K is

$$I = \chi H$$

$$= 2 \cdot 1 \times 10^{-5} \times 2000 \text{ A/m}.$$

$$= 4 \cdot 2 \times 10^{-2} \text{ A/m}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. When a dielectric is placed in an electric field, it gets polarized. The electric field in a polarized material is less than the applied field. When a paramagnetic substance is kept in a magnetic field, the field in the substance is more than the applied field. Explain the reason of this opposite behaviour.

2. The property of diamagnetism is said to be present in all materials. Then, why are some materials paramagnetic or ferromagnetic ?

3. Do permeability and relative permeability have the same dimensions ?

4. A rod when suspended in a magnetic field stays in east-west direction. Can we be sure that the field is in the east-west direction ? Can it be in the north-south direction ?

5. Why cannot we make permanent magnets from paramagnetic materials ?

6. Can we have magnetic hysteresis in paramagnetic or diamagnetic substances ?

7. When a ferromagnetic material goes through a hysteresis loop, its thermal energy is increased. Where does this energy come from ?

8. What are the advantages of using soft iron as a core, instead of steel, in the coils of galvanometers ?

9. To keep valuable instruments away from the earth's magnetic field, they are enclosed in iron boxes. Explain.

## OBJECTIVE I

1. A paramagnetic material is placed in a magnetic field. Consider the following statements:

(A) If the magnetic field is increased, the magnetization is increased.

(B) If the temperature is increased, the magnetization is increased.
- (a) Both A and B are true.
- (b) A is true but B is false.
- (c) B is true but A is false.
- (d) Both A and B are false.

2. A paramagnetic material is kept in a magnetic field. The field is increased till the magnetization becomes constant. If the temperature is now decreased, the magnetization
- (a) will increase
- (b) decrease
- (c) remain constant
- (d) may increase or decrease.

3. A ferromagnetic material is placed in an external magnetic field. The magnetic domains
- (a) increase in size
- (b) decrease in size
- (c) may increase or decrease in size
- (d) have no relation with the field.

4. A long, straight wire carries a current $i$. The magnetizing field intensity $H$ is measured at a point $P$ close to the wire. A long, cylindrical iron rod is brought close to the wire so that the point $P$ is at the centre of the rod. The value of $H$ at $P$ will
- (a) increase many times
- (b) decrease many times
- (c) remain almost constant
- (d) become zero.

5. The magnetic susceptibility is negative for
- (a) paramagnetic materials only
- (b) diamagnetic materials only
- (c) ferromagnetic materials only
- (d) paramagnetic and ferromagnetic materials.

6. The desirable properties for making permanent magnets are
- (a) high retentivity and high coercive force
- (b) high retentivity and low coercive force
- (c) low retentivity and high coercive force
- (d) low retentivity and low coercive force.

7. Electromagnets are made of soft iron because soft iron has
- (a) high retentivity and high coercive force
- (b) high retentivity and low coercive force
- (c) low retentivity and high coercive force
- (d) low retentivity and low coercive force.

## OBJECTIVE II

1. Pick the correct options.
- (a) All electrons have magnetic moment.
- (b) All protons have magnetic moment.
- (c) All nuclei have magnetic moment.
- (d) All atoms have magnetic moment.

2. The permanent magnetic moment of the atoms of a material is not zero. The material
- (a) must be paramagnetic
- (b) must be diamagnetic
- (c) must be ferromagnetic
- (d) may be paramagnetic.

3. The permanent magnetic moment of the atoms of a material is zero. The material
- (a) must be paramagnetic
- (b) must be diamagnetic
- (c) must be ferromagnetic
- (d) may be paramagnetic.

4. Which of the following pairs has quantities of the same dimensions ?
- (a) magnetic field $B$ and magnetizing field intensity $H$
- (b) magnetic field $B$ and intensity of magnetization $I$
- (c) magnetizing field intensity $H$ and intensity of magnetization $I$
- (d) longitudinal strain and magnetic susceptibility.

5. When a ferromagnetic material goes through a hysteresis loop, the magnetic susceptibility
- (a) has a fixed value
- (b) may be zero
- (c) may be infinity
- (d) may be negative.

6. Mark out the correct options.
- (a) Diamagnetism occurs in all materials.
- (b) Diamagnetism results from the partial alignment of permanent magnetic moment.
- (c) The magnetizing field intensity $H$ is always zero in free space.
- (d) The magnetic field of induced magnetic moment is opposite to the applied field.

## EXERCISES

1. The magnetic intensity $H$ at the centre of a long solenoid carrying a current of 2·0 A, is found to be 1500 A/m. Find the number of turns per centimetre of the solenoid.

2. A rod is inserted as the core in the current-carrying solenoid of the previous problem. (a) What is the magnetic intensity $H$ at the centre ? (b) If the magnetization $I$ of the core is found to be 0·12 A/m, find the susceptibility of the material of the rod. (c) Is the material paramagnetic, diamagnetic or ferromagnetic ?

3. The magnetic field inside a long solenoid having 50 turns/cm is increased from $2·5 \times 10^{-3}$ T to 2·5 T when an iron core of cross-sectional area 4 cm² is inserted into it. Find (a) the current in the solenoid, (b) the

magnetization $I$ of the core and (c) the pole strength developed in the core.

4. A bar magnet of length 1 cm and cross-sectional area 1·0 cm² produces a magnetic field of $1·5 \times 10^{-4}$ T at a point in end-on position at a distance 15 cm away from the centre. (a) Find the magnetic moment $M$ of the magnet. (b) Find the magnetization $I$ of the magnet. (c) Find the magnetic field $B$ at the centre of the magnet.

5. The susceptibility of annealed iron at saturation is 5500. Find the permeability of annealed iron at saturation.

6. The magnetic field $B$ and the magnetic intensity $H$ in a material are found to be 1·6 T and 1000 A/m

respectively. Calculate the relative permeability $\mu_r$ and the susceptibility $\chi$ of the material.

7. The susceptibility of magnesium at 300 K is $1.2 \times 10^{-5}$. At what temperature will the susceptibility increase to $1.8 \times 10^{-5}$?

8. Assume that each iron atom has a permanent magnetic moment equal to 2 Bohr magnetons (1 Bohr magneton equals $9.27 \times 10^{-24}$ A-m$^2$). The density of atoms in iron is $8.52 \times 10^{28}$ atoms/m$^3$. (a) Find the maximum magnetization $I$ in a long cylinder of iron. (b) Find the maximum magnetic field $B$ on the axis inside the cylinder.

9. The coercive force for a certain permanent magnet is $4.0 \times 10^{4}$ A/m. This magnet is placed inside a long solenoid of 40 turns/cm and a current is passed in the solenoid to demagnetise it completely. Find the current.

□

# ANSWERS

## OBJECTIVE I

1. (b)  2. (c)  3. (c)  4. (c)  5. (b)  6. (a)
7. (d)

## OBJECTIVE II

1. (a), (b)  2. (d)  3. (b)  4. (c), (d)
5. (b), (c), (d)  6. (a), (d)

## EXERCISES

1. 7.5

2. (a) 1500 A/m (b) $8.0 \times 10^{-5}$ (c) paramagnetic
3. (a) 0.4 A  (b) $2.0 \times 10^{6}$ A/m (c) 800 A-m
4. (a) 2.5 A-m$^2$ (b) $2.5 \times 10^{6}$ A/m (c) 1.2 T
5. $6.9 \times 10^{-3}$
6. $1.3 \times 10^{3}$ each
7. 200 K
8. (a) $1.58 \times 10^{6}$ A/m (b) 2.0 T
9. 10 A

□

# ELECTROMAGNETIC INDUCTION

## 38.1 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION
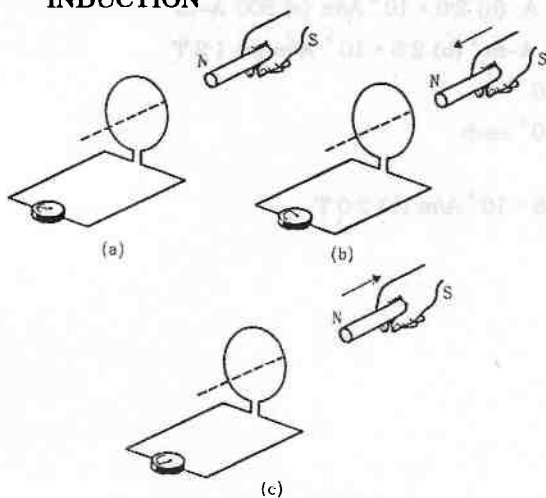


Figure 38.1

Figure (38.1a) shows a bar magnet placed along the axis of a conducting loop containing a galvanometer. There is no current in the loop and correspondingly no deflection in the galvanometer. If we move the magnet towards the loop (figure 38.1b), there is a deflection in the galvanometer showing that there is an electric current in the loop. If the magnet is moved away from the loop (figure 38.1c), again there is a current but the current is in the opposite direction. The current exists as long as the magnet is moving. Faraday studied this behaviour in detail by performing a number of experiments and discovered the following law of nature:

*Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by*

$$\mathcal{E} = -\frac{d\Phi}{dt} \qquad \ldots (38.1)$$

*where $\Phi = \int \vec{B} \cdot \vec{dS}$ is the flux of the magnetic field through the area.*

We shall call the quantity $\Phi$ *magnetic flux.* The SI unit of magnetic flux is called weber which is equivalent to tesla–metre$^2$.

The law described by equation (38.1) is called *Faraday's law of electromagnetic induction.* The flux may be changed in a number of ways. One can change the magnitude of the magnetic field $\vec{B}$ at the site of the loop, the area of the loop or the angle between the area-vector $\vec{dS}$ and the magnetic field $\vec{B}$. In any case, as long as the flux keeps changing, the emf is present. The emf so produced drives an electric current through the loop. If the resistance of the loop is $R$, the current is

$$i = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}. \qquad \ldots (38.2)$$

The emf developed by a changing flux is called *induced emf* and the current produced by this emf is called *induced current.*

**Direction of Induced Current**

The direction of the induced current in a loop may be obtained using equation (38.1) or (38.2). The procedure to decide the direction is as follows:

Put an arrow on the loop to choose the positive sense of current. This choice is arbitrary. Using right-hand thumb rule find the positive direction of the normal to the area bounded by the loop. If the fingers curl along the loop in the positive sense, the thumb represents the positive direction of the normal. Calculate the flux $\Phi = \int \vec{B} \cdot \vec{dS}$ through the area bounded by the loop. If the flux increases with time, $\frac{d\Phi}{dt}$ is positive and $\mathcal{E}$ is negative from equation (38.1). Correspondingly, the current is negative. It is, therefore, in the direction opposite to the arrow put on the loop. If $\Phi$ decreases with time, $\frac{d\Phi}{dt}$ is negative, $\mathcal{E}$ is positive and the current is along the arrow.

Putting the numerical values in (i), the energy at $t = 10$ ms is

$$\frac{1}{2} \times (1 \cdot 0 \text{ H}) \times [0 \cdot 12 \text{ A}(1 - 1/e)]^2$$

$$= 2 \cdot 8 \text{ mJ}.$$

**28.** *An inductance $L$ and a resistance $R$ are connected in series with a battery of emf $\mathcal{E}$. Find the maximum rate at which the energy is stored in the magnetic field.*

**Solution :**

The energy stored in the magnetic field at time $t$ is

$$U = \frac{1}{2} L i^2 = \frac{1}{2} L i_0^2 (1 - e^{-t/\tau})^2.$$

The rate at which the energy is stored is

$$P = \frac{dU}{dt} = L i_0^2 (1 - e^{-t/\tau})(-e^{-t/\tau})\left(-\frac{1}{\tau}\right)$$

$$= \frac{L i_0^2}{\tau}(e^{-t/\tau} - e^{-2t/\tau}). \qquad \ldots \text{(i)}$$

This rate will be maximum when $\frac{dP}{dt} = 0$

or, $\qquad \frac{L i_0^2}{\tau}\left(-\frac{1}{\tau}e^{-t/\tau} + \frac{2}{\tau}e^{-2t/\tau}\right) = 0$

or, $\qquad e^{-t/\tau} = \frac{1}{2}.$

Putting in (i),

$$P_{max} = \frac{L i_0^2}{\tau}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{L \mathcal{E}^2}{4R^2(L/R)} = \frac{\mathcal{E}^2}{4R}.$$

**29.** *Two conducting circular loops of radii $R_1$ and $R_2$ are placed in the same plane with their centres coinciding. Find the mutual inductance between them assuming $R_2 \ll R_1$.*

**Solution :** Suppose a current $i$ is established in the outer loop. The magnetic field at the centre will be

$$B = \frac{\mu_0 i}{2R_1}.$$

As the radius $R_2$ of the inner coil is small compared to $R_1$, the flux of magnetic field through it will be approximately

$$\Phi = \frac{\mu_0 i}{2R_1} \pi R_2^2$$

so that the mutual inductance is

$$M = \frac{\Phi}{i} = \frac{\mu_0 \pi R_2^2}{2R_1}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. A metallic loop is placed in a nonuniform magnetic field. Will an emf be induced in the loop ?

2. An inductor is connected to a battery through a switch. Explain why the emf induced in the inductor is much larger when the switch is opened as compared to the emf induced when the switch is closed.

3. The coil of a moving-coil galvanometer keeps on oscillating for a long time if it is deflected and released. If the ends of the coil are connected together, the oscillation stops at once. Explain.

4. A short magnet is moved along the axis of a conducting loop. Show that the loop repels the magnet if the magnet is approaching the loop and attracts the magnet if it is going away from the loop.

5. Two circular loops are placed coaxially but separated by a distance. A battery is suddenly connected to one of the loops establishing a current in it. Will there be a current induced in the other loop ? If yes, when does the current start and when does it end ? Do the loops attract each other or do they repel ?

6. The battery discussed in the previous question is suddenly disconnected. Is a current induced in the other loop ? If yes, when does it start and when does it end ? Do the loops attract each other or repel ?

7. If the magnetic field outside a copper box is suddenly changed, what happens to the magnetic field inside the box ? Such low-resistivity metals are used to form enclosures which shield objects inside them against varying magnetic fields.

8. Metallic (nonferromagnetic) and nonmetallic particles in a solid waste may be separated as follows. The waste is allowed to slide down an incline over permanent magnets. The metallic particles slow down as compared to the nonmetallic ones and hence are separated. Discuss the role of eddy currents in the process.

9. A pivoted aluminium bar falls much more slowly through a small region containing a magnetic field than a similar bar of an insulating material. Explain.

10. A metallic bob $A$ oscillates through the space between the poles of an electromagnet (figure 38-Q1). The oscillations are more quickly damped when the circuit
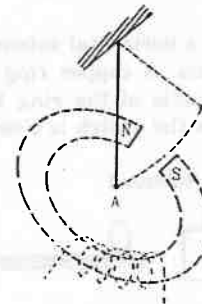


Figure 38-Q1

is on, as compared to the case when the circuit is off. Explain.

11. Two circular loops are placed with their centres separated by a fixed distance. How would you orient the loops to have (a) the largest mutual inductance (b) the smallest mutual inductance ?

12. Consider the self-inductance per unit length of a solenoid at its centre and that near its ends. Which of the two is greater ?

13. Consider the energy density in a solenoid at its centre and that near its ends. Which of the two is greater ?

## OBJECTIVE I

1. A rod of length $l$ rotates with a small but uniform angular velocity $\omega$ about its perpendicular bisector. A uniform magnetic field $B$ exists parallel to the axis of rotation. The potential difference between the centre of the rod and an end is

   (a) zero    (b) $\frac{1}{8}\omega Bl^2$    (c) $\frac{1}{2}\omega Bl^.$    (d) $B\omega l^.$.

2. A rod of length $l$ rotates with a uniform angular velocity $\omega$ about its perpendicular bisector. A uniform magnetic field $B$ exists parallel to the axis of rotation. The potential difference between the two ends of the rod is

   (a) zero    (b) $\frac{1}{2}Bl\omega^2$    (c) $Bl\omega^.$    (d) $2Bl\omega^.$.

3. Consider the situation shown in figure (38-Q2). If the switch is closed and after some time it is opened again, the closed loop will show
   (a) an anticlockwise current-pulse
   (b) a clockwise current-pulse
   (c) an anticlockwise current-pulse and then a clockwise current-pulse
   (d) a clockwise current-pulse and then an anticlockwise current-pulse.
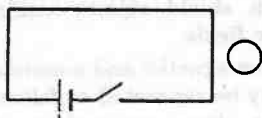


Figure 38-Q2

4. Solve the previous question if the closed loop is completely enclosed in the circuit containing the switch.

5. A bar magnet is released from rest along the axis of a very long, vertical copper tube. After some time the magnet
   (a) will stop in the tube
   (b) will move with almost contant speed
   (c) will move with an acceleration $g$
   (d) will oscillate.

6. Figure (38-Q3) shows a horizontal solenoid connected to a battery and a switch. A copper ring is placed on a frictionless track, the axis of the ring being along the axis of the solenoid. As the switch is closed, the ring will
   (a) remain stationary
   (b) move towards the solenoid
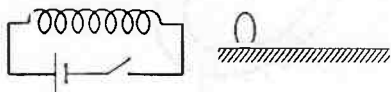


Figure 38-Q3

(c) move away from the solenoid
(d) move towards the solenoid or away from it depending on which terminal (positive or negative) of the battery is connected to the left end of the solenoid.

7. Consider the following statements:
   (A) An emf can be induced by moving a conductor in a magnetic field.
   (B) An emf can be induced by changing the magnetic field.
   (a) Both A and B are true.    (b) A is true but B is false.
   (c) B is true but A is false.    (d) Both A and B are false.

8. Consider the situation shown in figure (38-Q4). The wire $AB$ is slid on the fixed rails with a constant velocity. If the wire $AB$ is replaced by a semicircular wire, the magnitude of the induced current will
   (a) increase    (b) remain the same    (c) decrease
   (d) increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.
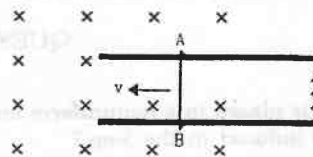


Figure 38-Q4

9. Figure (38-Q5a) shows a conducting loop being pulled out of a magnetic field with a speed $v$. Which of the four plots shown in figure (38-Q5b) may represent the power delivered by the pulling agent as a function of the speed $v$ ?



(a)                          (b)
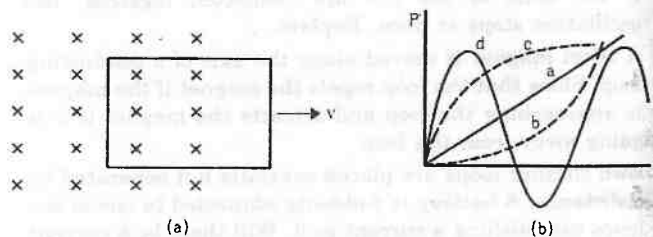
Figure 38-Q5

10. Two circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature. During this period, the two loops
    (a) attract each other        (b) repel each other
    (c) do not exert any force on each other
    (d) attract or repel each other depending on the sense of the current.

11. A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is
(a) clockwise      (b) anticlockwise      (c) zero
(d) clockwise or anticlockwise depending on whether the resistance is increased or decreased.

12. A conducting square loop of side $l$ and resistance $R$ moves in its plane with a uniform velocity $v$ perpendicular to one of its sides. A uniform and constant magnetic field $B$ exists along the perpendicular to the plane of the loop as shown in figure (38-Q6). The current induced in the loop is
(a) $Blv/R$ clockwise          (b) $Blv/R$ anticlockwise
(c) $2Blv/R$ anticlockwise     (d) zero.
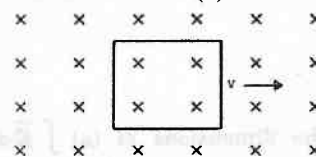


Figure 38-Q6

## OBJECTIVE II

1. A bar magnet is moved along the axis of a copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true ?

(a) The south pole faces the ring and the magnet moves towards it.
(b) The north pole faces the ring and the magnet moves towards it.
(c) The south pole faces the ring and the magnet moves away from it.
(d) The north pole faces the ring and the magnet moves away from it.

2. A conducting rod is moved with a constant velocity $v$ in a magnetic field. A potential difference appears across the two ends
(a) if $\vec{v} \parallel \vec{l}$      (b) if $\vec{v} \parallel \vec{B}$      (c) if $\vec{l} \parallel \vec{B}$
(d) none of these.

3. A conducting loop is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the loop if
(a) it is translated
(b) it is rotated about its axis
(c) it is rotated about a diameter
(d) it is deformed.

4. A metal sheet is placed in front of a strong magnetic pole. A force is needed to
(a) hold the sheet there if the metal is magnetic
(b) hold the sheet there if the metal is nonmagnetic
(c) move the sheet away from the pole with uniform velocity if the metal is magnetic
(d) move the sheet away from the pole with uniform velocity if the metal is nonmagnetic.
Neglect any effect of paramagnetism, diamagnetism and gravity.

5. A constant current $i$ is maintained in a solenoid. Which of the following quantities will increase if an iron rod is inserted in the solenoid along its axis ?
(a) magnetic field at the centre
(b) magnetic flux linked with the solenoid
(c) self-inductance of the solenoid
(d) rate of Joule heating.

6. Two solenoids have identical geometrical construction but one is made of thick wire and the other of thin wire. Which of the following quantities are different for the two solenoids ?
(a) self-inductance
(b) rate of Joule heating if the same current goes through them
(c) magnetic field energy if the same current goes through them
(d) time constant if one solenoid is connected to one battery and the other is connected to another battery.

7. An $LR$ circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the connection ?
(a) current in the circuit
(b) magnetic field energy in the inductor
(c) power delivered by the battery
(d) emf induced in the inductor.

8. A rod $AB$ moves with a uniform velocity $v$ in a uniform magnetic field as shown in figure (38-Q7).
(a) The rod becomes electrically charged.
(b) The end $A$ becomes positively charged.
(c) The end $B$ becomes positively charged.
(d) The rod becomes hot because of Joule heating.
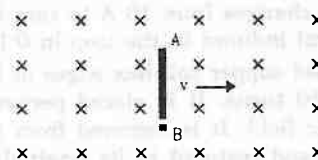


Figure 38-Q7

9. $L$, $C$ and $R$ represent the physical quantities inductance, capacitance and resistance respectively. Which of the following combinations have dimensions of frequency ?
(a) $\dfrac{1}{RC}$      (b) $\dfrac{R}{L}$      (c) $\dfrac{1}{\sqrt{LC}}$      (d) $C/L$.

10. The switches in figure (38-Q8a) and (38-8b) are closed at $t = 0$ and reopened after a long time at $t = t_0$.
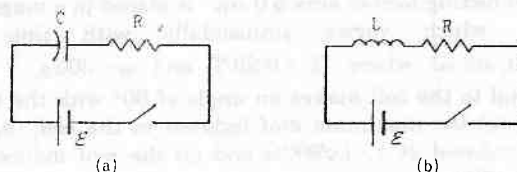


Figure 38-Q8

(a) The charge on $C$ just after $t = 0$ is $\mathcal{E}C$.

(b) The charge on $C$ long after $t = 0$ is $\mathcal{E}C$.

(c) The current in $L$ just before $t = t_0$ is $\mathcal{E}/R$.

(d) The current in $L$ long after $t = t_0$ is $\mathcal{E}/R$.

## EXERCISES

1. Calculate the dimensions of (a) $\int \vec{E} \cdot d\vec{l}$. (b) $vBl$ and (c) $\dfrac{d\Phi_B}{dt}$. The symbols have their usual meanings.

2. The flux of magnetic field through a closed conducting loop changes with time according to the equation, $\Phi = at^2 + bt + c$. (a) Write the SI units of $a$, $b$ and $c$. (b) If the magnitudes of $a$, $b$ and $c$ are 0·20, 0·40 and 0·60 respectively, find the induced emf at $t = 2$ s.

3. (a) The magnetic field in a region varies as shown in figure (38-E1). Calculate the average induced emf in a conducting loop of area $2·0 \times 10^{-3}$ m$^2$ placed perpendicular to the field in each of the 10 ms intervals shown. (b) In which intervals is the emf not constant? Neglect the behaviour near the ends of 10 ms intervals.
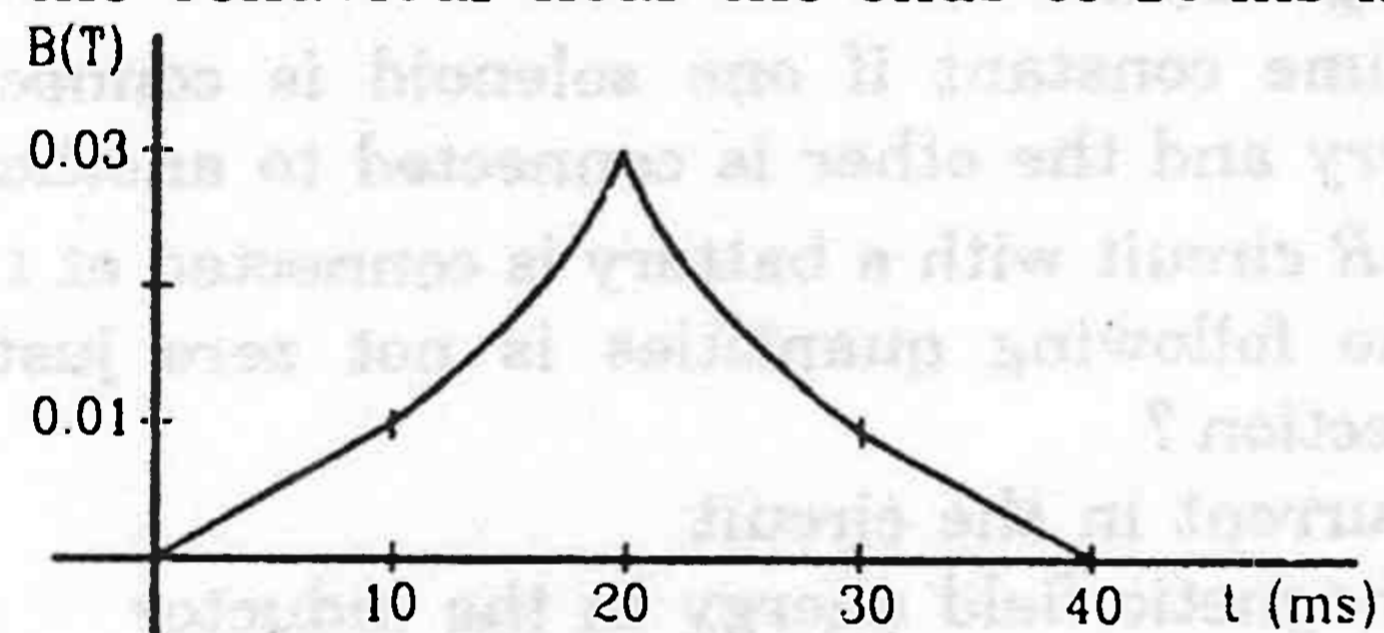


Figure 38-E1

4. A conducting circular loop having a radius of 5·0 cm, is placed perpendicular to a magnetic field of 0·50 T. It is removed from the field in 0·50 s. Find the average emf produced in the loop during this time.

5. A conducting circular loop of area 1 mm$^2$ is placed coplanarly with a long, straight wire at a distance of 20 cm from it. The straight wire carries an electric current which changes from 10 A to zero in 0·1 s. Find the average emf induced in the loop in 0·1 s.

6. A square-shaped copper coil has edges of length 50 cm and contains 50 turns. It is placed perpendicular to a 1·0 T magnetic field. It is removed from the magnetic field in 0·25 s and restored in its original place in the next 0·25 s. Find the magnitude of the average emf induced in the loop during (a) its removal, (b) its restoration and (c) its motion.

7. Suppose the resistance of the coil in the preivous problem is 25 Ω. Assume that the coil moves with uniform velocity during its removal and restoration. Find the thermal energy developed in the coil during (a) its removal, (b) its restoration and (c) its motion.

8. A conducting loop of area 5·0 cm$^2$ is placed in a magnetic field which varies sinusoidally with time as $B = B_0 \sin \omega t$ where $B_0 = 0·20$ T and $\omega = 300$ s$^{-1}$. The normal to the coil makes an angle of 60° with the field. Find (a) the maximum emf induced in the coil, (b) the emf induced at $\tau = (\pi/900)$s and (c) the emf induced at $t = (\pi/600)$ s.

9. Figure (38-E2) shows a conducting square loop placed parallel to the pole-faces of a ring magnet. The pole-faces have an area of 1 cm$^2$ each and the field between the poles is 0·10 T. The wires making the loop are all outside the magnetic field. If the magnet is removed in 1·0 s, what is the average emf induced in the loop?
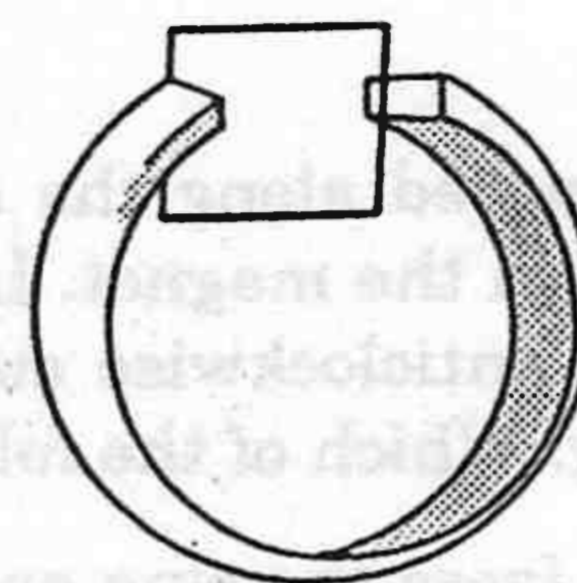


Figure 38-E2

10. A conducting square loop having edges of length 2·0 cm is rotated through 180° about a diagonal in 0·20 s. A magnetic field $B$ exists in the region which is perpendicular to the loop in its initial position. If the average induced emf during the rotation is 20 mV, find the magnitude of the magnetic field.

11. A conducting loop of face-area $A$ and resistance $R$ is placed perpendicular to a magnetic field $B$. The loop is withdrawn completely from the field. Find the charge which flows through any cross-section of the wire in the process. Note that it is independent of the shape of the loop as well as the way it is withdrawn.

12. A long solenoid of radius 2 cm has 100 turns/cm and carries a current of 5 A. A coil of radius 1 cm having 100 turns and a total resistance of 20 Ω is placed inside the solenoid coaxially. The coil is connected to a galvanometer. If the current in the solenoid is reversed in direction, find the charge flown through the galvanometer.

13. Figure (38-E3) shows a metallic square frame of edge $a$ in a vertical plane. A uniform magnetic field $B$ exists in the space in a direction perpendicular to the plane of the figure. Two boys pull the opposite corners of the square to deform it into a rhombus. They start pulling the corners at $t = 0$ and displace the corners at a uniform speed $u$. (a) Find the induced emf in the frame at the instant when the angles at these corners reduce to 60°. (b) Find the induced current in the frame at this instant if the total resistance of the frame is $R$. (c) Find the total charge which flows through a side of the frame by the time the square is deformed into a straight line.
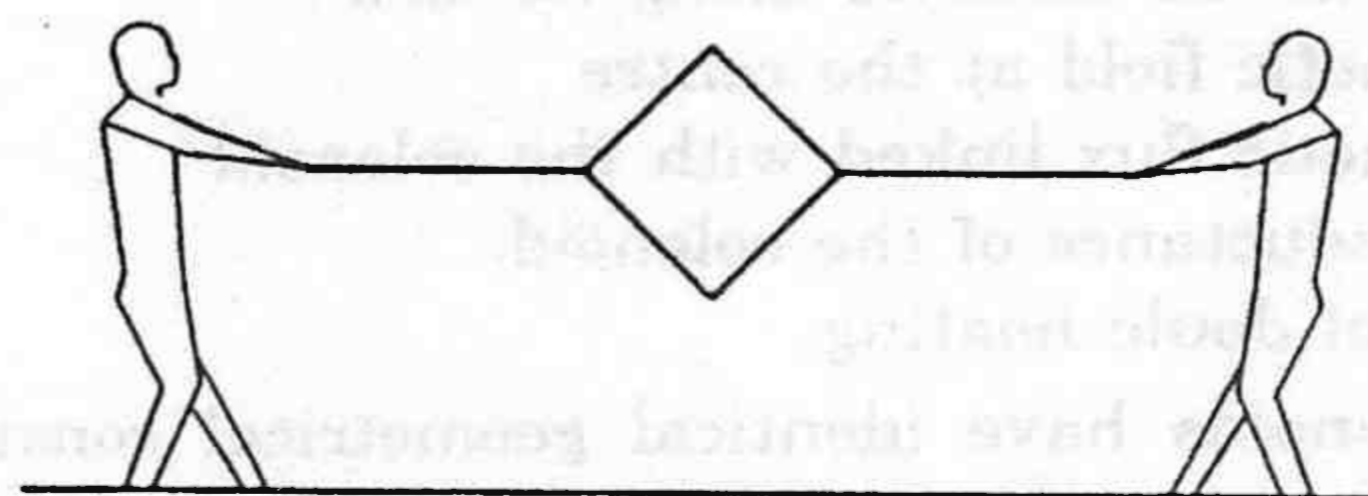


Figure 38-E3

14. The north pole of a magnet is brought down along the axis of a horizontal circular coil (figure 38-E4). As a result, the flux through the coil changes from 0·35 weber to 0·85 weber in an interval of half a second. Find the average emf induced during this period. Is the induced current clockwise or anticlockwise as you look into the coil from the side of the magnet?
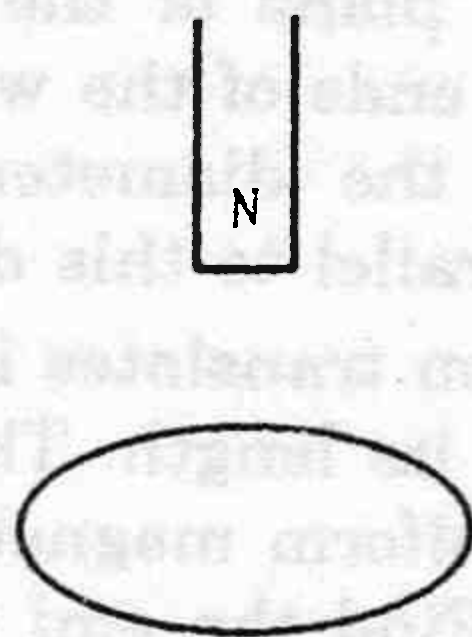
Figure 38-E4

15. A wire-loop confined in a plane is rotated in its own plane with some angular velocity. A uniform magnetic field exists in the region. Find the emf induced in the loop.

16. Figure (38-E5) shows a square loop of side 5 cm being moved towards right at a constant speed of 1 cm/s. The front edge enters the 20 cm wide magnetic field at $t = 0$. Find the emf induced in the loop at (a) $t = 2$ s, (b) $t = 10$ s, (c) $t = 22$ s and (d) $t = 30$ s.
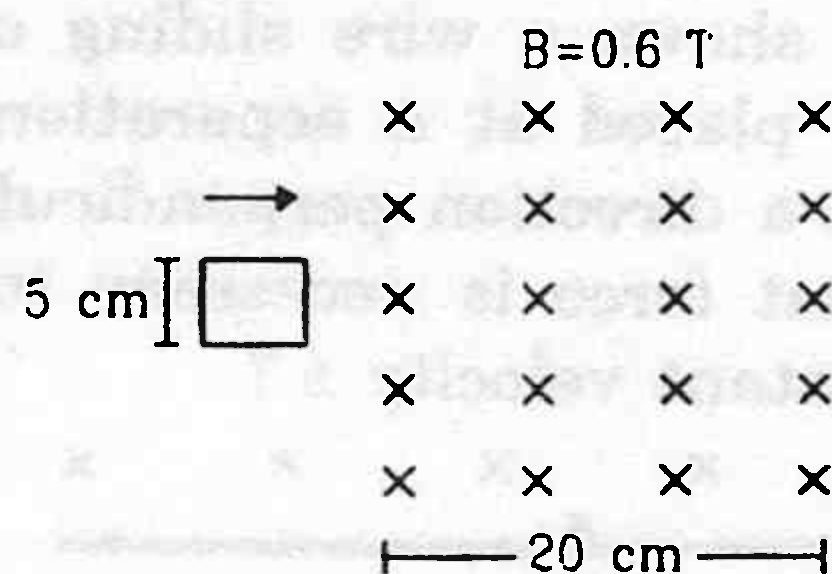
Figure 38-E5

17. Find the total heat produced in the loop of the previous problem during the interval 0 to 30 s if the resistance of the loop is 4·5 mΩ.

18. A uniform magnetic field $B$ exists in a cylindrical region of radius 10 cm as shown in figure (38-E6). A uniform wire of length 80 cm and resistance 4·0 Ω is bent into a square frame and is placed with one side along a diameter of the cylindrical region. If the magnetic field increases at a constant rate of 0·010 T/s, find the current induced in the frame.
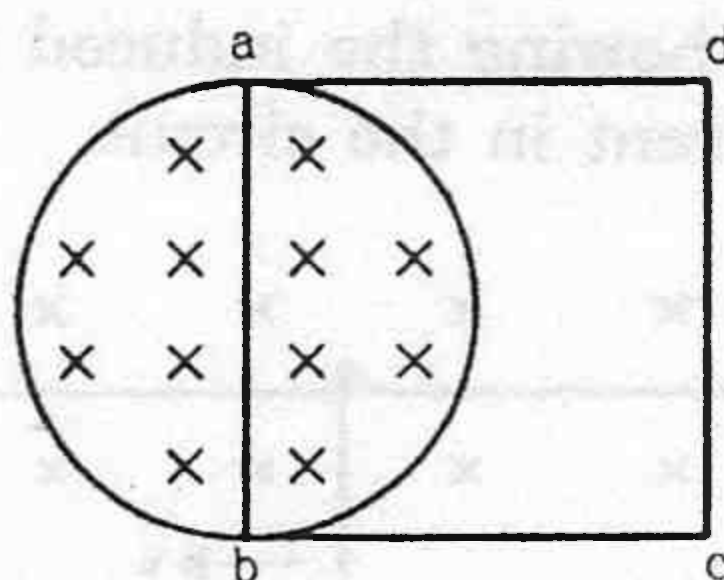
Figure 38-E6

19. The magnetic field in the cylindrical region shown in figure (38-E7) increases at a constant rate of 20·0 mT/s. Each side of the square loop abcd and defa has a length of 1·00 cm and a resistance of 4·00 Ω. Find the current (magnitude and sense) in the wire ad if (a) the switch $S_1$ is closed but $S_2$ is open, (b) $S_1$ is open but $S_2$ is closed, (c) both $S_1$ and $S_2$ are open and (d) both $S_1$ and $S_2$ are closed.
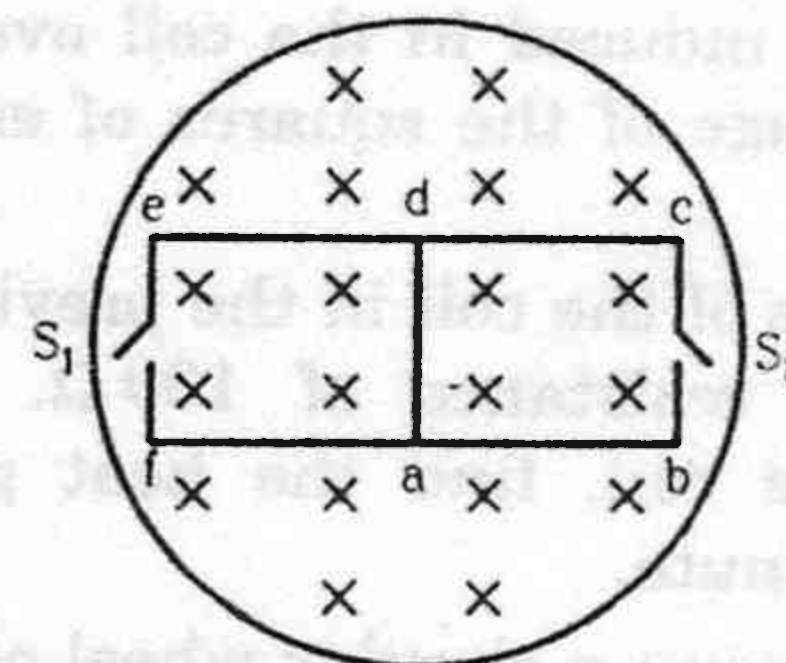
Figure 38-E7

20. Figure (38-E8) shows a circular coil of $N$ turns and radius $a$, connected to a battery of emf $\mathcal{E}$ through a rheostat. The rheostat has a total length $L$ and resistance $R$. The resistance of the coil is $r$. A small circular loop of radius $a'$ and resistance $r'$ is placed coaxially with the coil. The centre of the loop is at a distance $x$ from the centre of coil. the In the beginning, the sliding contact of the rheostat is at the left end and then onwards it is moved towards right at a constant speed $v$. Find the emf induced in the small circular loop at the instant (a) the contact begins to slide and (b) it has slid through half the length of the rheostat.
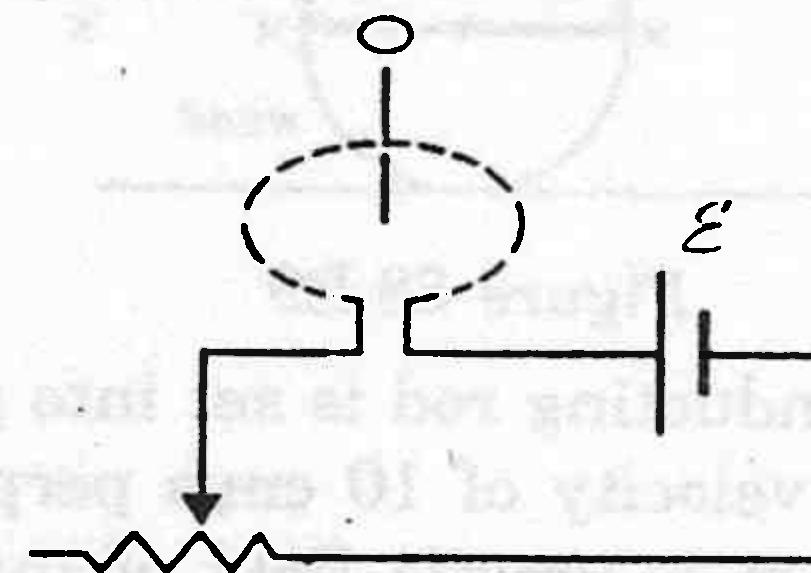
Figure 38-E8

21. A circular coil of radius 2·00 cm has 50 turns. A uniform magnetic field $B = 0·200$ T exists in the space in a direction parallel to the axis of the loop. The coil is now rotated about a diameter through an angle of 60·0°. The operation takes 0·100 s. (a) Find the average emf induced in the coil. (b) If the coil is a closed one (with the two ends joined together) and has a resistance of 4·00 Ω, calculate the net charge crossing a cross-section of the wire of the coil.

22. A closed coil having 100 turns is rotated in a uniform magnetic field $B = 4·0 \times 10^{-4}$ T about a diameter which is perpendicular to the field. The angular velocity of rotation is 300 revolutions per minute. The area of the coil is 25 cm$^2$ and its resistance is 4·0 Ω. Find (a) the average emf developed in half a turn from a position where the coil is perpendicular to the magnetic field, (b) the average emf in a full turn and (c) the net charge displaced in part (a).

23. A coil of radius 10 cm and resistance 40 Ω has 1000 turns. It is placed with its plane vertical and its axis parallel to the magnetic meridian. The coil is connected to a galvanometer and is rotated about the vertical diameter through an angle of 180°. Find the charge which flows through the galvanometer if the horizontal component of the earth's magnetic field is $B_H = 3·0 \times 10^{-5}$ T.

24. A circular coil of one turn of radius 5·0 cm is rotated about a diameter with a constant angular speed of 80 revolutions per minute. A uniform magnetic field $B = 0·010$ T exists in a direction perpendicular to the axis of rotation. Find (a) the maximum emf induced, (b)

the average emf induced in the coil over a long period and (c) the average of the squares of emf induced over a long period.

25. Suppose the ends of the coil in the previous problem are connected to a resistance of $100\,\Omega$. Neglecting the resistance of the coil, find the heat produced in the circuit in one minute.

26. Figure (38-E9) shows a circular wheel of radius $10.0$ cm whose upper half, shown dark in the figure, is made of iron and the lower half of wood. The two junctions are joined by an iron rod. A uniform magnetic field $B$ of magnitude $2.00 \times 10^{-4}$ T exists in the space above the central line as suggested by the figure. The wheel is set into pure rolling on the horizontal surface. If it takes $2.00$ seconds for the iron part to come down and the wooden part to go up, find the average emf induced during this period.
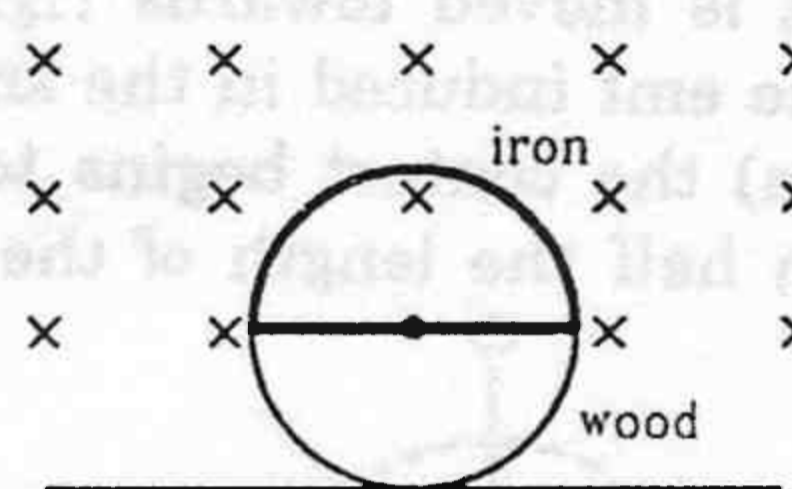


Figure 38-E9

27. A 20 cm long conducting rod is set into pure translation with a uniform velocity of 10 cm/s perpendicular to its length. A uniform magnetic field of magnitude $0.10$ T exists in a direction perpendicular to the plane of motion. (a) Find the average magnetic force on the free electrons of the rod. (b) For what electric field inside the rod, the electric force on a free elctron will balance the magnetic force ? How is this electric field created ? (c) Find the motional emf between the ends of the rod.

28. A metallic metre stick moves with a velocity of 2 m/s in a direction perpendicular to its length and perpendicular to a uniform magnetic field of magnitude $0.2$ T. Find the emf induced between the ends of the stick.

29. A 10 m wide spacecraft moves through the interstellar space at a speed $3 \times 10^7$ m/s. A magnetic field $B = 3 \times 10^{-10}$ T exists in the space in a direction perpendicular to the plane of motion. Treating the spacecraft as a conductor, calculate the emf induced across its width.

30. The two rails of a railway track, insulated from each other and from the ground, are connected to a millivoltmeter. What will be the reading of the millivoltmeter when a train travels on the track at a speed of 180 km/h? The vertical component of earth's magnetic field is $0.2 \times 10^{-4}$ T and the rails are separated by 1 m.

31. A right-angled triangle $abc$, made from a metallic wire, moves at a uniform speed $v$ in its plane as shown in
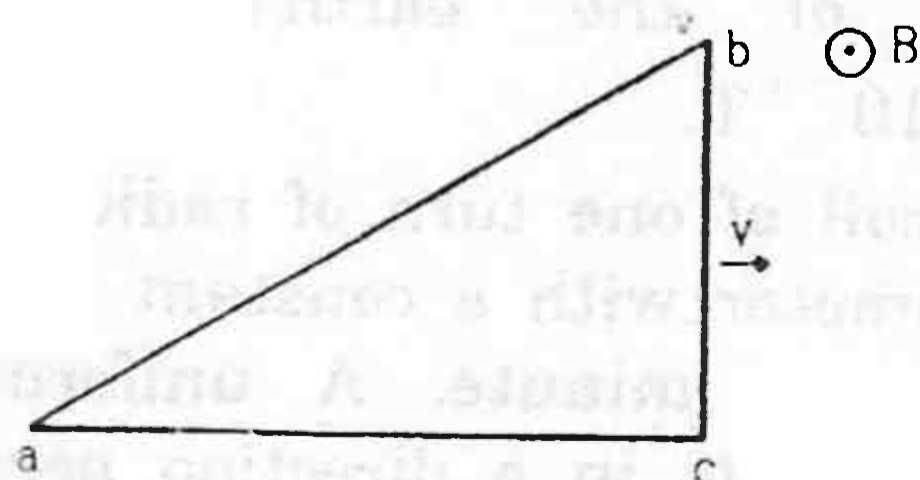


Figure 38-E10

figure (38-E10). A uniform magnetic field $B$ exists in the perpendicular direction. Find the emf induced (a) in the loop $abc$, (b) in the segment $bc$, (c) in the segment $ac$ and (d) in the segment $ab$.

32. A copper wire bent in the shape of a semicircle of radius $r$ translates in its plane with a constant velocity $v$. A uniform magnetic field $B$ exists in the direction perpendicular to the plane of the wire. Find the emf induced between the ends of the wire if (a) the velocity is perpendicular to the diameter joining free ends, (b) the velocity is parallel to this diameter.

33. A wire of length 10 cm translates in a direction making an angle of 60° with its length. The plane of motion is perpendicular to a uniform magnetic field of $1.0$ T that exists in the space. Find the emf induced between the ends of the rod if the speed of translation is 20 cm/s.

34. A circular copper-ring of radius $r$ translates in its plane with a constant velocity $v$. A uniform magnetic field $B$ exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring. (a) Between which pair of points is the emf maximum ? What is the value of this maximum emf ? (b) Between which pair of points is the emf minimum ? What is the value of this minimum emf ?

35. Figure (38-E11) shows a wire sliding on two parallel, conducting rails placed at a separation $l$. A magnetic field $B$ exists in a direction perpendicular to the plane of the rails. What force is necessary to keep the wire moving at a constant velocity $v$ ?
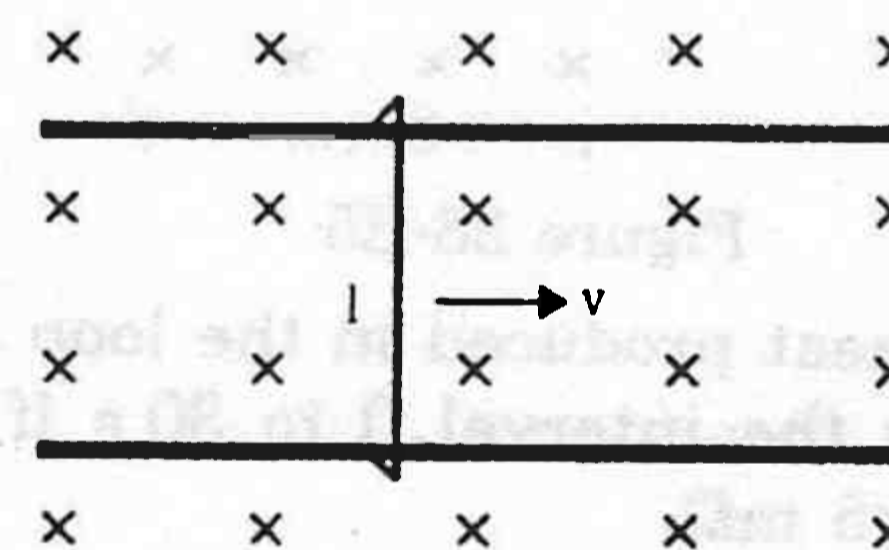


Figure 38-E11

36. Figure (38-E12) shows a long U-shaped wire of width $l$ placed in a perpendicular magnetic field $B$. A wire of length $l$ is slid on the U-shaped wire with a constant velocity $v$ towards right. The resistance of all the wires is $r$ per unit length. At $t = 0$, the sliding wire is close to the left edge of the U-shaped wire. Draw an equivalent circuit diagram, showing the induced emf as a battery. Calculate the current in the circuit.
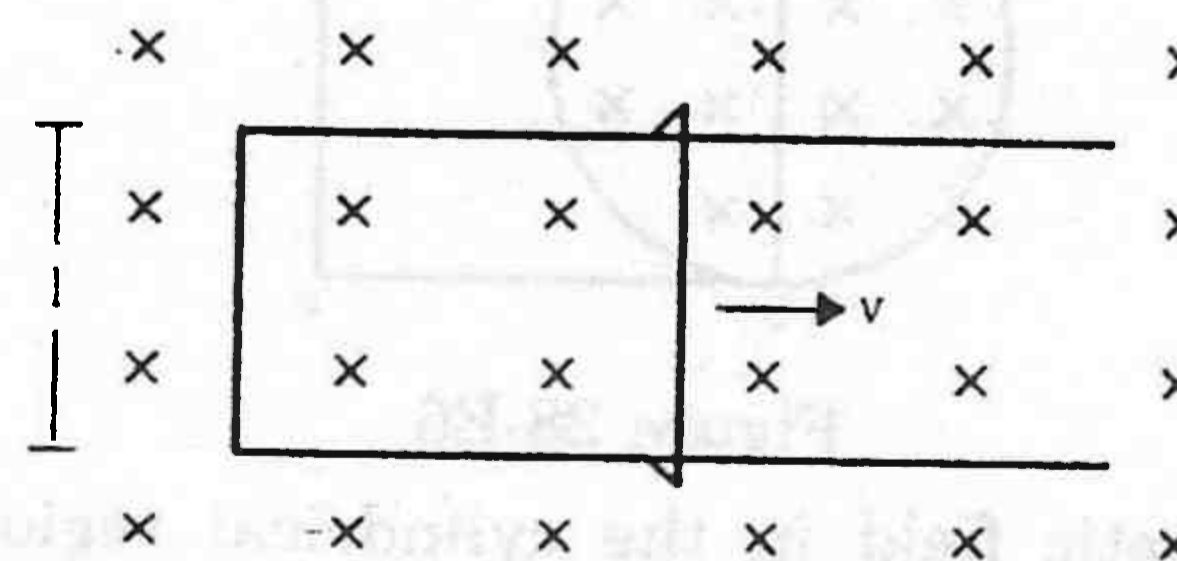


Figure 38-E12

37. Consider the situation of the previous problem. (a) Calculate the force needed to keep the sliding wire moving with a constant velocity $v$. (b) If the force needed just after $t = 0$ is $F_0$, find the time at which the force needed will be $F_0/2$.

**38.** Consider the situation shown in figure (38-E13). The wire $PQ$ has mass $m$, resistance $r$ and can slide on the smooth, horizontal parallel rails separated by a distance $l$. The resistance of the rails is negligible. A uniform magnetic field $B$ exists in the rectangular region and a resistance $R$ connects the rails outside the field region. At $t = 0$, the wire $PQ$ is pushed towards right with a speed $v_0$. Find (a) the current in the loop at an instant when the speed of the wire $PQ$ is $v$, (b) the acceleration of the wire at this instant, (c) the velocity $v$ as a function of $x$ and (d) the maximum distance the wire will move.
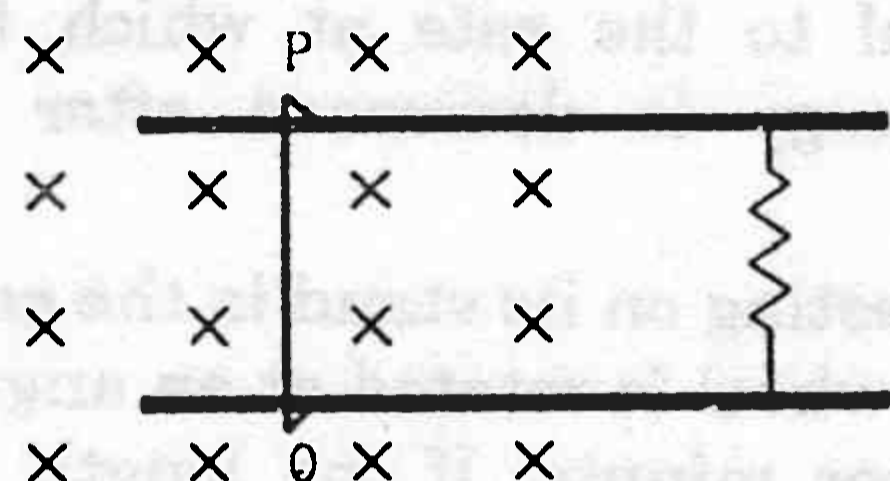


Figure 38-E13

**39.** A rectangular frame of wire $abcd$ has dimensions $32$ cm $\times$ $8.0$ cm and a total resistance of $2.0 \, \Omega$. It is pulled out of a magnetic field $B = 0.020$ T by applying a force of $3.2 \times 10^{-5}$ N (figure 38-E14). It is found that the frame moves with constant speed. Find (a) this constant speed, (b) the emf induced in the loop, (c) the potential difference between the points $a$ and $b$ and (d) the potential difference between the points $c$ and $d$.
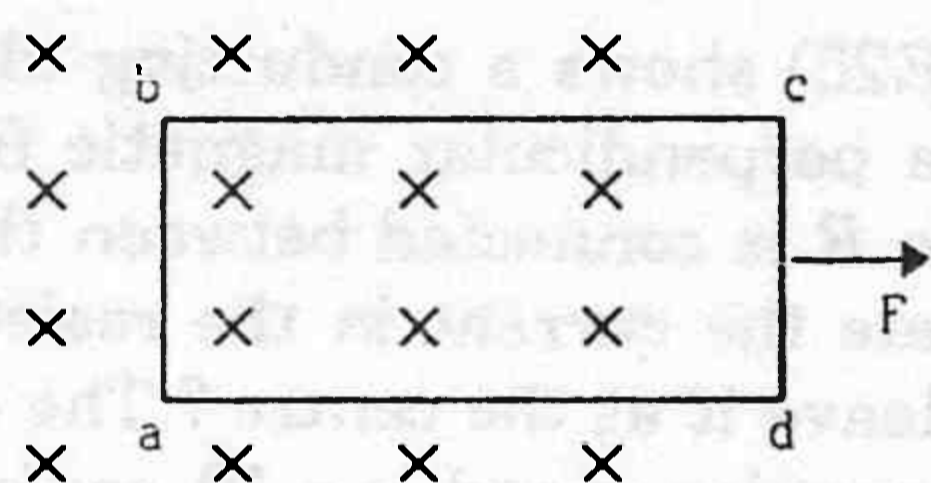


Figure 38-E14

**40.** Figure (38-E15) shows a metallic wire of resistance $0.20 \, \Omega$ sliding on a horizontal, U-shaped metallic rail. The separation between the parallel arms is 20 cm. An electric current of $2.0 \, \mu A$ passes through the wire when it is slid at a rate of 20 cm/s. If the horizontal component of the earth's magnetic field is $3.0 \times 10^{-5}$ T, calculate the dip at the place.
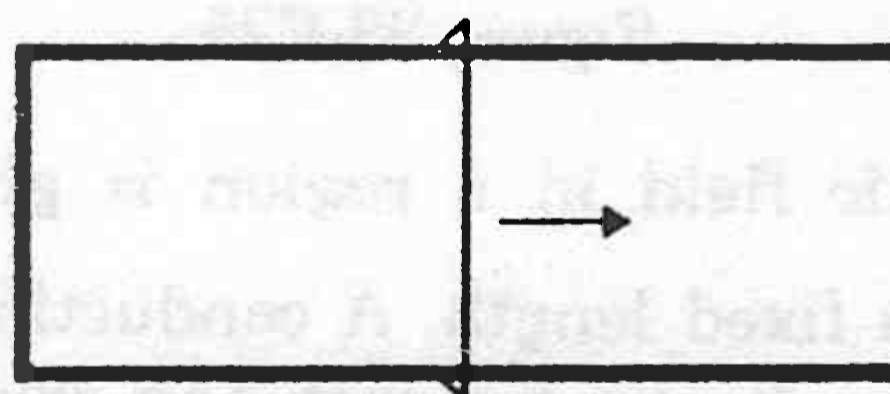


Figure 38-E15

**41.** A wire $ab$ of length $l$, mass $m$ and resistance $R$ slides on a smooth, thick pair of metallic rails joined at the bottom as shown in figure (38-E16). The plane of the rails makes an angle $\theta$ with the horizontal. A vertical magnetic field $B$ exists in the region. If the wire slides on the rails at a constant speed $v$, show that

$$B = \sqrt{\frac{mg R \sin\theta}{vl^2\cos^2\theta}} \, .$$
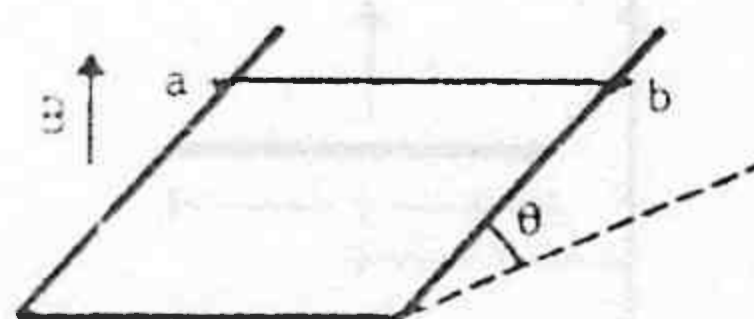


Figure 38-E16

**42.** Consider the situation shown in figure (38-E17). The wires $P_1Q_2$ and $P_2Q_2$ are made to slide on the rails with the same speed 5 cm/s. Find the electric current in the $19 \, \Omega$ resistor if (a) both the wires move towards right and (b) if $P_1Q_1$ moves towards left but $P_2Q_2$ moves towards right.
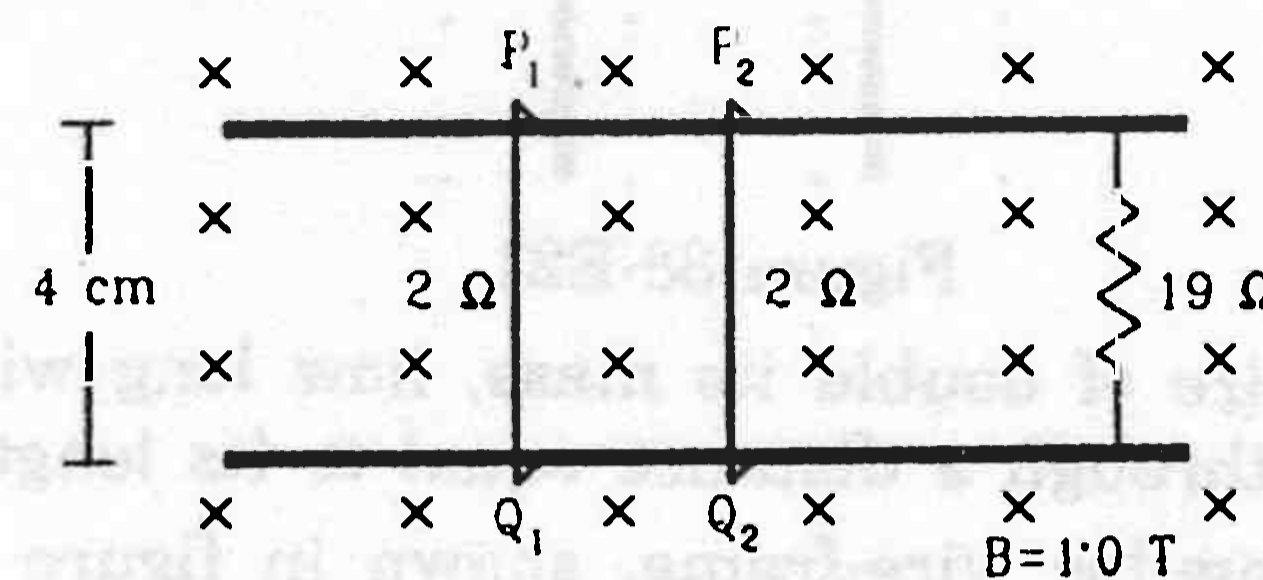


Figure 38-E17

**43.** Suppose the $19 \, \Omega$ resistor of the previous problem is disconnected. Find the current through $P_2Q_2$ in the two situations (a) and (b) of that problem.

**44.** Consider the situation shown in figure (38-E18). The wire $PQ$ has a negligible resistance and is made to slide on the three rails with a constant speed of 5 cm/s. Find the current in the $10 \, \Omega$ resistor when the switch $S$ is thrown to (a) the middle rail (b) the bottom rail.
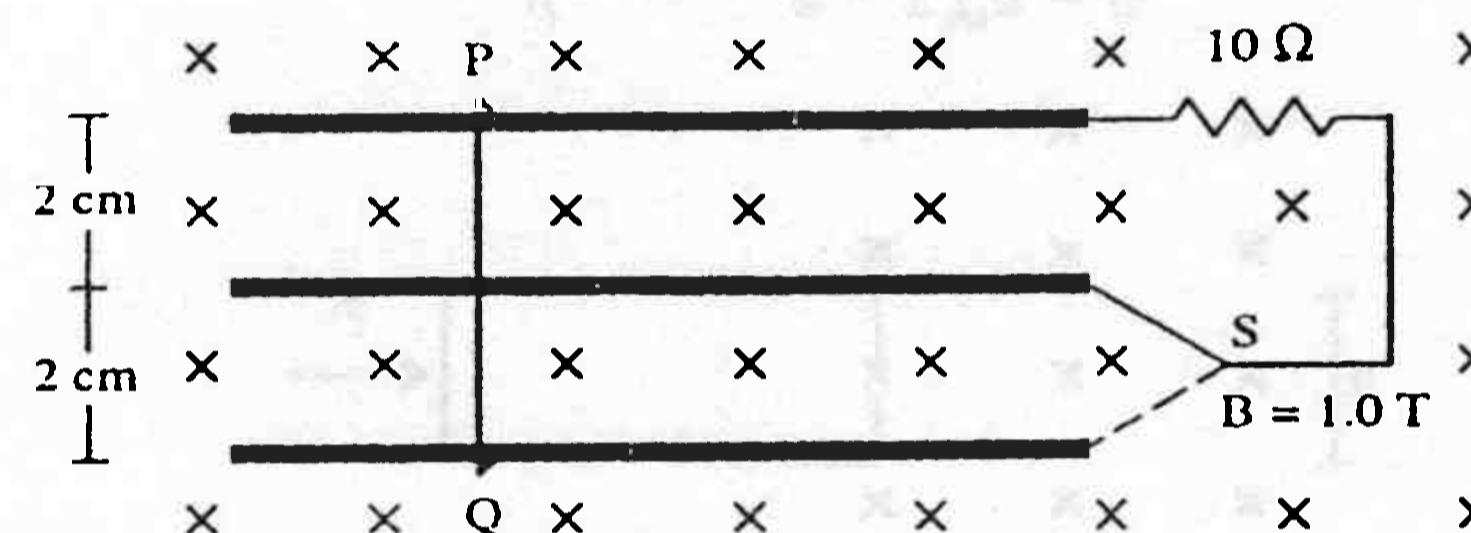


Figure 38-E18

**45.** The current generator $I_g$, shown in figure (38-E19), sends a constant current $i$ through the circuit. The wire $cd$ is fixed and $ab$ is made to slide on the smooth, thick rails with a constant velocity $v$ towards right. Each of these wires has resistance $r$. Find the current through the wire $cd$.
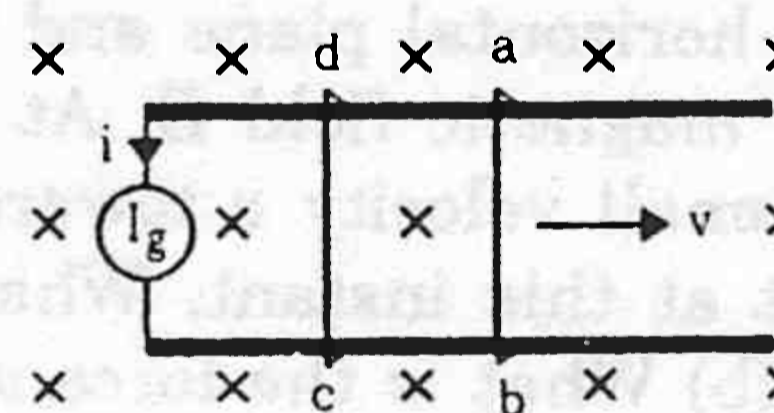


Figure 38-E19

**46.** The current generator $I_g$, shown in figure (38-E20), sends a constant current $i$ through the circuit. The wire $ab$ has a length $l$ and mass $m$ and can slide on the smooth, horizontal rails connected to $I_g$. The enitre system lies in a vertical magnetic field $B$. Find the velocity of the wire as a function of time.
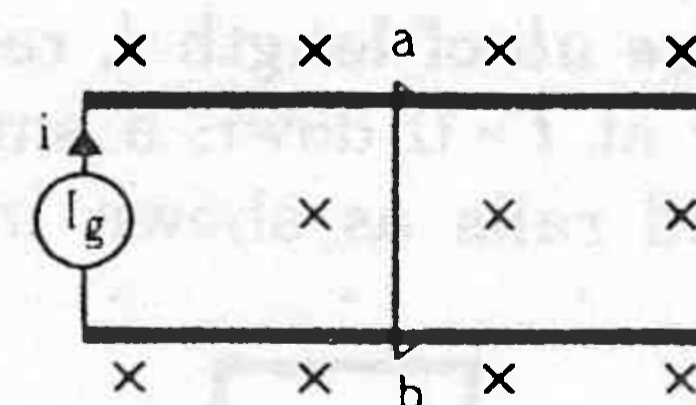


Figure 38-E20

**47.** The system containing the rails and the wire of the previous problem is kept vertically in a uniform horizontal magnetic field $B$ that is perpendicular to the plane of the rails (figure 38-E21). It is found that the wire stays in equilibrium. If the wire $ab$ is replaced by
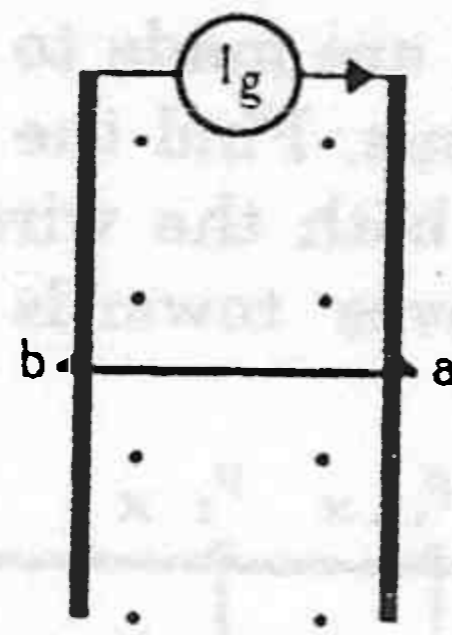
Figure 38-E21

another wire of double its mass, how long will it take in falling through a distance equal to its length ?

**48.** The rectangular wire-frame, shown in figure (38-E22), has a width $d$, mass $m$, resistance $R$ and a large length. A uniform magnetic field $B$ exists to the left of the frame. A constant force $F$ starts pushing the frame into the magnetic field at $t = 0$. (a) Find the acceleration of the frame when its speed has increased to $v$. (b) Show that after some time the frame will move with a constant velocity till the whole frame enters into the magnetic field. Find this velocity $v_0$. (c) Show that the velocity at time $t$ is given by

$$v = v_0(1 - e^{-Ft/mv_0}).$$
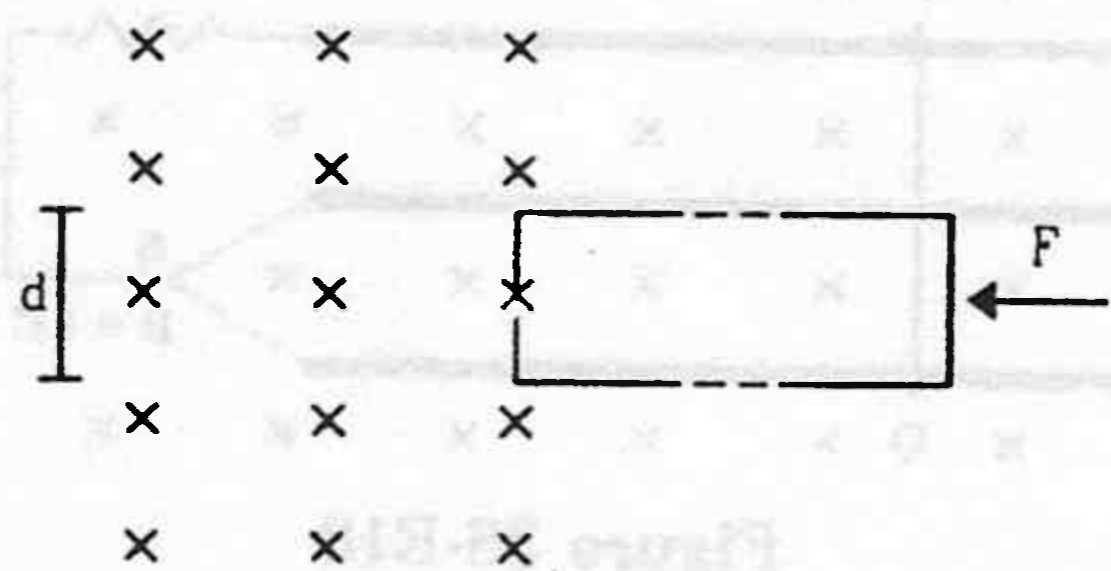


Figure 38-E22

**49.** Figure (38-E23) shows a smooth pair of thick metallic rails connected across a battery of emf $\mathcal{E}$ having a negligible internal resistance. A wire $ab$ of length $l$ and resistance $r$ can slide smoothly on the rails. The entire system lies in a horizontal plane and is immersed in a uniform vertical magnetic field $B$. At an instant $t$, the wire is given a small velocity $v$ towards right. (a) Find the current in it at this instant. What is the direction of the current ? (b) What is the force acting on the wire at this instant ? (c) Show that after some time the wire $ab$ will slide with a constant velocity. Find this velocity.
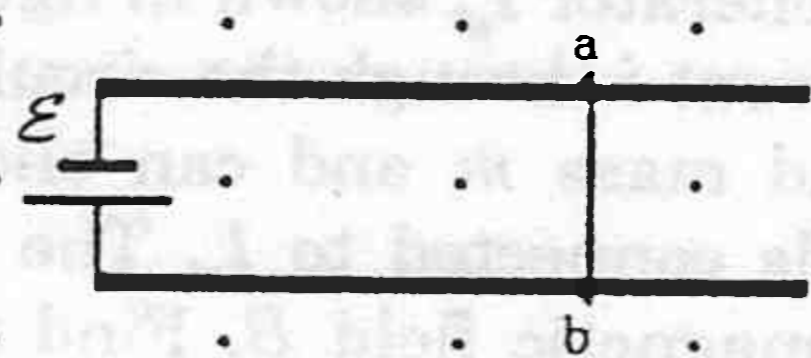


Figure 38-E23

**50.** A conducting wire $ab$ of length $l$, resistance $r$ and mass $m$ starts sliding at $t = 0$ down a smooth, vertical, thick pair of connected rails as shown in figure (38-E24). A
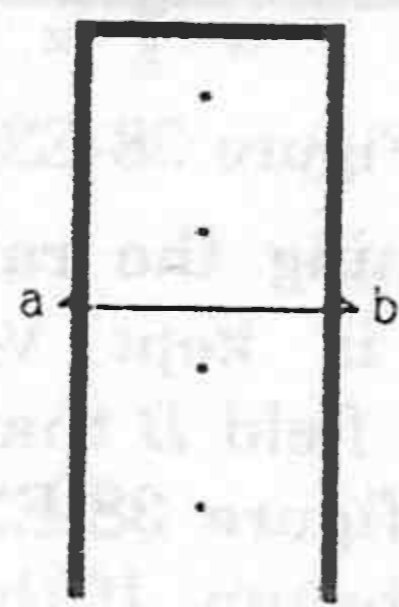


Figure 38-E24

uniform magnetic field $B$ exists in the space in a direction perpendicular to the plane of the rails. (a) Write the induced emf in the loop at an instant $t$ when the speed of the wire is $v$. (b) What would be the magnitude and direction of the induced current in the wire ? (c) Find the downward acceleration of the wire at this instant. (d) After sufficient time, the wire starts moving with a constant velocity. Find this velocity $v_m$. (e) Find the velocity of the wire as a function of time. (f) Find the displacement of the wire as a function of time. (g) Show that the rate of heat developed in the wire is equal to the rate at which the gravitational potential energy is decreased after steady state is reached.

**51.** A bicycle is resting on its stand in the east-west direction and the rear wheel is rotated at an angular speed of 100 revolutions per minute. If the length of each spoke is 30·0 cm and the horizontal component of the earth's magnetic field is $2·0 \times 10^{-5}$ T, find the emf induced between the axis and the outer end of a spoke. Neglect centripetal force acting on the free electrons of the spoke.

**52.** A conducting disc of radius $r$ rotates with a small but constant angular velocity $\omega$ about its axis. A uniform magnetic field $B$ exists parallel to the axis of rotation. Find the motional emf between the centre and the periphery of the disc.

**53.** Figure (38-E25) shows a conducting disc rotating about its axis in a perpendicular magnetic field $B$. A resistor of resistance $R$ is connected between the centre and the rim. Calculate the current in the resistor. Does it enter the disc or leave it at the centre ? The radius of the disc is 5·0 cm, angular speed $\omega = 10$ rad/s, $B = 0·40$ T and $R = 10 \, \Omega$.
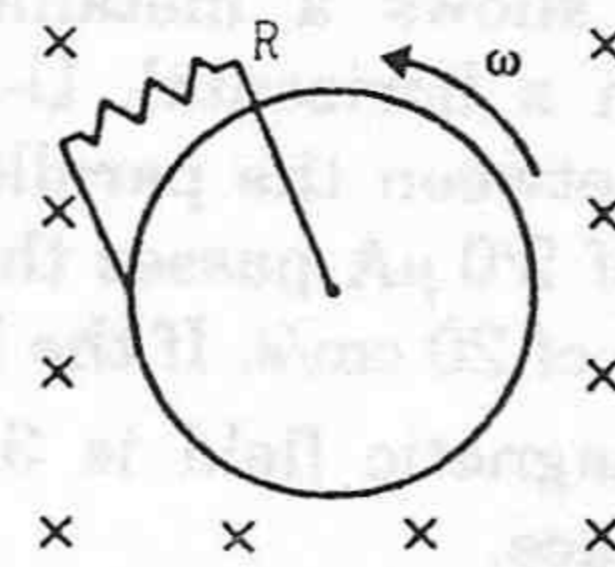


Figure 38-E25

**54.** The magnetic field in a region is given by $\vec{B} = \vec{k}\dfrac{B_0}{L}y$ where $L$ is a fixed length. A conducting rod of length $L$ lies along the Y-axis between the origin and the point $(0, L, 0)$. If the rod moves with a velocity $v = v_0 \vec{i}$, find the emf induced between the ends of the rod.

**55.** Figure (38-E26) shows a straight, long wire carrying a current $i$ and a rod of length $l$ coplanar with the wire and perpendicular to it. The rod moves with a constant velocity $v$ in a direction parallel to the wire. The distance of the wire from the centre of the rod is $x$. Find the motional emf induced in the rod.
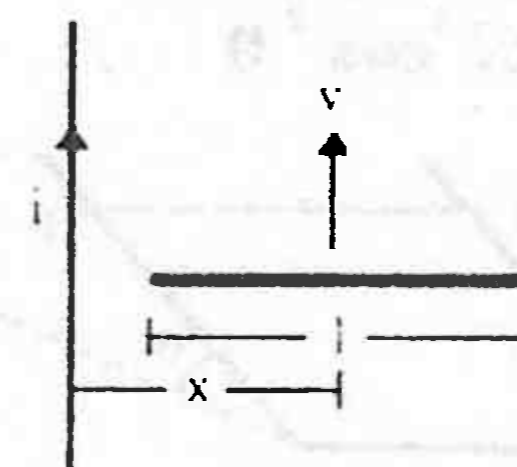


Figure 38-E26

56. Consider a situation similar to that of the previous problem except that the ends of the rod slide on a pair of thick metallic rails laid parallel to the wire. At one end the rails are connected by resistor of resistance $R$. (a) What force is needed to keep the rod sliding at a constant speed $v$? (b) In this situation what is the current in the resistance $R$? (c) Find the rate of heat developed in the resistor. (d) Find the power delivered by the external agent exerting the force on the rod.

57. Figure (38-E27) shows a square frame of wire having a total resistance $r$ placed coplanarly with a long, straight wire. The wire carries a current $i$ given by $i = i_0 \sin \omega t$. Find (a) the flux of the magnetic field through the square frame, (b) the emf induced in the frame and (c) the heat developed in the frame in the time interval 0 to $\dfrac{20\pi}{\omega}$.
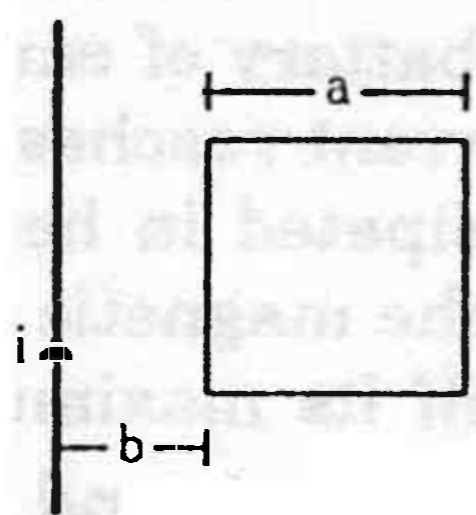


Figure 38-E27

58. A rectangular metallic loop of length $l$ and width $b$ is placed coplanarly with a long wire carrying a current $i$ (figure 38-E29). The loop is moved perpendicular to the wire with a speed $v$ in the plane containing the wire and the loop. Calculate the emf induced in the loop when the rear end of the loop is at a distance $a$ from the wire. Solve by using Faraday's law for the flux through the loop and also by replacing different segments with equivalent batteries.
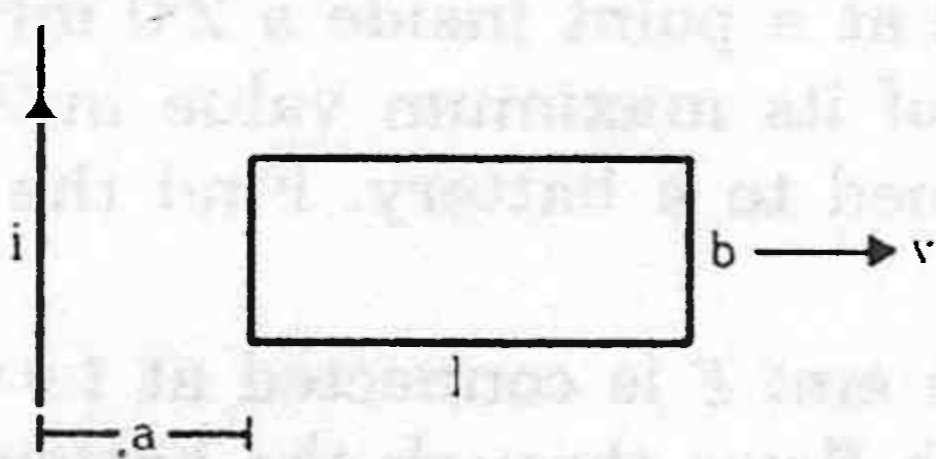


Figure 38-E28

59. Figure (38-E29) shows a conducting circular loop of radius $a$ placed in a uniform, perpendicular magnetic field $B$. A thick metal rod $OA$ is pivoted at the centre $O$. The other end of the rod touches the loop at $A$. The centre $O$ and a fixed point $C$ on the loop are connected by a wire $OC$ of resistance $R$. A force is applied at the middle point of the rod $OA$ perpendicularly, so that the rod rotates clockwise at a uniform angular velocity $\omega$. Find the force.
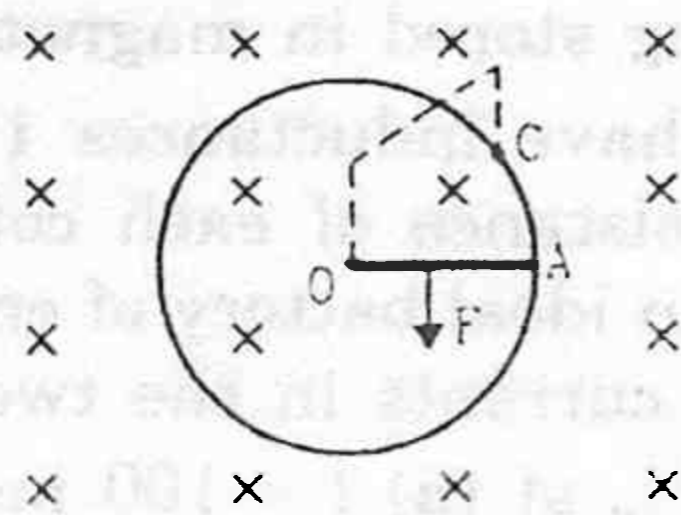


Figure 38-E29

60. Consider the situation shown in the figure of the previous problem. Suppose the wire connecting $O$ and $C$ has zero resistance but the circular loop has a resistance $R$ uniformly distributed along its length. The rod $OA$ is made to rotate with a uniform angular speed $\omega$ as shown in the figure. Find the current in the rod when $\angle AOC = 90°$.

61. Consider a variation of the previous problem (figure 38-E29). Suppose the circular loop lies in a vertical plane. The rod has a mass $m$. The rod and the loop have negligible resistances but the wire connecting $O$ and $C$ has a resistance $R$. The rod is made to rotate with a uniform angular velocity $\omega$ in the clockwise direction by applying a force at the midpoint of $OA$ in a direction perpendicular to it. Find the magnitude of this force when the rod makes an angle $\theta$ with the vertical.

62. Figure (38-E30) shows a situation similar to the previous problem. All parameters are the same except that a battery of emf $\mathcal{E}$ and a variable resistance $R$ are connected between $O$ and $C$. The connecting wires have zero resistance. No external force is applied on the rod (except gravity, forces by the magnetic field and by the pivot). In what way should the resistance $R$ be changed so that the rod may rotate with uniform angular velocity in the clockwise direction? Express your answer in terms of the given quantities and the angle $\theta$ made by the rod $OA$ with the horizontal.
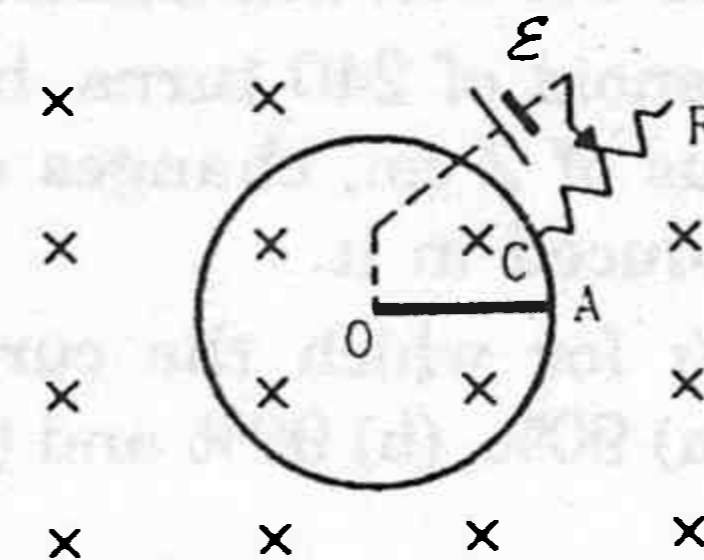


Figure 38-E30

63. A wire of mass $m$ and length $l$ can slide freely on a pair of smooth, vertical rails (figure 38-E31). A magnetic field $B$ exists in the region in the direction perpendicular to the plane of the rails. The rails are connected at the top end by a capacitor of capacitance $C$. Find the acceleration of the wire neglecting any electric resistance.
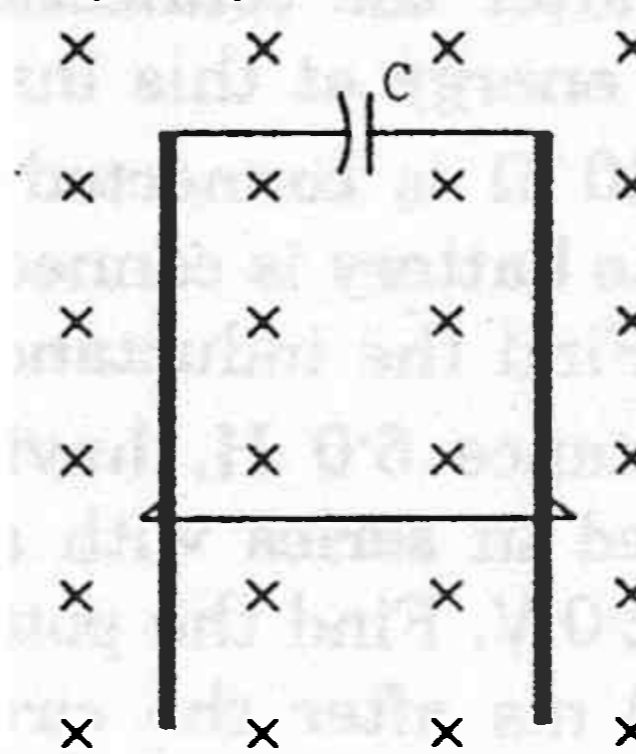


Figure 38-E31

64. A uniform magnetic field $B$ exists in a cylindrical region, shown dotted in figure (38-E32). The magnetic field increases at a constant rate $\dfrac{dB}{dt}$. Consider a circle of
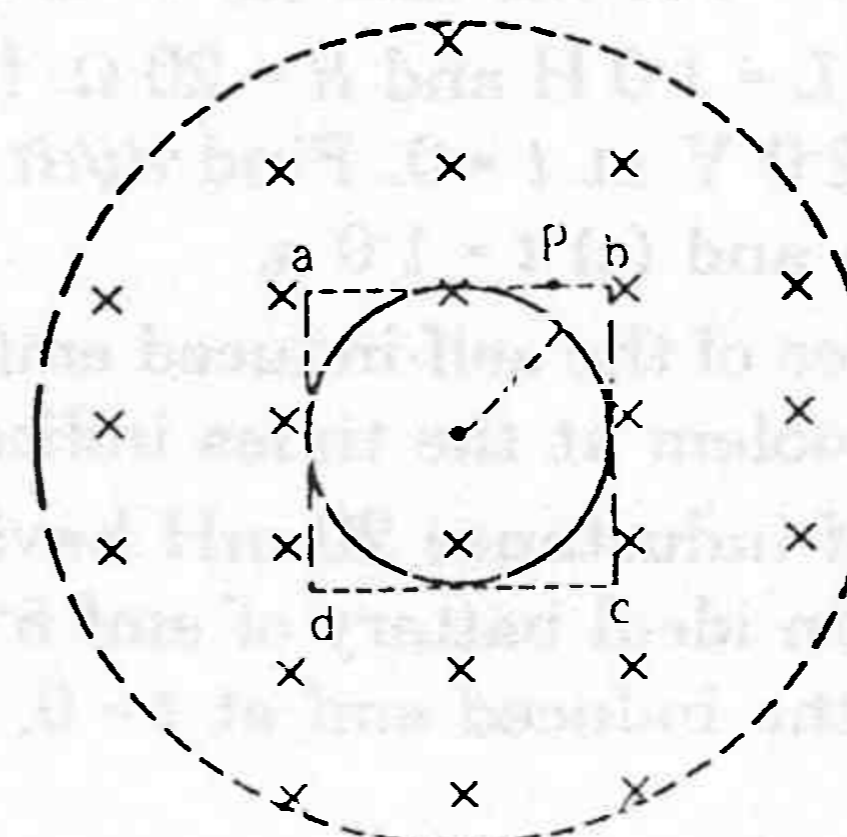


Figure 38-E32

radius $r$ coaxial with the cylindrical region. (a) Find the magnitude of the electric field $E$ at a point on the circumference of the circle. (b) Consider a point $P$ on the side of the square circumscribing the circle. Show that the component of the induced electric field at $P$ along $ba$ is the same as the magnitude found in part (a).

65. The current in an ideal, long solenoid is varied at a uniform rate of $0.01$ A/s. The solenoid has 2000 turns/m and its radius is $6.0$ cm. (a) Consider a circle of radius $1.0$ cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in $2.0$ seconds. (b) Find the electric field induced at a point on the circumference of the circle. (c) Find the electric field induced at a point outside the solenoid at a distance $8.0$ cm from its axis.

66. An average emf of 20 V is induced in an inductor when the current in it is changed from $2.5$ A in one direction to the same value in the opposite direction in $0.1$ s. Find the self-inductance of the inductor.

67. A magnetic flux of $8 \times 10^{-4}$ weber is linked with each turn of a 200-turn coil when there is an electric current of 4 A in it. Calculate the self-inductance of the coil.

68. The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at a rate of $0.8$ A/s. Find the emf induced in it.

69. Find the value of $t/\tau$ for which the current in an $LR$ circuit builds up to (a) 90%, (b) 99% and (c) 99.9% of the steady-state value.

70. An inductor-coil carries a steady-state current of $2.0$ A when connected across an ideal battery of emf $4.0$ V. If its inductance is $1.0$ H, find the time constant of the circuit.

71. A coil having inductance $2.0$ H and resistance 20 $\Omega$ is connected to a battery of emf $4.0$ V. Find (a) the current at the instant $0.20$ s after the connection is made and (b) the magnetic field energy at this instant.

72. A coil of resistance 40 $\Omega$ is connected across a $4.0$ V battery. $0.10$ s after the battery is connected, the current in the coil is 63 mA. Find the inductance of the coil.

73. An inductor of inductance $5.0$ H, having a negligible resistance, is connected in series with a 100 $\Omega$ resistor and a battery of emf $2.0$ V. Find the potential difference across the resistor 20 ms after the circuit is switched on.

74. The time constant of an $LR$ circuit is 40 ms. The circuit is connected at $t = 0$ and the steady-state current is found to be $2.0$ A. Find the current at (a) $t = 10$ ms (b) $t = 20$ ms, (c) $t = 100$ ms and (d) $t = 1$ s.

75. An $LR$ circuit has $L = 1.0$ H and $R = 20$ $\Omega$. It is connected across an emf of $2.0$ V at $t = 0$. Find $di/dt$ at (a) $t = 100$ ms, (b) $t = 200$ ms and (c) $t = 1.0$ s.

76. What are the values of the self-induced emf in the circuit of the previous problem at the times indicated therein ?

77. An inductor-coil of inductance 20 mH having resistance 10 $\Omega$ is joined to an ideal battery of emf $5.0$ V. Find the rate of change of the induced emf at $t = 0$, (b) $t = 10$ ms and (c) $t = 1.0$ s.

78. An $LR$ circuit contains an inductor of 500 mH, a resistor of $25.0$ $\Omega$ and an emf of $5.00$ V in series. Find the potential difference across the resistor at $t =$ (a) $20.0$ ms, (b) 100 ms and (c) $1.00$ s.

79. An inductor-coil of resistance 10 $\Omega$ and inductance 120 mH is connected across a battery of emf 6 V and internal resistance 2 $\Omega$. Find the charge which flows through the inductor in (a) 10 ms, (b) 20 ms and (c) 100 ms after the connections are made.

80. An inductor-coil of inductance 17 mH is constructed from a copper wire of length 100 m and cross-sectional area 1 mm$^2$. Calculate the time constant of the circuit if this inductor is joined across an ideal battery. The resistivity of copper $= 1.7 \times 10^{-8}$ $\Omega$–m.

81. An $LR$ circuit having a time constant of 50 ms is connected with an ideal battery of emf $\mathcal{E}$. Find the time elapsed before (a) the current reaches half its maximum value, (b) the power dissipated in heat reaches half its maximum value and (c) the magnetic field energy stored in the circuit reaches half its maximum value.

82. A coil having an inductance $L$ and a resistance $R$ is connected to a battery of emf $\mathcal{E}$. Find the time taken for the magnetic energy stored in the circuit to change from one fourth of the steady-state value to half of the steady-state value.

83. A solenoid having inductance $4.0$ H and resistance 10 $\Omega$ is connected to a $4.0$ V battery at $t = 0$. Find (a) the time constant, (b) the time elapsed before the current reaches $0.63$ of its steady-state value, (c) the power delivered by the battery at this instant and (d) the power dissipated in Joule heating at this instant.

84. The magnetic field at a point inside a $2.0$ mH inductor-coil becomes $0.80$ of its maximum value in 20 $\mu$s when the inductor is joined to a battery. Find the resistance of the circuit.

85. An $LR$ circuit with emf $\mathcal{E}$ is connected at $t = 0$. (a) Find the charge $Q$ which flows through the battery during 0 to $t$. (b) Calculate the work done by the battery during this period. (c) Find the heat developed during this period. (d) Find the magnetic field energy stored in the circuit at time $t$. (e) Verify that the results in the three parts above are consistent with energy conservation.

86. An inductor of inductance $2.00$ H is joined in series with a resistor of resistance 200 $\Omega$ and a battery of emf $2.00$ V. At $t = 10$ ms, find (a) the current in the circuit, (b) the power delivered by the battery, (c) the power dissipated in heating the resistor and (d) the rate at which energy is being stored in magnetic field.

87. Two coils $A$ and $B$ have inductances $1.0$ H and $2.0$ H respectively. The resistance of each coil is 10 $\Omega$. Each coil is connected to an ideal battery of emf $2.0$ V at $t = 0$. Let $i_A$ and $i_B$ be the currents in the two circuit at time $t$. Find the ratio $i_A / i_B$ at (a) $t = 100$ ms, (b) $t = 200$ ms and (c) $t = 1$ s.

88. The current in a discharging $LR$ circuit without the battery drops from $2.0$ A to $1.0$ A in $0.10$ s. (a) Find the time constant of the circuit. (b) If the inductance of the circuit is $4.0$ H, what is its resistance ?

89. A constant current exists in an inductor-coil connected to a battery. The coil is short-circuited and the battery is removed. Show that the charge flown through the coil after the short-circuiting is the same as that which flows in one time constant before the short-circuiting.

90. Consider the circuit shown in figure (38-E33). (a) Find the current through the battery a long time after the switch $S$ is closed. (b) Suppose the switch is again opened at $t = 0$. What is the time constant of the discharging circuit ? (c) Find the current through the inductor after one time constant.
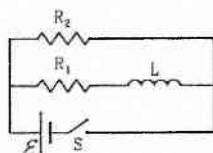


Figure 38-E33

91. A current of $1.0$ A is established in a tightly wound solenoid of radius 2 cm having 1000 turns/metre. Find the magnetic energy stored in each metre of the solenoid.

92. Consider a small cube of volume 1 mm³ at the centre of a circular loop of radius 10 cm carrying a current of 4 A. Find the magnetic energy stored inside the cube.

93. A long wire carries a current of $4.00$ A. Find the energy stored in the magnetic field inside a volume of $1.00$ mm³ at a distance of $10.0$ cm from the wire.

94. The mutual inductance between two coils is $2.5$ H. If the current in one coil is changed at the rate of 1 A/s, what will be the emf induced in the other coil ?

95. Find the mutual inductance between the straight wire and the square loop of figure (38-E27).

96. Find the mutual inductance between the circular coil and the loop shown in figure (38-E8).

97. A solenoid of length 20 cm, area of cross-section $4.0$ cm² and having 4000 turns is placed inside another solenoid of 2000 turns having a cross-sectional area $8.0$ cm² and length 10 cm. Find the mutual inductance between the solenoids.

98. The current in a long solenoid of radius $R$ and having $n$ turns per unit length is given by $i = i_0 \sin \omega t$. A coil having $N$ turns is wound around it near the centre. Find (a) the induced emf in the coil and (b) the mutual inductance between the solenoid and the coil.

□

# ANSWERS

## OBJECTIVE I

| 1. (b) | 2. (a) | 3. (d) | 4. (c) | 5. (b) | 6. (c) |
|--------|--------|--------|--------|--------|--------|
| 7. (a) | 8. (b) | 9. (b) | 10. (a) | 11. (c) | 12. (d) |

## OBJECTIVE II

1. (b), (c)  2. (d)  3. (c), (d)
4. (a), (c), (d)  5. (a), (b), (c)  6. (b), (d)
7. (d)  8. (b)  9. (a), (b), (c)
10. (b), (c)

## EXERCISE

1. $ML^2I^{-1}T^{-3}$ in each case
2. (a) volt/sec, volt, volt-sec (or weber) (b) $1.2$ volt
3. (a) $-2.0$ mV, $-4.0$ mV, $4.0$ mV, $2.0$ mV
   (b) 10 ms to 20 ms and 20 ms to 30 ms
4. $7.8 \times 10^{-3}$ V
5. $1 \times 10^{-10}$ V
6. (a) 50 V (b) 50 V (c) zero
7. (a) 25 J (b) 25 J (c) 50 J
8. (a) $0.015$ V (b) $7.5 \times 10^{-3}$ V (c) zero
9. 10 μV
10. $5.0$ T

11. $BA/R$
12. $2 \times 10^{-4}$ C
13. (a) $2Bav$ (b) $2Bav/R$ (c) $a^2 B/R$
14. $\mathcal{E} = 1.0$ V, anticlockwise
15. zero
16. (a) $3 \times 10^{-4}$ V, (b) zero, (c) $3 \times 10^{-4}$ V and (d) zero
17. $2 \times 10^{-4}$ J
18. $3.9 \times 10^{-5}$ A
19. (a) $1.25 \times 10^{-7}$ A, $a$ to $d$ (b) $1.25 \times 10^{-7}$ A, $d$ to $a$, (c) zero (d) zero
20. $\dfrac{\pi\mu_0 Na^2a'^2\mathcal{E}Rv}{2L(a^2 + x^2)^{3/2}(R' + r)}$ where $R' = R$ for part (a) and $R/2$ for part (b)
21. (a) $6.28 \times 10^{-2}$ V (b) $1.57 \times 10^{-3}$ C
22. (a) $2.0 \times 10^{-3}$ V (b) zero (c) $5.0 \times 10^{-5}$ C
23. $4.7 \times 10^{-5}$ C
24. (a) $6.6 \times 10^{-4}$ V (b) zero (c) $2.2 \times 10^{-7}$ V²
25. $1.3 \times 10^{-7}$ J
26. $1.57 \times 10^{-6}$ V
27. (a) $1.6 \times 10^{-4}$ N (b) $1.0 \times 10^{-2}$ V/m (c) $2.0 \times 10^{-3}$ V
28. $0.4$ V

29. $0.09$ V

30. $1$ mV

31. (a) zero    (b) $vB(bc)$, positive at $c$ (c) zero
    (d) $vB(bc)$, positive at $a$

32. (a) $2rvB$    (b) zero

33. $17 \times 10^{-3}$ V

34. (a) at the ends of the diameter perpendicular to the
velocity, $2\,rvB$ (b) at the ends of the diameter parallel to the
velocity, zero

35. zero

36. $\dfrac{Blv}{2r(l + vt)}$

37. (a) $\dfrac{B^2 l^2 v}{2r(l + vt)}$    (b) $l/v$

38. (a) $\dfrac{Blv}{R + r}$ (b) $\dfrac{B^2 l^2 v}{m(R + r)}$ towards left (c) $v = v_0 - \dfrac{B^2 l^2 x}{m(R + r)}$
   (d) $\dfrac{m v_0 (R + r)}{B^2 l^2}$

39. (a) $25$ m/s (b) $4.0 \times 10^{-2}$ V (c) $3.6 \times 10^{-2}$ V
   (d) $4.0 \times 10^{-3}$ V

40. $\tan^{-1}(1/3)$

42. (a) $0.1$ mA (b) zero

43. (a) zero (b) $1$ mA

44. (a) $0.1$ mA (b) $0.2$ mA

45. $\dfrac{ir - Blv}{2r}$

46. $ilBt/m$, away from the generator

47. $2\sqrt{l/g}$

48. (a) $\dfrac{RF - vl^2 B^2}{mR}$ (b) $\dfrac{RF}{l^2 B^2}$

49. (a) $\dfrac{1}{r}(E - vBl)$ from $b$ to $a$ (b) $\dfrac{lB}{r}(E - vBl)$ towards
right (c) $\dfrac{E}{Bl}$

50. (a) $vBl$    (b) $\dfrac{vBl}{r}$, $b$ to $a$    (c) $g - \dfrac{B^2 l^2}{mr}v$    (d) $\dfrac{mgr}{B^2 l^2}$

    (e) $v_m(1 - e^{-gt/v_m})$         (f) $v_m t - \dfrac{v_m^2}{g}(1 - e^{-gt/v_m})$

51. $9.4 \times 10^{-6}$ V

52. $\dfrac{1}{2}\omega r^2 B$

53. $0.5$ mA, leaves

54. $\dfrac{B_0 v_0 l}{2}$

55. $\dfrac{\mu_0 \, iv}{2\pi} \ln\left(\dfrac{2x + l}{2x - l}\right)$

56. (a) $\dfrac{v}{R}\left\{\dfrac{\mu_0 i}{2\pi}\ln\dfrac{2x + l}{2x - l}\right\}^2$ (b) $\dfrac{\mu_0\, iv}{2\pi R}\ln\dfrac{2x + l}{2x - l}$

    (c) $\dfrac{1}{R}\left\{\dfrac{\mu_0\, iv}{2\pi}\ln\dfrac{2x + l}{2x - l}\right\}^2$    (d) same as (c)

57. (a) $\dfrac{\mu_0\, ia}{2\pi}\ln\left(1 + \dfrac{a}{b}\right)$ (b) $\dfrac{\mu_0\, a i_m \omega \cos \omega t}{2\pi}\ln\left(1 + \dfrac{a}{b}\right)$

    (c) $\dfrac{5\mu_0{}^2 a^2 i_0{}^2 \omega}{2\pi r}\ln^2\left(1 + \dfrac{a}{b}\right)$

58. $\dfrac{\mu\, ilvb}{2\pi a(a + l)}$

59. $\dfrac{\omega a^3 B^2}{2R}$ to the right of $OA$ in the figure.

60. $\dfrac{8}{3}\dfrac{\omega a^2 B}{R}$

61. $\dfrac{\omega a^3 B^2}{2R} - mg \sin\theta$

62. $\dfrac{aB(2\varepsilon + \omega a^2 B)}{2\, mg \cos\theta}$

63. $\dfrac{mg}{m + CB^2 l^2}$

64. (a) $\dfrac{r}{2}\dfrac{dB}{dt}$

65. (a) $1.6 \times 10^{-8}$ weber (b) $1.2 \times 10^{-7}$ V/m
    (c) $5.6 \times 10^{-7}$ V/m

66. $0.4$ H

67. $4 \times 10^{-2}$ H

68. $6 \times 10^{-4}$ V

69. $2.3,\ 4.6,\ 6.9$

70. $0.50$ s

71. (a) $0.17$ A (b) $0.03$ J

72. $4.0$ H

73. $0.66$ V

74. (a) $0.44$ A (b) $0.79$ A (c) $1.8$ A and (d) $2.0$ A

75. (a) $0.27$ A/s (b) $0.036$ A/s and (c) $4.1 \times 10^{-9}$ A/s

76. (a) $0.27$ V (b) $0.036$ V (c) $4.1 \times 10^{-9}$ V

77. (a) $2.5 \times 10^3$ V/s (b) $17$ V/s and (c) $0.00$ V/s

78. (a) $3.16$ V (b) $4.97$ V and (c) $5.00$ V

79. (a) $1.8$ mC (b) $5.7$ mC and (c) $45$ mC

80. $10$ ms

81. (a) $35$ ms (b) $61$ ms (c) $61$ ms

82. $\tau \ln \dfrac{1}{2 - \sqrt{2}}$

83. (a) $0.40$ s (b) $0.40$ s (c) $1.0$ W and (d) $0.64$ W

84. $160\ \Omega$

85. (a) $\dfrac{\varepsilon}{R}\{t - \dfrac{L}{R}(1 - x)\}$

    (b) $\dfrac{\varepsilon^2}{R}\{t - \dfrac{L}{R}(1 - x)\}$

    (c) $\dfrac{\varepsilon^2}{R}\{t - \dfrac{L}{2R}(3 - 4x + x^2)\}$

    (d) $\dfrac{L\varepsilon^2}{2R^2}(1 - x)^2$ where $x = e^{-Rt/L}$

86. (a) $6.3$ mA (b) $12.6$ mW (c) $8.0$ mW and (d) $4.6$ mW

87. (a) 1·6 (b) 1·4 (c) 1·0
88. (a) 0·14 s (b) 28 Ω
90. (a) $\dfrac{\mathcal{E}(R_1 + R_2)}{R_1 R_2}$     (b) $\dfrac{L}{R_1 + R_2}$     (c) $\dfrac{\mathcal{E}}{R_1 e}$
91. $7{\cdot}9 \times 10^{-4}$ J
92. $8\pi \times 10^{-14}$ J
93. $2{\cdot}55 \times 10^{-14}$ J
94. 2·5 V

95. $\dfrac{\mu_0 a}{2\pi} \ln\left(1 + \dfrac{a}{b}\right)$
96. $N \dfrac{\mu_0 \pi a^2 a'^{12}}{2(a^2 + x^2)^{3/2}}$
97. $2{\cdot}0 \times 10^{-6}$ H
98. (a) $\pi\mu_0 i_0 nN\omega R^2 \cos \omega t$ (b) $\pi\mu_0 nNR^2$

**6.** *A series AC circuit contains an inductor (20 mH), a capacitor (100 μF), a resistor (50 Ω) and an AC source of 12 V, 50 Hz. Find the energy dissipated in the circuit in 1000 s.*

**Solution :**

The time period of the source is

$$T = 1/v = 20 \text{ ms}.$$

The given time 1000 s is much larger than the time period. Hence we can write the average power dissipated as

$$P_{av} = V_{rms} \, i_{rms} \cos\varphi$$

where $\cos\varphi = R/Z$ is the power factor. Thus,

$$P_{av} = V_{rms} \frac{V_{rms}}{Z} \frac{R}{Z} = \frac{R \, V_{rms}^2}{Z^2}$$

$$= \frac{(50 \ \Omega)(12 \ V)^2}{Z^2}$$

$$= \frac{7200}{Z^2} \ \Omega\text{-V}^2. \qquad \dots \text{(i)}$$

The capacitive reactance $\dfrac{1}{\omega C} = \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}} \ \Omega$

$$= \frac{100}{\pi} \ \Omega.$$

The inductive reactance $= \omega L$

$$= 2\pi \times 50 \times 20 \times 10^{-3} \ \Omega = 2\pi \ \Omega.$$

The net reactance is $X = \dfrac{1}{\omega C} - \omega L$

$$= \frac{100}{\pi} \ \Omega - 2\pi \ \Omega \approx 25 \cdot 5 \ \Omega.$$

Thus,

$$Z^2 = (50 \ \Omega)^2 + (25 \cdot 5 \ \Omega)^2 = 3150 \ \Omega^2.$$

From (i), average power $P_{av} = \dfrac{7200 \ \Omega\text{-V}^2}{3150 \ \Omega^2} = 2 \cdot 286 \ \text{W}.$

The energy dissipated in 1000 s $= P_{av} \times 1000 \text{ s}$

$$\approx 2 \cdot 3 \times 10^3 \text{ J}.$$

**7.** *An inductor of inductance 100 mH is connected in series with a resistance, a variable capacitance and an AC source of frequency 2·0 kHz. What should be the value of the capacitance so that maximum current may be drawn into the circuit ?*

**Solution :**

This is an *LCR* series circuit. The current will be maximum when the net reactance is zero. For this,

$$\frac{1}{\omega C} = \omega L$$

or, $\quad C = \dfrac{1}{\omega^2 L} = \dfrac{1}{4\pi^2 \times (2 \cdot 0 \times 10^3 \text{ s}^{-1})^2 (0 \cdot 1 \text{ H})}$

$$= 63 \text{ nF}.$$

**8.** *An inductor coil joined to a 6 V battery draws a steady current of 12 A. This coil is connected to a capacitor and an AC source of rms voltage 6 V in series. If the current in the circuit is in phase with the emf, find the rms current.*

**Solution :**

The resistance of the coil is $R = \dfrac{6 \text{ V}}{12 \text{ A}} = 0 \cdot 5 \ \Omega.$

In the AC circuit, the current is in phase with the emf. This means that the net reactance of the circuit is zero. The impedance is equal to the resistance, i.e., $Z = 0 \cdot 5 \ \Omega.$

The rms current $= \dfrac{\text{rms voltage}}{Z} = \dfrac{6 \text{ V}}{0 \cdot 5 \ \Omega} = 12 \text{ A}.$

□

# QUESTIONS FOR SHORT ANSWER

**1.** What is the reactance of a capacitor connected to a constant DC source ?

**2.** The voltage and current in a series AC circuit are given by

$$V = V_0 \cos \omega t \quad \text{and} \quad i = i_0 \sin \omega t.$$

What is the power dissipated in the circuit ?

**3.** Two alternating currents are given by

$$i_1 = i_0 \sin \omega t \quad \text{and} \quad i_2 = i_0 \sin\left(\omega t + \frac{\pi}{3}\right).$$

Will the rms values of the currents be equal or different ?

**4.** Can the peak voltage across the inductor be greater than the peak voltage of the source in an *LCR* circuit ?

**5.** In a circuit containing a capacitor and an AC source, the current is zero at the instant the source voltage is maximum. Is it consistent with Ohm's law ?

**6.** An AC source is connected to a capacitor. Will the rms current increase, decrease or remain constant if a dielectric slab is inserted into the capacitor ?

**7.** When the frequency of the AC source in an *LCR* circuit equals the resonant frequency, the reactance of the circuit is zero. Does it mean that there is no current through the inductor or the capacitor ?

**8.** When an AC source is connected to a capacitor there is a steady-state current in the circuit. Does it mean that

the charges jump from one plate to the other to complete the circuit ?

9. A current $i_1 = i_0 \sin \omega t$ passes through a resistor of resistance $R$. How much thermal energy is produced in one time period ? A current $i_2 = -i_0 \sin \omega t$ passes through the resistor. How much thermal energy is produced in one time period ? If $i_1$ and $i_2$ both pass through the resistor simultaneously, how much thermal energy is produced ? Is the principle of superposition obeyed in this case ?

10. Is energy produced when a transformer steps up the voltage ?

11. A transformer is designed to convert an AC voltage of 220 V to an AC voltage of 12 V. If the input terminals are connected to a DC voltage of 220 V, the transformer usually burns. Explain.

12. Can you have an AC series circuit in which there is a phase difference of 180° between the emf and the current ? 120° ?

13. A resistance is connected to an AC source. If a capacitor is included in the series circuit, will the average power absorbed by the resistance increase or decrease ? If an inductor of small inductance is also included in the series circuit, will the average power absorbed increase or decrease further ?

14. Can a hot-wire ammeter be used to measure a direct current having a constant value ? Do we have to change the graduations ?

## OBJECTIVE I

1. A capacitor acts as an infinite resistance for
   (a) DC
   (b) AC
   (c) DC as well as AC
   (d) neither AC nor DC.

2. An AC source producing emf
   $$\mathcal{E} = \mathcal{E}_0 \left[ \cos(100 \pi \text{ s}^{-1})t + \cos(500 \pi \text{ s}^{-1})t \right]$$
   is connected in series with a capacitor and a resistor. The steady-state current in the circuit is found to be
   $$i = i_1 \cos \left[ (100 \pi \text{ s}^{-1})t + \varphi_1 \right] + i_2 \cos \left[ (500 \pi \text{ s}^{-1})t + \varphi_2 \right]$$
   (a) $i_1 > i_2$
   (b) $i_1 = i_2$
   (c) $i_1 < i_2$
   (d) the information is insufficient to find the relation between $i_1$ and $i_2$.

3. The peak voltage in a 220 V AC source is
   (a) 220 V
   (b) about 160 V
   (c) about 310 V
   (d) 440 V.

4. An AC source is rated 220 V, 50 Hz. The average voltage is calculated in a time interval of 0·01 s. It
   (a) must be zero
   (b) may be zero
   (c) is never zero
   (d) is $(220/\sqrt{2})$V.

5. The magnetic field energy in an inductor changes from maximum value to minimum value in 5·0 ms when connected to an AC source. The frequency of the source is
   (a) 20 Hz
   (b) 50 Hz
   (c) 200 Hz
   (d) 500 Hz.

6. Which of the following plots may represent the reactance of a series $LC$ combination ?
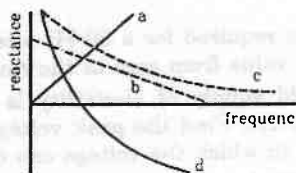


Figure 39-Q1

7. A series AC circuit has a resistance of 4 Ω and a reactance of 3 Ω. The impedance of the circuit is
   (a) 5 Ω
   (b) 7 Ω
   (c) 12/7 Ω
   (d) 7/12 Ω.

8. Transformers are used
   (a) in DC circuits only
   (b) in AC circuits only
   (c) in both DC and AC circuits
   (d) neither in DC nor in AC circuits.

9. An alternating current is given by
   $$i = i_1 \cos \omega t + i_2 \sin \omega t.$$
   The rms current is given by
   (a) $\dfrac{i_1 + i_2}{\sqrt 2}$
   (b) $\dfrac{|i_1 + i_2|}{\sqrt 2}$
   (c) $\sqrt{\dfrac{i_1^2 + i_2^2}{2}}$
   (d) $\sqrt{\dfrac{i_1^2 + i_2^2}{\sqrt 2}}$.

10. An alternating current having peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current $i$ can be used where $i$ is
    (a) 14 A
    (b) about 20 A
    (c) 7 A
    (d) about 10 A.

11. A constant current of 2·8 A exists in a resistor. The rms current is
    (a) 2·8 A
    (b) about 2 A
    (c) 1·4 A
    (d) undefined for a direct current.

## OBJECTIVE II

1. An inductor, a resistor and a capacitor are joined in series with an AC source. As the frequency of the source is slightly increased from a very low value, the reactance
   (a) of the inductor increases
   (b) of the resistor increases
   (c) of the capacitor increases
   (d) of the circuit increases.

2. The reactance of a circuit is zero. It is possible that the circuit contains
   (a) an inductor and a capacitor
   (b) an inductor but no capacitor

(c) a capacitor but no inductor

(d) neither an inductor nor a capacitor.

3. In an AC series circuit, the instantaneous current is zero when the instantaneous voltage is maximum. Connected to the source may be a

(a) pure inductor          (b) pure capacitor

(c) pure resistor

(d) combination of an inductor and a capacitor.

4. An inductor-coil having some resistance is connected to an AC source. Which of the following quantities have zero average value over a cycle ?

(a) current          (b) induced emf in the inductor

(c) Joule heat

(d) magnetic energy stored in the inductor.

5. The AC voltage across a resistance can be measured using

(a) a potentiometer          (b) a hot-wire voltmeter

(c) a moving-coil galvanometer

(d) a moving-magnet galvanometer.

6. To convert mechanical energy into electrical energy, one can use

(a) DC dynamo          (b) AC dynamo

(c) motor'          (d) transformer.

7. An AC source rated 100 V (rms) supplies a current of 10 A (rms) to a circuit. The average power delivered by the source

(a) must be 1000 W          (b) may be 1000 W

(c) may be greater than 1000 W

(d) may be less than 1000 W.

## EXERCISES

1. Find the time required for a 50 Hz alternating current to change its value from zero to the rms value.

2. The household supply of electricity is at 220 V (rms value) and 50 Hz. Find the peak voltage and the least possible time in which the voltage can change from the rms value to zero.

3. A bulb rated 60 W at 220 V is connected across a household supply of alternating voltage of 220 V. Calculate the maximum instantaneous current through the filament.

4. An electric bulb is designed to operate at 12 volts DC. If this bulb is connected to an AC source and gives normal brightness, what would be the peak voltage of the source ?

5. The peak power consumed by a resistive coil when connected to an AC source is 80 W. Find the energy consumed by the coil in 100 seconds which is many times larger than the time period of the source.

6. The dielectric strength of air is $3.0 \times 10^{6}$ V/m. A parallel-plate air-capacitor has area 20 cm$^{2}$ and plate separation 0·10 mm. Find the maximum rms voltage of an AC source which can be safely connected to this capacitor.

7. The current in a discharging LR circuit is given by $i = i_0 e^{-t/\tau}$ where $\tau$ is the time constant of the circuit. Calculate the rms current for the period $t = 0$ to $t = \tau$.

8. A capacitor of capacitance 10 μF is connected to an oscillator giving an output voltage $\mathcal{E} = (10\ V)\sin \omega t$. Find the peak currents in the circuit for $\omega = 10\ s^{-1}$, $100\ s^{-1}$, $500\ s^{-1}$, $1000\ s^{-1}$.

9. A coil of inductance 5·0 mH and negligible resistance is connected to the oscillator of the previous problem. Find the peak currents in the circuit for $\omega = 100\ s^{-1}$, $500\ s^{-1}$, $1000\ s^{-1}$.

10. A coil has a resistance of 10 Ω and an inductance of 0·4 henry. It is connected to an AC source of 6·5 V, $\dfrac{30}{\pi}$ Hz. Find the average power consumed in the circuit.

11. A resistor of resistance 100 Ω is connected to an AC source $\mathcal{E} = (12\ V)\sin (250\ \pi\ s^{-1})t$. Find the energy dissipated as heat during $t = 0$ to $t = 1.0$ ms.

12. In a series RC circuit with an AC source, $R = 300\ \Omega$, $C = 25$ μF, $\mathcal{E}_0 = 50$ V and $v = 50/\pi$ Hz. Find the peak current and the average power dissipated in the circuit.

13. An electric bulb is designed to consume 55 W when operated at 110 volts. It is connected to a 220 V, 50 Hz line through a choke coil in series. What should be the inductance of the coil for which the bulb gets correct voltage ?

14. In a series LCR circuit with an AC source, $R = 300\ \Omega$, $C = 20$ μF, $L = 1.0$ henry, $\mathcal{E}_0 = 50$ V and $v = 50/\pi$ Hz. Find (a) the rms current in the circuit and (b) the rms potential differences across the capacitor, the resistor and the inductor. Note that the sum of the rms potential differences across the three elements is greater than the rms voltage of the source.

15. Consider the situation of the previous problem. Find the average electric field energy stored in the capacitor and the average magnetic field energy stored in the coil.

16. An inductance of 2·0 H, a capacitance of 18 μF and a resistance of 10 kΩ are connected to an AC source of 20 V with adjustable frequency. (a) What frequency should be chosen to maximise the current in the circuit ? (b) What is the value of this maximum current ?

17. An inductor-coil, a capacitor and an AC source of rms voltage 24 V are connected in series. When the frequency of the source is varied, a maximum rms current of 6·0 A is observed. If this inductor coil is connected to a battery of emf 12 V and internal resistance 4·0 Ω, what will be the current ?

18. Figure (39-E1) shows a typical circuit for low-pass filter. An AC input $V_i = 10$ mV is applied at the left end and
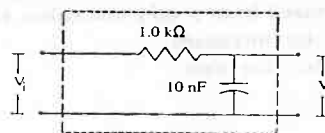


Figure 39-E1

the output $V_0$ is received at the right end. Find the output voltages for $v = 10$ kHz, 100kHz, 1·0 MHz and 10·0 MHz. Note that as the frequency is increased the output decreases and hence the name low-pass filter.

19. A transformer has 50 turns in the primary and 100 in the secondary. If the primary is connected to a 220 V DC supply, what will be the voltage across the secondary ?

□

# ANSWERS

## OBJECTIVE I

| 1. (a) | 2. (c) | 3. (c) | 4. (b) | 5. (b) | 6. (d) |
|---|---|---|---|---|---|
| 7. (a) | 8. (b) | 9. (c) | 10. (d) | 11. (a) | |

## OBJECTIVE II

1. (a)          2. (a), (d)          3. (a), (b), (d)
4. (a), (b)     5. (b)               6. (a), (b)
7. (b), (d)

## EXERCISES

1. 2·5 ms
2. 311 V, 2·5 ms
3. 0·39 A
4. 17 volts
5. 4·0 kJ

6. 210 V

7. $\dfrac{i_o}{e}\sqrt{(e^2 - 1)/2}$

8. $1·0 \times 10^{-5}$ A, 0·01 A, 0·05 A, 0·1 A
9. 20 A, 4·0 A, 0·20 A
10. 5/8 W
11. 2·61 × 10$^{-5}$ J
12. 0·10 A, 1·5 W
13. 1·2 H
14. (a) 0·10 A (b) 50 V, 30 V, 10 V
15. 25 mJ, 5 mJ
16. (a) 27 Hz (b) 2 mA
17. 1·5 A
18. 8·5 mV, 1 6 mV, 0 16 mV, 16 μV
19. zero.

# CHAPTER 40

# ELECTROMAGNETIC WAVES

## 40.1 INTRODUCTION

We have seen that in certain situations light may be described as a wave. The wave equation for light propagating in $x$-direction in vacuum may be written as

$$E = E_0 \sin \omega(t - x/c)$$

where $E$ is the sinusoidally varying electric field at the position $x$ at time $t$. The constant $c$ is the speed of light in vacuum. The electric field $E$ is in the $Y$–$Z$ plane, that is, perpendicular to the direction of propagation.

There is also a sinusoidally varying magnetic field associated with the electric field when light propagates. This magnetic field is perpendicular to the direction of propagation as well as to the electric field $E$. It is given by

$$B = B_0 \sin \omega(t - x/c).$$

Such a combination of mutually perpendicular electric and magnetic fields is referred to as an *electromagnetic wave* in vacuum. The theory of electromagnetic wave was mainly developed by Maxwell around 1864. We give a brief discussion of this theory.

## 40.2 MAXWELL'S DISPLACEMENT CURRENT

We have stated Ampere's law as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i \qquad \ldots (40.1)$$

where $i$ is the electric current crossing a surface bounded by a closed curve and the line integral of $\vec{B}$ (circulation) is calculated along that closed curve. This equation is valid only when the electric field at the surface does not change with time. This law tells us that an electric current produces magnetic field and gives a method to calculate the field.

Ampere's law in this form is not valid if the electric field at the surface varies with time. As an example, consider a parallel-plate capacitor with circular plates, being charged by a battery (figure 40.1). If we place a compass needle in the space between the plates, the

needle, in general, deflects. This shows that there is a magnetic field in this region. Figure (40.1) also shows a closed curve $\gamma$ which lies completely in the region between the plates. The plane surface $S$ bounded by this curve is also parallel to the plates and lies completely inside the region between the plates.
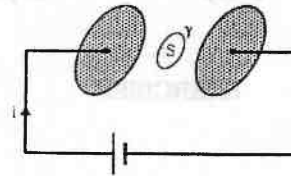


Figure 40.1

During the charging process, there is an electric current through the connecting wires. Charge is accummulated on the plates and the electric field at the points on the surface $S$ changes. It is found that there is a magnetic field at the points on the curve $\gamma$ and the circulation

$$\oint \vec{B} \cdot d\vec{l}$$

has a nonzero value. As no charge crosses the surface $S$, the electric current $i$ through the surface is zero. Hence,

$$\oint \vec{B} \, d\vec{l} \neq \mu_0 \, i. \qquad \ldots (i)$$

Now, Ampere's law (40.1) can be deduced from Biot–Savart law. We can calculate the magnetic field due to each current element from Biot–Savart law and then its circulation along the closed curve $\gamma$. The circulation of the magnetic field due to these current elements must satisfy equation (40.1). If we denote this magnetic field by $\vec{B}'$,

$$\oint \vec{B}' \cdot d\vec{l} = 0. \qquad \ldots (ii)$$

This shows that the actual magnetic field $\vec{B}$ is different from the field $\vec{B}'$ produced by the electric currents only. So, there must be some other source of magnetic field. This other source is nothing but the

$= (1.6 \times 10^{-19} \text{ C}) \times (2.0 \times 10^7 \text{ m/s}) \times (10^{-6} \text{ T})$

$= 3.2 \times 10^{-18} \text{ N}.$

**4.** *Find the energy stored in a 60 cm length of a laser beam operating at 4 mW.*

**Solution :**
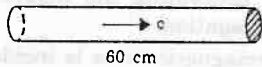


60 cm

Figure 40-W1

The time taken by the electromagnetic wave to move through a distance of 60 cm is $t = \dfrac{60 \text{ cm}}{c} = 2 \times 10^{-9}$ s. The energy contained in the 60 cm length passes through a cross-section of the beam in $2 \times 10^{-9}$ s (figure 40-W1). But the energy passing through any cross-section in $2 \times 10^{-9}$ s is

$$U = (4 \text{ mW}) \times (2 \times 10^{-9} \text{ s})$$

$= (4 \times 10^{-3} \text{ J/s}) \times (2 \times 10^{-9} \text{ s})$

$= 8 \times 10^{-12} \text{ J}.$

This is the energy contained in 60 cm length.

**5.** *Find the amplitude of the electric field in a parallel beam of light of intensity* $2.0 \text{ W/m}^2$.

**Solution :**

The intensity of a plane electromagnetic wave is

$$I = u_{av} c = \frac{1}{2} \varepsilon_0 E_0^2 c$$

or, $\quad E_0 = \sqrt{\dfrac{2I}{\varepsilon_0 c}}$

$$= \sqrt{\dfrac{2 \times (2.0 \text{ W/m}^2)}{\left(8.85 \times 10^{-12} \dfrac{\text{C}^2}{\text{N-m}^2}\right) \times (3 \times 10^8 \text{ m/s})}}$$

$= 38.8 \text{ N/C}.$

□

# QUESTIONS FOR SHORT ANSWER

**1.** In a microwave oven, the food is kept in a plastic container and the microwave is directed towards the food. The food is cooked without melting or igniting the plastic container. Explain.

**2.** A metal rod is placed along the axis of a solenoid carrying a high-frequency alternating current. It is found that the rod gets heated. Explain why the rod gets heated.

**3.** Can an electromagnetic wave be deflected by an electric field ? By a magnetic field ?

**4.** A wire carries an alternating current $i = i_0 \sin \omega t$. Is there an electric field in the vicinity of the wire ?

**5.** A capacitor is connected to an alternating-current source. Is there a magnetic field between the plates ?

**6.** Can an electromagnetic wave be polarized ?

**7.** A plane electromagnetic wave is passing through a region. Consider the quantities (a) electric field, (b) magnetic field, (c) electrical energy in a small volume and (d) magnetic energy in a small volume. Construct pairs of the quantities that oscillate with equal frequencies.

# OBJECTIVE I

**1.** A magnetic field can be produced by
(a) a moving charge        (b) a changing electric field
(c) none of them        (d) both of them.

**2.** A compass needle is placed in the gap of a parallel plate capacitor. The capacitor is connected to a battery through a resistance. The compass needle
(a) does not deflect
(b) deflects for a very short time and then comes back to the original position
(c) deflects and remains deflected as long as the battery is connected
(d) deflects and gradually comes to the original position in a time which is large compared to the time constant.

**3.** Dimensions of $1/(\mu_0 \varepsilon_0)$ is
(a) L/T        (b) T/L        (c) $L^2/T^2$        (d) $T^2/L^2$.

**4.** Electromagnetic waves are produced by
(a) a static charge        (b) a moving charge
(c) an accelerating charge        (d) chargeless particles.

**5.** An electromagnetic wave going through vacuum is described by
$$E = E_0 \sin(kx - \omega t); \quad B = B_0 \sin(kx - \omega t).$$
Then
(a) $E_0 k = B_0 \omega$        (b) $E_0 B_0 = \omega k$
(c) $E_0 \omega = B_0 k$        (d) none of these.

6. An electric field $\vec{E}$ and a magnetic field $\vec{B}$ exist in a region. The fields are not perpendicular to each other.
   (a) This is not possible.
   (b) No electromagnetic wave is passing through the region.
   (c) An electromagnetic wave may be passing through the region.
   (d) An electromagnetic wave is certainly passing through the region.

7. Consider the following two statements regarding a linearly polarized, plane electromagnetic wave:
   (A) The electric field and the magnetic field have equal average values.
   (B) The electric energy and the magnetic energy have equal average values.
   (a) Both A and B are true.   (b) A is false but B is true.
   (c) B is false but A is true.   (d) Both A and B are false.

8. A free electron is placed in the path of a plane electromagnetic wave. The electron will start moving
   (a) along the electric field
   (b) along the magnetic field
   (c) along the direction of propagation of the wave
   (d) in a plane containing the magnetic field and the direction of propagation.

9. A plane electromagnetic wave is incident on a material surface. The wave delivers momentum $p$ and energy $E$.
   (a) $p = 0, E \neq 0$.   (b) $p \neq 0, E = 0$.
   (c) $p \neq 0, E \neq 0$.   (d) $p = 0, E = 0$.

## OBJECTIVE II

1. An electromagnetic wave going through vacuum is described by
   $$E = E_0 \sin(kx - \omega t).$$
   Which of the following is/are independent of the wavelength ?
   (a) $k$   (b) $\omega$   (c) $k/\omega$   (d) $k\omega$.

2. Displacement current goes through the gap between the plates of a capacitor when the charge of the capacitor
   (a) increases   (b) decreases
   (c) does not change   (d) is zero.

3. Speed of electromagnetic waves is the same

   (a) for all wavelengths   (b) in all media
   (c) for all intensities   (d) for all frequencies.

4. Which of the following have zero average value in a plane electromagnetic wave ?
   (a) electric field   (b) magnetic field
   (c) electric energy   (d) magnetic energy.

5. The energy contained in a small volume through which an electromagnetic wave is passing oscillates with
   (a) zero frequency   (b) the frequency of the wave
   (c) half the frequency of the wave
   (d) double the frequency of the wave.

## EXERCISES

1. Show that the dimensions of the displacement current $\varepsilon_0 \dfrac{d\Phi_E}{dt}$ are that of an electric current.

2. A point charge is moving along a straight line with a constant velocity $v$. Consider a small area $A$ perpendicular to the direction of motion of the charge (figure 40-E1). Calculate the displacement current through the area when its distance from the charge is $x$. The value of $x$ is not large so that the electric field at any instant is essentially given by Coulomb's law.
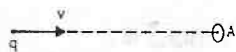


Figure 40-E1

3. A parallel-plate capacitor having plate-area $A$ and plate separation $d$ is joined to a battery of emf $\mathcal{E}$ and internal resistance $R$ at $t = 0$. Consider a plane surface of area $A/2$, parallel to the plates and situated symmetrically between them. Find the displacement current through this surface as a function of time.

4. Consider the situation of the previous problem. Define displacement resistance $R_d = V/i_d$ of the space between the plates where $V$ is the potential difference between the plates and $i_d$ is the displacement current. Show that $R_d$ varies with time as
   $$R_d = R(e^{t/\tau} - 1).$$

5. Using $B = \mu_0 H$ find the ratio $E_0/H_0$ for a plane electromagnetic wave propagating through vacuum. Show that it has the dimensions of electric resistance. This ratio is a universal constant called the *impedance of free space*.

6. The sunlight reaching the earth has maximum electric field of 810 V/m. What is the maximum magnetic field in this light ?

7. The magnetic field in a plane electromagnetic wave is given by
   $$B = (200 \, \mu T) \sin [(4.0 \times 10^{15} \, s^{-1})(t - x/c)].$$
   Find the maximum electric field and the average energy density corresponding to the electric field.

8. A laser beam has intensity $2.5 \times 10^{14}$ W/m$^2$. Find the amplitudes of electric and magnetic fields in the beam.

9. The intensity of the sunlight reaching the earth is 1380 W/m$^2$. Assume this light to be a plane, monochromatic wave. Find the amplitudes of electric and magnetic fields in this wave.

□

# ANSWERS

OBJECTIVE I

1. (d)    2. (d)    3. (c)    4. (c)    5. (a)    6. (c)
7. (a)    8. (a)    9. (c)

OBJECTIVE II

1. (c)    2. (a), (b)         3. (c),    4. (a), (b)
5. (d)

EXERCISES

2. $\dfrac{q\,Av}{2\pi x^{3}}$

3. $\dfrac{\mathcal{E}}{2R}\,e^{-\frac{Id}{\varepsilon AR}}$

5. $377\ \Omega$

6. $2\cdot7\ \mu T$

7. $6\times10^{4}\ \text{N/C},\ 0\cdot016\ \text{J/m}^{3}$

8. $4\cdot3\times10^{8}\ \text{N/C},\ 1\cdot44\ \text{T}$

9. $1\cdot02\times10^{3}\ \text{N/C},\ 3\cdot40\times10^{-5}\ \text{T}$

□

$$n = \frac{7.93 \times 10^{-19}\,C}{1.6 \times 10^{-19}\,C} = 4.96.$$

It is clear that 5 electrons are attached to the drop.

7. *Show that the dynamic plate resistance of a diode is* $\frac{2}{3}\frac{V}{i}$ *where V and i are the plate voltage and the plate current respectively. Assume Langmuir–Child equation to hold.*

**Solution :**

The dynamic plate resistance of the diode is $R = \frac{dV}{di}$.

The Langmuir–Child equation is
$$i = cV^{3/2} \qquad \ldots \text{(i)}$$
where $c$ is a constant for a given diode. This gives
$$\frac{di}{dV} = \frac{3}{2}cV^{1/2}. \qquad \ldots \text{(ii)}$$
Dividing (ii) by (i), $\frac{1}{i}\frac{di}{dV} = \frac{3}{2V}$
or, $\frac{dV}{di} = \frac{2V}{3i}$.

8. *The mutual conductance of a triode valve is 2·5 millimho. Find the change in the plate current if the grid voltage is changed from − 2·0 V to − 4·5 V.*

**Solution :**

The mutual conductance of a triode valve is
$$g_m = \left[\frac{\Delta i_p}{\Delta V_g}\right]_{\Delta V_p = 0}$$
or, $\Delta i_p = g_m \Delta V_g$
$$= (2.5 \times 10^{-3}\,\Omega^{-1}) \times (-4.5\,V + 2.0\,V)$$
$$= -6.25 \times 10^{-3}\,A.$$

9. *A triode valve has amplification factor 21 and dynamic plate resistance 10 kΩ. This is used as an amplifier with a load of 20 kΩ. Find the gain factor of the amplifier.*

**Solution :**

The gain factor of a triode valve amplifier is
$$A = \frac{\mu}{1 + \frac{r_p}{R_L}}$$
where $\mu$ is the amplification factor, $r_p$ is the plate resistance and $R_L$ is the load resistance. Thus,
$$A = \frac{21}{1 + \frac{10\,k\Omega}{20\,k\Omega}} = 14.$$

□

## QUESTIONS FOR SHORT ANSWER

1. Why is conduction easier in gases if the pressure is low ? Will the conduction continue to improve if the pressure is made as low as nearly zero ?
2. An AC source is connected to a diode and a resistor in series. Is the current through the resistor AC or DC ?
3. How will the thermionic current vary if the filament current is increased ?
4. Would you prefer a material having a high melting point or a low melting point to be used as a cathode in a diode ?
5. Would you prefer a material having a high work function or a low work function to be used as a cathode in a diode ?
6. An isolated metal sphere is heated to a high temperature. Will it become positively charged due to thermionic emission ?
7. A diode valve is connected to a battery and a load resistance. The filament is heated so that a constant current is obtained in the circuit. As the cathode continuously emits electrons, does it get more and more positively charged ?
8. Why does thermionic emission not take place in nonconductors ?
9. The cathode of a diode valve is replaced by another cathode of double the surface area. Keeping the voltage and temperature conditions the same, will the plate current decrease, increase or remain the same ?
10. Why is the linear portion of the triode characteristic chosen to operate the triode as an amplifier ?

## OBJECTIVE I

1. Cathode rays constitute a stream of
   (a) electrons  (b) protons
   (c) positive ions  (d) negative ions.
2. Cathode rays are passing through a discharge tube. In the tube, there is
   (a) an electric field but no magnetic field

(b) a magnetic field but no electric field
(c) an electric as well as a magnetic field
(d) neither an electric nor a magnetic field.

3. Let $i_0$ be the thermionic current from a metal surface when the absolute temperature of the surface is $T_0$. The temperature is slowly increased and the thermionic current is measured as a function of temperature. Which of the following plots may represent the variation in $(i/i_0)$ against $(T/T_0)$?
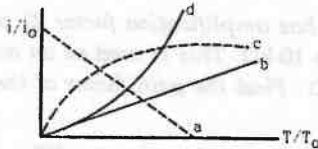


Figure 41-Q1

4. When the diode shows saturated current, dynamic plate resistance is
(a) zero                  (b) infinity
(c) indeterminate    (d) different for different diodes.

5. The anode of a thermionic diode is connected to the negative terminal of a battery and the cathode to its positive terminal.
(a) No appreciable current will pass through the diode.

(b) A large current will pass through the diode from the anode to the cathode.
(c) A large current will pass through the diode from the cathode to the anode.
(d) The diode will be damaged.

6. A diode, a resistor and a 50 Hz AC source are connected in series. The number of current pulses per second through the resistor is
(a) 25          (b) 50          (c) 100          (d) 200.

7. A triode is operated in the linear region of its characteristics. If the plate voltage is slightly increased, the dynamic plate resistance will
(a) increase                              (b) decrease
(c) remain almost the same
(d) become zero.

8. The plate current in a triode valve is maximum when the potential of the grid is
(a) positive     (b) zero     (c) negative     (d) nonpositive.

9. The amplification factor of a triode operating in the linear region depends strongly on
(a) the temperature of the cathode
(b) the plate potential          (c) the grid potential
(d) the separations of the grid from the cathode and the anode.

## OBJECTIVE II

1. Electric conduction takes place in a discharge tube due to the movement of
(a) positive ions                  (b) negative ions
(c) electrons                       (d) protons.

2. Which of the following are true for cathode ray?
(a) It travels along straight lines.
(b) It emits X-ray when strikes a metal.
(c) It is an electromagnetic wave.
(d) It is not deflected by magnetic field.

3. Because of the space charge in a diode valve,
(a) the plate current decreases
(b) the plate voltage increases
(c) the rate of emission of thermions increases
(d) the saturation current increases.

4. The saturation current in a triode valve can be changed by changing
(a) the grid voltage
(b) the plate voltage

(c) the separation between the grid and the cathode
(d) the temperature of the cathode.

5. Mark the correct options.
(a) A diode valve can be used as a rectifier.
(b) A triode valve can be used as a rectifier.
(c) A diode valve can be used as an amplifier.
(d) A triode valve can be used as an amplifier.

6. The plate current in a diode is zero. It is possible that
(a) the plate voltage is zero
(b) the plate voltage is slightly negative
(c) the plate voltage is slightly positive
(d) the temperature of the filament is low.

7. The plate current in a triode valve is zero. The temperature of the filament is high. It is possible that
(a) $V_g > 0$, $V_p > 0$              (b) $V_g > 0$, $V_p < 0$
(c) $V_g < 0$, $V_p > 0$              (d) $V_g < 0$, $V_p < 0$.

## EXERCISES

1. A discharge tube contains helium at a low pressure. A large potential difference is applied across the tube. Consider a helium atom that has just been ionized due to the detachment of an atomic electron. Find the ratio of the distance travelled by the free electron to that by the positive ion in a short time $dt$ after the ionization.

2. A molecule of a gas, filled in a discharge tube, gets ionized when an electron is detached from it. An electric field of $5.0$ kV/m exists in the vicinity of the event. (a) Find the distance travelled by the free electron in 1 µs assuming no collision. (b) If the mean free path of the electron is $1.0$ mm, estimate the time of transit of the free electron between successive collisions.

3. The mean free path of electrons in the gas in a discharge tube is inversely proportional to the pressure inside it. The Crookes dark space occupies half the length of the discharge tube when the pressure is 0·02 mm of mercury. Estimate the pressure at which the dark space will fill the whole tube.

4. Two discharge tubes have identical material structure and the same gas is filled in them. The length of one tube is 10 cm and that of the other tube is 20 cm. Sparking starts in both the tubes when the potential difference between the cathode and the anode is 100 V. If the pressure in the shorter tube is 1·0 mm of mercury, what is the pressure in the longer tube ?

5. Calculate $n(T)/n(1000 \text{ K})$ for tungsten emitter at $T = 300$ K, 2000 K and 3000 K where $n(T)$ represents the number of thermions emitted per second by the surface at temperature $T$. Work function of tungsten is 4·52 eV.

6. The saturation current from a thoriated-tungsten cathode at 2000 K is 100 mA. What will be the saturation current for a pure-tungsten cathode of the same surface area operating at the same temperature ? The constant $A$ in the Richardson–Dushman equation is $60 \times 10^4$ A/m$^2$-K$^2$ for pure tungsten and $3·0 \times 10^4$ A/m$^2$-K$^2$ for thoriated tungsten. The work function of pure tungsten is 4·5 eV and that of thoriated tungsten is 2·6 eV.

7. A tungsten cathode and a thoriated-tungsten cathode have the same geometrical dimensions and are operated at the same temperature. The thoriated-tungsten cathode gives 5000 times more current than the other one. Find the operating temperature. Take relevant data from the previous problem.

8. If the temperature of a tungsten filament is raised from 2000 K to 2010 K, by what factor does the emission current change ? Work function of tungsten is 4·5 eV.

9. The constant $A$ in the Richardson–Dushman equation for tungsten is $60 \times 10^4$ A/m$^2$-K$^2$. The work function of tungsten is 4·5 eV. A tungsten cathode having a surface area $2·0 \times 10^{-5}$ m$^2$ is heated by a 24 W electric heater. In steady state, the heat radiated by the cathode equals the energy input by the heater and the temperature becomes constant. Assuming that the cathode radiates like a blackbody, calculate the saturation current due to thermions. Take Stefan constant $= 6 \times 10^{-8}$ W/m$^2$-K$^4$. Assume that the thermions take only a small fraction of the heat supplied.

10. A plate current of 10 mA is obtained when 60 volts are applied across a diode tube. Assuming the Langmuir–Child equation $i_p \propto V_p^{3/2}$ to hold, find the dynamic resistance $r_p$ in this operating condition.

11. The plate current in a diode is 20 mA when the plate voltage is 50 V or 60 V. What will be the current if the plate voltage is 70 V ?

12. The power delivered in the plate circuit of a diode is 1·0 W when the plate voltage is 36 V. Find the power delivered if the plate voltage is increased to 49 V. Assume Langmuir–Child equation to hold.

13. A triode valve operates at $V_p = 225$ V and $V_g = -0·5$ V. The plate current remains unchanged if the plate voltage is increased to 250 V and the grid voltage is decreased to $-2·5$ V. Calculate the amplification factor.

14. Calculate the amplification factor of a triode valve which has plate resistance of 2 kΩ and transconductance of 2 millimho.

15. The dynamic plate resistance of a triode valve is 10 kΩ. Find the change in the plate current if the plate voltage is changed from 200 V to 220 V.

16. Find the values of $r_p$, $\mu$ and $g_m$ of a triode operating at plate voltage 200 V and grid voltage $-6$ V. The plate characteristics are shown in figure (41-E1).
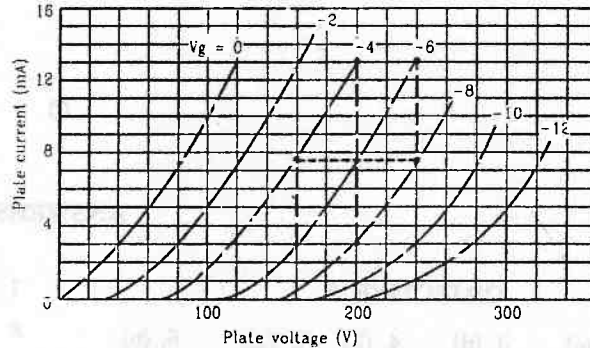


Figure 41-E1

17. The plate resistance of a triode is 8 kΩ and the transconductance is 2·5 millimho. (a) If the plate voltage is increased by 48 V, and the grid voltage is kept constant, what will be the increase in the plate current ? (b) With plate voltage kept constant at this increased value, how much should the grid voltage be decreased in order to bring the plate current back to its initial value ?

18. The plate resistance and the amplification factor of a triode are 10 kΩ and 20. The tube is operated at plate voltage 250 V and grid voltage $-7·5$ V. The plate current is 10 mA. (a) To what value should the grid voltage be changed so as to increase the plate current to 15 mA ? (b) To what value should the plate voltage be changed to take the plate current back to 10 mA ?

19. The plate current, plate voltage and grid voltage of a 6F6 triode tube are related as

$$i_p = 41(V_p + 7 V_g)^{1·41}$$

where $V_p$ and $V_g$ are in volts and $i_p$ in microamperes. The tube is operated at $V_p = 250$ V, $V_g = -20$ V. Calculate (a) the tube current, (b) the plate resistance, (c) the mutual conductance and (d) the amplification factor.

20. The plate current in a triode can be written as

$$i_p = k\left(V_g + \frac{V_p}{\mu}\right)^{3/2}.$$

Show that the mutual conductance is proportional to the cube root of the plate current.

21. A triode has mutual conductance $= 2·0$ millimho and plate resistance $= 20$ kΩ. It is desired to amplify a signal

by a factor of 30. What load resistance should be added in the circuit?

22. The gain factor of an amplifier is increased from 10 to 12 as the load resistance is changed from 4 kΩ to 8 kΩ. Calculate (a) the amplification factor and (b) the plate resistance.

23. Figure (41-E2) shows two identical triode tubes connected in parallel. The anodes are connected together, the grids are connected together and the cathodes are connected together. Show that the equivalent plate resistance is half of the individual plate resistance, the equivalent mutual conductance is double
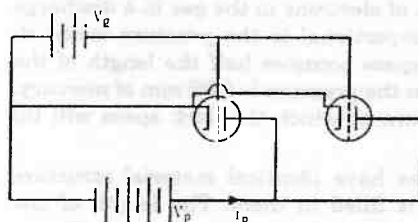


Figure 41-E2

the individual mutual conductance and the equivalent amplification factor is the same as the individual amplification factor.

# ANSWERS

## OBJECTIVE I

1. (a)  2. (c)  3. (d)  4. (b)  5. (a)  6. (b)
7. (c)  8. (a)  9. (d)

## OBJECTIVE II

1. (a), (b), (c)  2. (a), (b)  3. (a)
4. (d)  5. (a), (b), (d)  6. all
7. (b), (c), (d)

## EXERCISES

1. 7340

2. (a) 440 m  (b) 1·5 ns

3. 0·01 mm of mercury

4. 0·5 mm of mercury

5. $6·57 \times 10^{-55}$, $9·73 \times 10^{-11}$. $1·37 \times 10^{16}$

6. 33 µA

7. 1914 K
8. 1·14
9. 1·0 mA
10. 4 kΩ
11. 20 mA
12. 2·2 W
13. 12·5
14. 4
15. 2 mA
16. 8·0 kΩ, 20 and 2·5 millimho
17. (a) 6 mA (b) 2·4 V
18. (a) – 5·0 V (b) 200 V
19. (a) 30 mA (b) 2·53 kΩ (c) 2·77 millimho (d) 7
21. 60 kΩ
22. (a) 15 (b) 2 kΩ

13. *Light described at a place by the equation*

$E = (100 \text{ V/m}) [\sin (5 \times 10^{15} \text{ s}^{-1}) t + \sin (8 \times 10^{15} \text{ s}^{-1})t]$

*falls on a metal surface having work function 2·0 eV. Calculate the maximum kinetic energy of the photoelectrons.*

**Solution :** The light contains two different frequencies. The one with larger frequency will cause photoelectrons with largest kinetic energy. This larger frequency is

$\nu = \dfrac{\omega}{2\pi} = \dfrac{8 \times 10^{15} \text{ s}^{-1}}{2\pi}.$

The maximum kinetic energy of the photoelectrons is

$K_{max} = h\nu - \varphi$

$= (4·14 \times 10^{-15} \text{ eV-s}) \times \left(\dfrac{8 \times 10^{15}}{2\pi} \text{ s}^{-1}\right) - 2·0 \text{ eV}$

$= 5·27 \text{ eV} - 2·0 \text{ eV} = 3·27 \text{ eV}.$

□

## QUESTIONS FOR SHORT ANSWER

1. Can we find the mass of a photon by the definition $p = mv$ ?

2. Is it always true that for two sources of equal intensity, the number of photons emitted in a given time are equal ?

3. What is the speed of a photon with respect to another photon if (a) the two photons are going in the same direction and (b) they are going in opposite directions ?

4. Can a photon be deflected by an electric field ? By a magnetic field ?

5. A hot body is placed in a closed room maintained at a lower temperature. Is the number of photons in the room increasing ?

6. Should the energy of a photon be called its kinetic energy or its internal energy ?

7. In an experiment on photoelectric effect, a photon is incident on an electron from one direction and the photoelectron is emitted almost in the opposite direction. Does this violate conservation of momentum ?

8. It is found that yellow light does not eject photoelectrons from a metal. Is it advisable to try with orange light ? With green light ?

9. It is found that photosynthesis starts in certain plants when exposed to the sunlight but it does not start if the plant is exposed only to infrared light. Explain.

10. The threshold wavelength of a metal is $\lambda_0$. Light of wavelength slightly less than $\lambda_0$ is incident on an insulated plate made of this metal. It is found that photoelectrons are emitted for sometime and after that the emission stops. Explain.

11. Is $p = E/c$ valid for electrons ?

12. Consider the de Broglie wavelength of an electron and a proton. Which wavelength is smaller if the two particles have (a) the same speed (b) the same momentum (c) the same energy ?

13. If an electron has a wavelength, does it also have a colour ?

## OBJECTIVE I

1. Planck constant has the same dimensions as
   (a) force × time
   (b) force × distance
   (c) force × speed
   (d) force × distance × time.

2. Two photons having
   (a) equal wavelengths have equal linear momenta
   (b) equal energies have equal linear momenta
   (c) equal frequencies have equal linear momenta
   (d) equal linear momenta have equal wavelengths.

3. Let $p$ and $E$ denote the linear momentum and energy of a photon. If the wavelength is decreased,
   (a) both $p$ and $E$ increase
   (b) $p$ increases and $E$ decreases
   (c) $p$ decreases and $E$ increases
   (d) both $p$ and $E$ decrease.

4. Let $n_r$ and $n_b$ be respectively the number of photons emitted by a red bulb and a blue bulb of equal power in a given time.

   (a) $n_r = n_b$.
   (b) $n_r < n_b$.
   (c) $n_r > n_b$.
   (d) The information is insufficient to get a relation between $n_r$ and $n_b$.

5. The equation $E = pc$ is valid
   (a) for an electron as well as for a photon
   (b) for an electron but not for a photon
   (c) for a photon but not for an electron
   (d) neither for an electron nor for a photon.

6. The work function of a metal is $h\nu_0$. Light of frequency $\nu$ falls on this metal. The photoelectric effect will take place only if
   (a) $\nu > \nu_0$
   (b) $\nu > 2\nu_0$
   (c) $\nu < \nu_0$
   (d) $\nu < \nu_0/2$.

7. Light of wavelength $\lambda$ falls on a metal having work function $hc/\lambda_0$. Photoelectric effect will take place only if
   (a) $\lambda \geq \lambda_0$
   (b) $\lambda \geq 2\lambda_0$
   (c) $\lambda < \lambda_0$
   (d) $\lambda < \lambda_0/2$.

8. When stopping potential is applied in an experiment on photoelectric effect, no photocurrent is observed. This means that
   (a) the emission of photoelectrons is stopped
   (b) the photoelectrons are emitted but are reabsorbed by the emitter metal
   (c) the photoelectrons are accumulated near the collector plate
   (d) the photoelectrons are dispersed from the sides of the apparatus.

9. If the frequency of light in a photoelectric experiment is doubled, the stopping potential will
   (a) be doubled
   (b) be halved
   (c) become more than double
   (d) become less than double.

10. The frequency and intensity of a light source are both doubled. Consider the following statements.
    (A) The saturation photocurrent remains almost the same.
    (B) The maximum kinetic energy of the photoelectrons is doubled.
    (a) Both A and B are true.   (b) A is true but B is false.
    (c) A is false but B is true.   (d) Both A and B are false.

11. A point source of light is used in a photoelectric effect. If the source is removed farther from the emitting metal, the stopping potential
    (a) will increase
    (b) will decrease

(c) will remain constant
(d) will either increase or decrease.

12. A point source causes photoelectric effect from a small metal plate. Which of the following curves may represent the saturation photocurrent as a function of the distance between the source and the metal ?
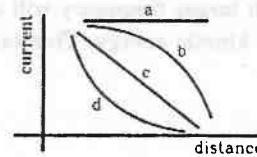


Figure 42-Q1

13. A nonmonochromatic light is used in an experiment on photoelectric effect. The stopping potential
    (a) is related to the mean wavelength
    (b) is related to the longest wavelength
    (c) is related to the shortest wavelength
    (d) is not related to the wavelength.

14. A proton and an electron are accelerated by the same potential difference. Let $\lambda_e$ and $\lambda_p$ denote the de Broglie wavelengths of the electron and the proton respectively.
    (a) $\lambda_e = \lambda_p$.   (b) $\lambda_e < \lambda_p$.   (c) $\lambda_e > \lambda_p$.
    (d) The relation between $\lambda_e$ and $\lambda_p$ depends on the accelerating potential difference.

## OBJECTIVE II

1. When the intensity of a light source is increased,
   (a) the number of photons emitted by the source in unit time increases
   (b) the total energy of the photons emitted per unit time increases
   (c) more energetic photons are emitted
   (d) faster photons are emitted.

2. Photoelectric effect supports quantum nature of light because
   (a) there is a minimum frequency below which no photoelectrons are emitted
   (b) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity
   (c) even when the metal surface is faintly illuminated the photoelectrons leave the surface immediately
   (d) electric charge of the photoelectrons is quantized.

3. A photon of energy $h\nu$ is absorbed by a free electron of a metal having work function $\varphi < h\nu$.
   (a) The electron is sure to come out.
   (b) The electron is sure to come out with a kinetic energy $h\nu - \varphi$.
   (c) Either the electron does not come out or it comes out with a kinetic energy $h\nu - \varphi$.
   (d) It may come out with a kinetic energy less than $h\nu - \varphi$.

4. If the wavelength of light in an experiment on photoelectric effect is doubled,

(a) the photoelectric emission will not take place
(b) the photoelectric emission may or may not take place
(c) the stopping potential will increase
(d) the stopping potential will decrease.

5. The photocurrent in an experiment on photoelectric effect increases if
   (a) the intensity of the source is increased
   (b) the exposure time is increased
   (c) the intensity of the source is decreased
   (d) the exposure time is decreased.

6. The collector plate in an experiment on photoelectric effect is kept vertically above the emitter plate. Light source is put on and a saturation photocurrent is recorded. An electric field is switched on which has a vertically downward direction.
   (a) The photocurrent will increase.
   (b) The kinetic energy of the electrons will increase.
   (c) The stopping potential will decrease.
   (d) The threshold wavelength will increase.

7. In which of the following situations the heavier of the two particles has smaller de Broglie wavelength ? The two particles
   (a) move with the same speed
   (b) move with the same linear momentum
   (c) move with the same kinetic energy
   (d) have fallen through the same height.

# EXERCISES

Use $h = 6.63 \times 10^{-34}$ J-s $= 4.14 \times 10^{-15}$ eV-s, $c = 3 \times 10^8$ m/s and $m_e = 9.1 \times 10^{-31}$ kg wherever needed.

1. Visible light has wavelengths in the range of 400 nm to 780 nm. Calculate the range of energy of the photons of visible light.

2. Calculate the momentum of a photon of light of wavelength 500 nm.

3. An atom absorbs a photon of wavelength 500 nm and emits another photon of wavelength 700 nm. Find the net energy absorbed by the atom in the process.

4. Calculate the number of photons emitted per second by a 10 W sodium vapour lamp. Assume that 60% of the consumed energy is converted into light. Wavelength of sodium light = 590 nm.

5. When the sun is directly overhead, the surface of the earth receives $1.4 \times 10^3$ W/m$^2$ of sunlight. Assume that the light is monochromatic with average wavelength 500 nm and that no light is absorbed in between the sun and the earth's surface. The distance between the sun and the earth is $1.5 \times 10^{11}$ m. (a) Calculate the number of photons falling per second on each square metre of earth's surface directly below the sun. (b) How many photons are there in each cubic metre near the earth's surface at any instant? (c) How many photons does the sun emit per second?

6. A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is $1.0 \times 10^{19}$. Calculate the force exerted by the light beam on the mirror.

7. A beam of white light is incident normally on a plane surface absorbing 70% of the light and reflecting the rest. If the incident beam carries 10 W of power, find the force exerted by it on the surface.

8. A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in figure (42-E1). The mass of the mirror is 20 g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Find the power of the source needed to support the weight of the mirror. Take $g = 10$ m/s$^2$.
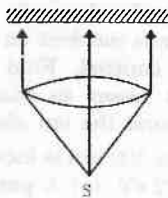


Figure 42-E1

9. A 100 W light bulb is placed at the centre of a spherical chamber of radius 20 cm. Assume that 60% of the energy supplied to the bulb is converted into light and that the surface of the chamber is perfectly absorbing. Find the pressure exerted by the light on the surface of the chamber.

10. A sphere of radius 1.00 cm is placed in the path of a parallel beam of light of large aperture. The intensity of the light is 0.50 W/cm$^2$. If the sphere completely absorbs the radiation falling on it, find the force exerted by the light beam on the sphere.

11. Consider the situation described in the previous problem. Show that the force on the sphere due to the light falling on it is the same even if the sphere is not perfectly absorbing.

12. Show that it is not possible for a photon to be completely absorbed by a free electron.

13. Two neutral particles are kept 1 m apart. Suppose by some mechanism some charge is transferred from one particle to the other and the electric potential energy lost is completely converted into a photon. Calculate the longest and the next smaller wavelength of the photon possible.

14. Find the maximum kinetic energy of the photoelectrons ejected when light of wavelength 350 nm is incident on a cesium surface. Work function of cesium = 1.9 eV.

15. The work function of a metal is $2.5 \times 10^{-19}$ J. (a) Find the threshold frequency for photoelectric emission. (b) If the metal is exposed to a light beam of frequency $6.0 \times 10^{14}$ Hz, what will be the stopping potential?

16. The work function of a photoelectric material is 4.0 eV. (a) What is the threshold wavelength? (b) Find the wavelength of light for which the stopping potential is 2.5 V.

17. Find the maximum magnitude of the linear momentum of a photoelectron emitted when light of wavelength 400 nm falls on a metal having work function 2.5 eV.

18. When a metal plate is exposed to a monochromatic beam of light of wavelength 400 nm, a negative potential of 1.1 V is needed to stop the photocurrent. Find the threshold wavelength for the metal.

19. In an experiment on photoelectric effect, the stopping potential is measured for monochromatic light beams corresponding to different wavelengths. The data collected are as follows:

wavelength (nm):         350   400   450   500   550
stopping potential(V): 1.45  1.00  0.66  0.38  0.16

Plot the stopping potential against inverse of wavelength $(1/\lambda)$ on a graph paper and find (a) the Planck constant, (b) the work function of the emitter and (c) the threshold wavelength.

20. The electric field associated with a monochromatic beam becomes zero $1.2 \times 10^{15}$ times per second. Find the maximum kinetic energy of the photoelectrons when this light falls on a metal surface whose work function is 2.0 eV.

21. The electric field associated with a light wave is given by

$$E = E_0 \sin [(1.57 \times 10^7 \text{ m}^{-1})(x - ct)].$$

Find the stopping potential when this light is used in

an experiment on photoelectric effect with the emitter having work function 1·9 eV.

22. The electric field at a point associated with a light wave is $E = (100 \text{ V/m}) \sin [(3\cdot0 \times 10^{15} \text{ s}^{-1})t] \sin [(6\cdot0 \times 10^{15} \text{ s}^{-1})t]$. If this light falls on a metal surface having a work function of 2·0 eV, what will be the maximum kinetic energy of the photoelectrons ?

23. A monochromatic light source of intensity 5 mW emits $8 \times 10^{15}$ photons per second. This light ejects photoelectrons from a metal surface. The stopping potential for this setup is 2·0 V. Calculate the work function of the metal.

24. Figure (42-E2) is the plot of the stopping potential versus the frequency of the light used in an experiment on photoelectric effect. Find (a) the ratio $h/e$ and (b) the work function.
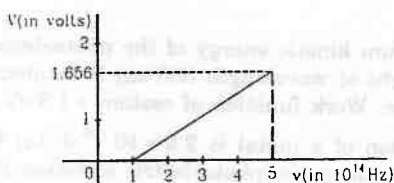


Figure 42-E2

25. A photographic film is coated with a silver bromide layer. When light falls on this film, silver bromide molecules dissociate and the film records the light there. A minimum of 0·6 eV is needed to dissociate a silver bromide molecule. Find the maximum wavelength of light that can be recorded by the film.

26. In an experiment on photoelectric effect, light of wavelength 400 nm is incident on a cesium plate at the rate of 5·0 W. The potential of the collector plate is made sufficiently positive with respect to the emitter so that the current reaches its saturation value. Assuming that on the average one out of every $10^6$ photons is able to eject a photoelectron, find the photocurrent in the circuit.

27. A silver ball of radius 4·8 cm is suspended by a thread in a vacuum chamber. Ultraviolet light of wavelength 200 nm is incident on the ball for some time during which a total light energy of $1\cdot0 \times 10^{-7}$ J falls on the surface. Assuming that on the average one photon out of every ten thousand is able to eject a photoelectron, find the electric potential at the surface of the ball assuming zero potential at infinity. What is the potential at the centre of the ball ?

28. In an experiment on photoelectric effect, the emitter and the collector plates are placed at a separation of 10 cm and are connected through an ammeter without any cell
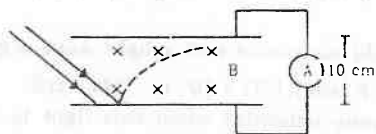


Figure 42-E3

(figure 42-E3). A magnetic field $B$ exists parallel to the plates. The work function of the emitter is 2·39 eV and the light incident on it has wavelengths between 400 nm and 600 nm. Find the minimum value of $B$ for which the current registered by the ammeter is zero. Neglect any effect of space charge.

29. In the arrangement shown in figure (42-E4), $y = 1\cdot0$ mm, $d = 0\cdot24$ mm and $D = 1\cdot2$ m. The work function of the material of the emitter is 2·2 eV. Find the stopping potential $V$ needed to stop the photocurrent.
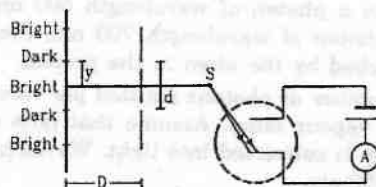


Figure 42-E4

30. In a photoelectric experiment, the collector plate is at 2·0 V with respect to the emitter plate made of copper ($\varphi = 4\cdot5$ eV). The emitter is illuminated by a source of monochromatic light of wavelength 200 nm. Find the minimum and maximum kinetic energy of the photoelectrons reaching the collector.

31. A small piece of cesium metal ($\varphi = 1\cdot9$ eV) is kept at a distance of 20 cm from a large metal plate having a charge density of $1\cdot0 \times 10^{-9}$ C/m$^2$ on the surface facing the cesium piece. A monochromatic light of wavelength 400 nm is incident on the cesium piece. Find the minimum and the maximum kinetic energy of the photoelectrons reaching the large metal plate. Neglect any change in electric field due to the small piece of cesium present.

32. Consider the situation of the previous problem. Consider the fastest electron emitted parallel to the large metal plate. Find the displacement of this electron parallel to its initial velocity before it strikes the large metal plate.

33. A horizontal cesium plate ($\varphi = 1\cdot9$ eV) is moved vertically downward at a constant speed $v$ in a room full of radiation of wavelength 250 nm and above. What should be the minimum value of $v$ so that the vertically upward component of velocity is nonpositive for each photoelectron ?

34. A small metal plate (work function $\varphi$) is kept at a distance $d$ from a singly ionized, fixed ion. A monochromatic light beam is incident on the metal plate and photoelectrons are emitted. Find the maximum wavelength of the light beam so that some of the photoelectrons may go round the ion along a circle.

35. A light beam of wavelength 400 nm is incident on a metal plate of work function 2·2 eV. (a) A particular electron absorbs a photon and makes two collisions before coming out of the metal. Assuming that 10% of the extra energy is lost to the metal in each collision, find the kinetic energy of this electron as it comes out of the metal. (b) Under the same assumptions, find the maximum number of collisions the electron can suffer before it becomes unable to come out of the metal.

# ANSWERS

## OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (d) | 2. (d) | 3. (a) | 4. (c) | 5. (c) | 6. (a) |
| 7. (c) | 8. (b) | 9. (c) | 10. (b) | 11. (c) | 12. (d) |
| 13. (c) | 14. (c) | | | | |

## OBJECTIVE II

1. (a), (b)       2. (a), (b), (c)       3. (d)
4. (b), (d)       5. (a)       6. (b)
7. (a), (c), (d)

## EXERCISES

1. $2.56 \times 10^{-19}$ J to $5.00 \times 10^{-19}$ J
2. $1.33 \times 10^{-27}$ kg-m/s
3. $1.1 \times 10^{-19}$ J
4. $1.77 \times 10^{19}$
5. (a) $3.5 \times 10^{21}$ (b) $1.2 \times 10^{13}$ (c) $9.9 \times 10^{44}$
6. $1.0 \times 10^{-8}$ N
7. $4.3 \times 10^{-8}$ N
8. 100 MW
9. $4.0 \times 10^{-7}$ Pa
10. $5.2 \times 10^{-9}$ N
13. 860 m, 215 m
14. $1.6$ eV

15. (a) $3.8 \times 10^{14}$ Hz (b) $0.91$ V
16. (a) 310 nm (b) 190 nm
17. $4.2 \times 10^{-25}$ kg-m/s
18. 620 nm
19. (a) $4.2 \times 10^{-15}$ eV-s (b) $2.15$ eV (c) 585 nm
20. $0.48$ eV
21. $1.2$ V
22. $3.93$ eV
23. $1.9$ eV
24. (a) $4.14 \times 10^{-15}$ V-s (b) $0.414$ eV
25. 2070 nm
26. $1.6$ μA
27. $0.3$ V in each case
28. $2.85 \times 10^{-5}$ T
29. $0.9$ V
30. $2.0$ eV, $3.7$ eV
31. $22.6$ eV, $23.8$ eV
32. $9.2$ cm
33. $1.04 \times 10^{6}$ m/s
34. $\dfrac{8\pi\varepsilon_0 dhc}{e^2 + 8\pi\varepsilon_0 \varphi d}$
35. (a) $0.31$ eV (b) 4

# BOHR'S MODEL AND PHYSICS OF THE ATOM

## 43.1 EARLY ATOMIC MODELS

The idea that all matter is made of very small indivisible particles is very old. It has taken a long time, intelligent reasoning and classic experiments to cover the journey from this idea to the present day atomic models.

We can start our discussion with the mention of English scientist Robert Boyle (1627–1691) who studied the expansion and compression of air. The fact that air can be compressed or expanded, tells that air is made of tiny particles with lot of empty space between the particles. When air is compressed, these particles get closer to each other, reducing the empty space. We mention Robert Boyle here, because, with him atomism entered a new phase, from mere reasoning to experimental observations. The smallest unit of an element, which carries all the properties of the element is called an atom. Experiments on discharge tube, measurement of $e/m$ by Thomson etc. established the existence of negatively charged electrons in the atoms. And then started the search for the structure of the positive charge inside an atom because the matter as a whole is electrically neutral.

### Thomson's Model of the Atom

Thomson suggested in 1898 that the atom is a positively charged solid sphere and electrons are embedded in it in sufficient number so as to make the atom electrically neutral. One can compare Thomson's atom to a birthday cake in which cherries are embedded. This model was quite attractive as it could explain several observations available at that time. It could explain why only negatively charged particles are emitted when a metal is heated and never the positively charged particles. It could also explain the formation of ions and ionic compounds of chemistry.

### Lenard's Suggestion

Lenard had noted that cathode rays could pass through materials of small thickness almost undeviated. If the atoms were solid spheres, most of the electrons in the cathode rays would hit them and would not be able to go ahead in the forward direction. Lenard, therefore, suggested in 1903 that the atom must have a lot of empty space in it. He proposed that the atom is made of electrons and similar tiny particles carrying positive charge. But then, the question was, why on heating a metal, these tiny positively charged particles were not ejected ?

### Rutherford's Model of the Atom

Thomson's model and Lenard's model, both had certain advantages and disadvantages. Thomson's model made the positive charge immovable by assuming it to be spread over the total volume of the atom. On the other hand, electrons were tiny particles and could be ejected on heating a metal. But the almost free passage of cathode rays through an atom was not consistent with Thomson's model. For that, the atom should have a lot of empty space as suggested by Lenard. So, the positive charge should be in the form of tiny particles occupying a very small volume, yet these particles should not be able to come out on heating.

It was Ernest Rutherford who solved the problem by doing a series of experiments from 1906 to 1911 on alpha particle scattering.

In these experiments, a beam of alpha particles was bombarded on a thin gold foil and their deflections were studied (figure 43.1). Most of the alpha particles passed through the gold foil either undeviated or with a small deviation. This was expected because an alpha particle is a heavy particle and will brush aside any tiny particle coming in its way. However, some of the alpha particles were deflected by large angles.

to the third excited state is

$$13.6 \text{ eV} - 0.85 \text{ eV} = 12.75 \text{ eV etc.}$$

Thus, 10.2 eV photons and 12.1 eV photons have large probability of being absorbed from the given range 6.21 eV to 12.42 eV. The corresponding wavelengths are

$$\frac{1242 \text{ eV-nm}}{10.2 \text{ eV}} = 122 \text{ nm}$$

and

$$\frac{1242 \text{ eV-nm}}{12.1 \text{ eV}} = 103 \text{ nm}$$

These wavelengths will have low intensity in the transmitted beam.

17. *A neutron moving with speed $v$ makes a head-on collision with a hydrogen atom kept at rest. Find the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron = mass of hydrogen = $1.67 \times 10^{-27}$ kg.*

**Solution :** Suppose the neutron and the hydrogen atom move at speeds $v_1$ and $v_2$ after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy $\Delta E$ is used in this way. Using conservation of linear momentum and energy,

$$mv = mv_1 + mv_2 \qquad \text{(i)}$$

and

$$\frac{1}{2} mv^2 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 + \Delta E \qquad \text{(ii)}$$

From (i), $\quad v^2 = v_1^2 + v_2^2 + 2 v_1 v_2.$

From (ii), $\quad v^2 = v_1^2 + v_2^2 + \dfrac{2\Delta E}{m}$

Thus, $\quad 2 v_1 v_2 = \dfrac{2\Delta E}{m}$

Hence, $(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4 v_1 v_2 = v^2 - \dfrac{4\Delta E}{m}$

As $v_1 - v_2$ must be real,

$$v^2 - \frac{4\Delta E}{m} > 0$$

or, $\quad \frac{1}{2} mv^2 > 2\Delta E.$

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state

is 10.2 eV. Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is

$$\frac{1}{2} mv_m^2 = 2 \times 10.2 \text{ eV} = 20.4 \text{ eV.}$$

18. *Light corresponding to the transition $n = 4$ to $n = 2$ in hydrogen atoms falls on cesium metal (work function = 1.9 eV). Find the maximum kinetic energy of the photoelectrons emitted.*

**Solution :** The energy of the photons emitted in transition $n = 4$ to $n = 2$ is

$$h\nu = 13.6 \text{ eV} \left[ \frac{1}{2^2} - \frac{1}{4^2} \right] = 2.55 \text{ eV.}$$

The maximum kinetic energy of the photoelectrons is

$$= 2.55 \text{ eV} - 1.9 \text{ eV} = 0.65 \text{ eV.}$$

19. *A small particle of mass $m$ moves in such a way that the potential energy $U = \frac{1}{2} m^2 \omega^2 r^2$ where $\omega$ is a constant and $r$ is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, show that radius of the $n$th allowed orbit is proportional to $\sqrt{n}$.*

**Solution :** The force at a distance $r$ is

$$F = -\frac{dU}{dr} = -m\omega^2 r. \qquad \text{... (i)}$$

Suppose the particle moves along a circle of radius $r$. The net force on it should be $mv^2/r$ along the radius. Comparing with (i),

$$\frac{mv^2}{r} = m\omega^2 r$$

or, $\quad v = \omega r.$ $\qquad$ ... (ii)

The quantization of angular momentum gives

$$mvr = \frac{nh}{2\pi}$$

or, $\quad v = \frac{nh}{2\pi mr}$ $\qquad$ ... (iii)

From (ii) and (iii),

$$r = \left[ \frac{nh}{2\pi m\omega} \right]^{1/2}$$

Thus, the radius of the $n$th orbit is proportional to $\sqrt{n}$.

□

## QUESTIONS FOR SHORT ANSWER

1. How many wavelengths are emitted by atomic hydrogen in visible range (380 nm–780 nm) ? In the range 50 nm to 100 nm ?

2. The first excited energy of a He$^+$ ion is the same as the ground state energy of hydrogen. Is it always true that one of the energies of any hydrogen-like ion will be the same as the ground state energy of a hydrogen atom ?

3. Which wavelengths will be emitted by a sample of atomic hydrogen gas (in ground state) if electrons of energy 12.2 eV collide with the atoms of the gas ?

4. When white radiation is passed through a sample of hydrogen gas at room temperature, absorption lines are observed in Lyman series only. Explain.

5. Balmer series was observed and analysed before the other series. Can you suggest a reason for such an order ?

6. What will be the energy corresponding to the first excited state of a hydrogen atom if the potential energy of the atom is taken to be 10 eV when the electron is widely separated from the proton ? Can we still write $E_n = E_1 / n^2$ ? $r_n = a_0 n^2$ ?

7. The difference in the frequencies of series limit of Lyman series and Balmer series is equal to the frequency of the first line of the Lyman series. Explain.

8. The numerical value of ionization energy in eV equals the ionization potential in volts. Does the equality hold if these quantities are measured in some other units ?

9. We have stimulated emission and spontaneous emission. Do we also have stimulated absorption and spontaneous absorption ?

10. An atom is in its excited state. Does the probability of its coming to ground state depend on whether the radiation is already present or not ? If yes, does it also depend on the wavelength of the radiation present ?

## OBJECTIVE I

1. The minimum orbital angular momentum of the electron in a hydrogen atom is
   (a) $h$ (b) $h/2$ (c) $h/2\pi$ (d) $h/\lambda$.

2. Three photons coming from excited atomic-hydrogen sample are picked up. Their energies are 12·1 eV, 10·2 eV and 1·9 eV. These photons must come from
   (a) a single atom (b) two atoms
   (c) three atoms (d) either two atoms or three atoms.

3. Suppose, the electron in a hydrogen atom makes transition from $n = 3$ to $n = 2$ in $10^{-8}$ s. The order of the torque acting on the electron in this period, using the relation between torque and angular momentum as discussed in the chapter on rotational mechanics is
   (a) $10^{-34}$ N-m (b) $10^{-24}$ N-m
   (c) $10^{-42}$ N-m (d) $10^{-8}$ N-m.

4. In which of the following transitions will the wavelength be minimum ?
   (a) $n = 5$ to $n = 4$ (b) $n = 4$ to $n = 3$
   (c) $n = 3$ to $n = 2$ (d) $n = 2$ to $n = 1$.

5. In which of the following systems will the radius of the first orbit ($n = 1$) be minimum ?
   (a) hydrogen atom (b) deuterium atom
   (c) singly ionized helium (d) doubly ionized lithium.

6. In which of the following systems will the wavelength corresponding to $n = 2$ to $n = 1$ be minimum ?
   (a) hydrogen atom (b) deuterium atom
   (c) singly ionized helium (d) doubly ionized lithium.

7. Which of the following curves may represent the speed of the electron in a hydrogen atom as a function of the principal quantum number $n$ ?
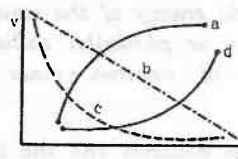


Figure 43-Q1

8. As one considers orbits with higher values of $n$ in a hydrogen atom, the electric potential energy of the atom
   (a) decreases (b) increases
   (c) remains the same (d) does not increase.

9. The energy of an atom (or ion) in its ground state is $-54\cdot4$ eV. It may be
   (a) hydrogen (b) deuterium (c) $He^+$ (d) $Li^{++}$.

10. The radius of the shortest orbit in a one-electron system is 18 pm. It may be
    (a) hydrogen (b) deuterium (c) $He^+$ (d) $Li^{++}$.

11. A hydrogen atom in ground state absorbs 10·2 eV of energy. The orbital angular momentum of the electron is increased by
    (a) $1\cdot05 \times 10^{-34}$ J-s (b) $2\cdot11 \times 10^{-34}$ J-s
    (c) $3\cdot16 \times 10^{-34}$ J-s (d) $4\cdot22 \times 10^{-34}$ J-s.

12. Which of the following parameters are the same for all hydrogen-like atoms and ions in their ground states ?
    (a) radius of the orbit (b) speed of the electron
    (c) energy of the atom
    (d) orbital angular momentum of the electron.

13. In a laser tube, all the photons
    (a) have same wavelength (b) have same energy
    (c) move in same direction (d) move with same speed.

## OBJECTIVE II

1. In a laboratory experiment on emission from atomic hydrogen in a discharge tube, only a small number of lines are observed whereas a large number of lines are present in the hydrogen spectrum of a star. This is because in a laboratory
   (a) the amount of hydrogen taken is much smaller than that present in the star

   (b) the temperature of hydrogen is much smaller than that of the star
   (c) the pressure of hydrogen is much smaller than that of the star
   (d) the gravitational pull is much smaller than that in the star.

2. An electron with kinetic energy 5 eV is incident on a hydrogen atom in its ground state. The collision
   (a) must be elastic          (b) may be partially elastic
   (c) must be completely inelastic
   (d) may be completely inelastic.

3. Which of the following products in a hydrogen atom are independent of the principal quantum number $n$? The symbols have their usual meanings.
   (a) $vn$          (b) $Er$          (c) $En$          (d) $vr$.

4. Let $A_n$ be the area enclosed by the $n$th orbit in a hydrogen atom. The graph of $\ln(A_n/A_1)$ against $\ln(n)$
   (a) will pass through the origin
   (b) will be a straight line with slope 4

   (c) will be a monotonically increasing nonlinear curve
   (d) will be a circle.

5. Ionization energy of a hydrogen-like ion $A$ is greater than that of another hydrogen-like ion $B$. Let $r$, $u$, $E$ and $L$ represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron respectively. In ground state
   (a) $r_A > r_B$    (b) $u_A > u_B$    (c) $E_A > E_B$    (d) $L_A > L_B$.

6. When a photon stimulates the emission of another photon, the two photons have
   (a) same energy          (b) same direction
   (c) same phase           (d) same wavelength.

## EXERCISES

Planck constant $h = 6.63 \times 10^{-34}$ J–s $= 4.14 \times 10^{-15}$ eV–s, first Bohr radius of hydrogen $a_0 = 53$ pm. Energy of hydrogen atom in ground state $= -13.6$ eV. Rydberg's constant $= 1.097 \times 10^7$ m$^{-1}$.

1. The Bohr radius is given by $a_0 = \dfrac{\varepsilon_0 h^2}{\pi m e^2}$. Verify that the RHS has dimensions of length.

2. Find the wavelength of the radiation emitted by hydrogen in the transitions (a) $n = 3$ to $n = 2$, (b) $n = 5$ to $n = 4$ and (c) $n = 10$ to $n = 9$.

3. Calculate the smallest wavelength of radiation that may be emitted by (a) hydrogen, (b) He$^+$ and (c) Li$^{++}$

4. Evaluate Rydberg constant by putting the values of the fundamental constants in its expression.

5. Find the binding energy of a hydrogen atom in the state $n = 2$.

6. Find the radius and energy of a He$^+$ ion in the states (a) $n = 1$, (b) $n = 4$ and (c) $n = 10$.

7. A hydrogen atom emits ultraviolet radiation of wavelength 102·5 nm. What are the quantum numbers of the states involved in the transition?

8. (a) Find the first excitation potential of He$^+$ ion. (b) Find the ionization potential of Li$^{++}$ ion.

9. A group of hydrogen atoms are prepared in $n = 4$ states. List the wavelengths that are emitted as the atoms make transitions and return to $n = 2$ states.

10. A positive ion having just one electron ejects it if a photon of wavelength 228 Å or less is absorbed by it. Identify the ion.

11. Find the maximum Coulomb force that can act on the electron due to the nucleus in a hydrogen atom.

12. A hydrogen atom in a state having a binding energy of 0·85 eV makes transition to a state with excitation energy 10·2 eV. (a) Identify the quantum numbers $n$ of the upper and the lower energy states involved in the transition. (b) Find the wavelength of the emitted radiation.

13. Whenever a photon is emitted by hydrogen in Balmer series, it is followed by another photon in Lyman series. What wavelength does this latter photon correspond to?

14. A hydrogen atom in state $n = 6$ makes two successive transitions and reaches the ground state. In the first transition a photon of 1·13 eV is emitted. (a) Find the energy of the photon emitted in the second transition. (b) What is the value of $n$ in the intermediate state?

15. What is the energy of a hydrogen atom in the first excited state if the potential energy is taken to be zero in the ground state?

16. A hot gas emits radiation of wavelengths 46·0 nm, 82·8 nm and 103·5 nm only. Assume that the atoms have only two excited states and the difference between consecutive energy levels decreases as energy is increased. Taking the energy of the highest energy state to be zero, find the energies of the ground state and the first excited state.

17. A gas of hydrogen-like ions is prepared in a particular excited state $A$. It emits photons having wavelength equal to the wavelength of the first line of the Lyman series together with photons of five other wavelengths. Identify the gas and find the principal quantum number of the state $A$.

18. Find the maximum angular speed of the electron of a hydrogen atom in a stationary orbit.

19. A spectroscopic instrument can resolve two nearby wavelengths $\lambda$ and $\lambda + \Delta\lambda$ if $\lambda/\Delta\lambda$ is smaller than 8000. This is used to study the spectral lines of the Balmer series of hydrogen. Approximately how many lines will be resolved by the instrument?

20. Suppose, in certain conditions only those transitions are allowed to hydrogen atoms in which the principal quantum number $n$ changes by 2. (a) Find the smallest wavelength emitted by hydrogen. (b) List the wavelengths emitted by hydrogen in the visible range (380 nm to 780 nm).

21. According to Maxwell's theory of electrodynamics, an electron going in a circle should emit radiation of frequency equal to its frequency of revolution. What

should be the wavelength of the radiation emitted by a hydrogen atom in ground state if this rule is followed ?

22. The average kinetic energy of molecules in a gas at temperature $T$ is $1.5 \, kT$. Find the temperature at which the average kinetic energy of the molecules of hydrogen equals the binding energy of its atoms. Will hydrogen remain in molecular form at this temperature ? Take $k = 8.62 \times 10^{-5}$ eV/K.

23. Find the temperature at which the average thermal kinetic energy is equal to the energy needed to take a hydrogen atom from its ground state to $n = 3$ state. Hydrogen can now emit red light of wavelength $653.1$ nm. Because of Maxwellian distribution of speeds, a hydrogen sample emits red light at temperatures much lower than that obtained from this problem. Assume that hydrogen molecules dissociate into atoms.

24. Average lifetime of a hydrogen atom excited to $n = 2$ state is $10^{-8}$ s. Find the number of revolutions made by the electron on the average before it jumps to the ground state.

25. Calculate the magnetic dipole moment corresponding to the motion of the electron in the ground state of a hydrogen atom.

26. Show that the ratio of the magnetic dipole moment to the angular momentum ($l = mvr$) is a universal constant for hydrogen-like atoms and ions. Find its value.

27. A beam of light having wavelengths distributed uniformly between 450 nm to 550 nm passes through a sample of hydrogen gas. Which wavelength will have the least intensity in the transmitted beam ?

28. Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms falls on helium ions in $n = 1$ and $n = 2$ states. What are the possible transitions of helium ions as they absorb energy from the radiation ?

29. A hydrogen atom in ground state absorbs a photon of ultraviolet radiation of wavelength 50 nm. Assuming that the entire photon energy is taken up by the electron, with what kinetic energy will the electron be ejected ?

30. A parallel beam of light of wavelength 100 nm passes through a sample of atomic hydrogen gas in ground state. (a) Assume that when a photon supplies some of its energy to a hydrogen atom, the rest of the energy appears as another photon moving in the same direction as the incident photon. Neglecting the light emitted by the excited hydrogen atoms in the direction of the incident beam, what wavelengths may be observed in the transmitted beam ? (b) A radiation detector is placed near the gas to detect radiation coming perpendicular to the incident beam. Find the wavelengths of radiation that may be detected by the detector.

31. A beam of monochromatic light of wavelength $\lambda$ ejects photoelectrons from a cesium surface ($\Phi = 1.9$ eV). These photoelectrons are made to collide with hydrogen atoms in ground state. Find the maximum value of $\lambda$ for which (a) hydrogen atoms may be ionized, (b) hydrogen atoms may get excited from the ground state to the first excited state and (c) the excited hydrogen atoms may emit visible light.

32. Electrons are emitted from an electron gun at almost zero velocity and are accelerated by an electric field $E$ through a distance of $1.0$ m. The electrons are now scattered by an atomic hydrogen sample in ground state. What should be the minimum value of $E$ so that red light of wavelength $656.3$ nm may be emitted by the hydrogen ?

33. A neutron having kinetic energy $12.5$ eV collides with a hydrogen atom at rest. Nelgect the difference in mass between the neutron and the hydrogen atom and assume that the neutron does not leave its line of motion. Find the possible kinetic energies of the neutron after the event.

34. A hydrogen atom moving at speed $v$ collides with another hydrogen atom kept at rest. Find the minimum value of $v$ for which one of the atoms may get ionized. The mass of a hydrogen atom = $1.67 \times 10^{-27}$ kg.

35. A neutron moving with a speed $v$ strikes a hydrogen atom in ground state moving towards it with the same speed. Find the minimum speed of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron $\approx$ mass of hydrogen = $1.67 \times 10^{-27}$ kg.

36. When a photon is emitted by a hydrogen atom, the photon carries a momentum with it. (a) Calculate the momentum carried by the photon when a hydrogen atom emits light of wavelength $656.3$ nm. (b) With what speed does the atom recoil during this transition ? Take the mass of the hydrogen atom = $1.67 \times 10^{-27}$ kg. (c) Find the kinetic energy of recoil of the atom.

37. When a photon is emitted from an atom, the atom recoils. The kinetic energy of recoil and the energy of the photon come from the difference in energies between the states involved in the transition. Suppose, a hydrogen atom changes its state from $n = 3$ to $n = 2$. Calculate the fractional change in the wavelength of light emitted, due to the recoil.

38. The light emitted in the transition $n = 3$ to $n = 2$ in hydrogen is called $H_\alpha$ light. Find the maximum work function a metal can have so that $H_\alpha$ light can emit photoelectrons from it.

39. Light from Balmer series of hydrogen is able to eject photoelectrons from a metal. What can be the maximum work function of the metal ?

40. Radiation from hydrogen discharge tube falls on a cesium plate. Find the maximum possible kinetic energy of the photoelectrons. Work function of cesium is $1.9$ eV.

41. A filter transmits only the radiation of wavelength greater than 440 nm. Radiation from a hydrogen-discharge tube goes through such a filter and is incident on a metal of work function $2.0$ eV. Find the stopping potential which can stop the photoelectrons.

42. The earth revolves round the sun due to gravitational attraction. Suppose that the sun and the earth are point particles with their existing masses and that Bohr's quantization rule for angular momentum is valid in the case of gravitation. (a) Calculate the minimum radius the earth can have for its orbit. (b) What is the value of the principal quantum number $n$ for the present

radius ? Mass of the earth $= 6 \cdot 0 \times 10^{24}$ kg, mass of the sun $= 2 \cdot 0 \times 10^{30}$ kg, earth–sun distance $= 1 \cdot 5 \times 10^{11}$ m.

43. Consider a neutron and an electron bound to each other due to gravitational force. Assuming Bohr's quantization rule for angular momentum to be valid in this case, derive an expression for the energy of the neutron–electron system.

44. A uniform magnetic field $B$ exists in a region. An electron projected perpendicular to the field goes in a circle. Assuming Bohr's quantization rule for angular momentum, calculate (a) the smallest possible radius of the electron (b) the radius of the $n$th orbit and (c) the minimum possible speed of the electron.

45. Suppose in an imaginary world the angular momentum is quantized to be even integral multiples of $h/2\pi$. What is the longest possible wavelength emitted by hydrogen atoms in visible range in such a world according to Bohr's model ?

46. Consider an excited hydrogen atom in state $n$ moving with a velocity $v(v \ll c)$. It emits a photon in the direction of its motion and changes its state to a lower state $m$. Apply momentum and energy conservation principles to calculate the frequency $v$ of the emitted radiation. Compare this with the frequency $v_0$ emitted if the atom were at rest.

□

# ANSWERS

## OBJECTIVE I

1. (c)   2. (d)   3. (b)   4. (d)   5. (d)   6. (d)
7. (c)   8. (b)   9. (c)   10. (d)   11. (a)   12. (d)
13. (d)

## OBJECTIVE II

1. (b)   2. (a)   3. (a), (b)       4. (a), (b)
5. (b)   6. all.

## EXERCISES

2. (a) 654 nm (b) 4050 nm (c) 38860 nm

3. (a) 91 nm (b) 23 nm (c) 10 nm

4. $1 \cdot 097 \times 10^7$ m$^{-1}$

5. 3·4 eV

6. (a) 0·265 A, – 54·4 eV (b) 4·24 A, – 3·4 eV
   (c) 26·5 A, – 0·544 eV

7. 1 and 3

8. (a) 40·8 V (b)  122·4 V

9. 487 nm, 654 nm, 1910 nm

10. He$^+$

11. $8 \cdot 2 \times 10^{-8}$ N

12. (a) 4, 2 (b) 487 nm

13. 122 nm

14. 12·1 eV, 3

15. 23·8 eV

16. – 27 eV, – 12 eV

17. He$^+$, 4

18. $4 \cdot 1 \times 10^{16}$ rad/s

19. 38

20. (a) 103 nm (b) 487 nm

21. 45·7 nm

22. $1 \cdot 05 \times 10^5$ K

23. $9 \cdot 4 \times 10^4$ K

24. $8 \cdot 2 \times 10^6$

25. $9 \cdot 2 \times 10^{-24}$ A–m$^2$

26. $\dfrac{e}{2m} = 8 \cdot 8 \times 10^{10}$ C/kg

27. 487 nm

28. $n = 2$ to $n = 3$ and $n = 2$ to $n = 4$

29. 11·24 eV

30. (a) 100 nm, 560 nm, 3880 nm
    (b) 103 nm, 121 nm, 654 nm

31. (a) 80 nm (b) 102 nm (c) 89 nm

32. 12·1 V/m

33. zero

34. $7 \cdot 2 \times 10^4$ m/s

35. $3 \cdot 13 \times 10^4$ m/s

36. (a) $1 \cdot 0 \times 10^{27}$ kg–m/s (b) 0·6 m/s
    (c) $1 \cdot 9 \times 10^{-9}$ eV

37. $10^{-9}$

38. 1·9 eV

39. 3·4 eV

40. 11·7 eV

41. 0·55 V

42. (a) $2 \cdot 3 \times 10^{-138}$ m (b) $2 \cdot 5 \times 10^{74}$

# X-RAYS

## 44.1 PRODUCTION OF X-RAYS

When highly energetic electrons are made to strike a metal target, electromagnetic radiation comes out. A large part of this radiation has wavelength of the order of 0.1 nm ($\approx$ 1 A) and is known as *X-ray*.

X-ray was discovered by the German physicist W.C. Roentgen in 1895. He found that photographic film wrapped light-tight in black paper became exposed when placed near a cathode-ray tube. He concluded that some invisible radiation was coming from the cathode-ray tube which penetrated the black paper to affect the photographic plate. He named this radiation as X-ray because its nature and properties could not be known at that time. In mathematics, we generally use the symbol $x$ for unknown quantities. However, after some calculation we finally get the value of this unknown $x$. Similarly, we now know about the nature and properties of X-rays.

A device used to produce X-rays is generally called an *X-ray tube*. Figure (41.1) shows a schematic diagram of such a device. This was originally designed by Coolidge and is known as *Coolidge tube* to produce X-rays.
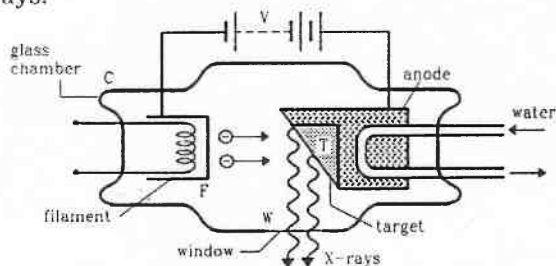


Figure 44.1

A filament $F$ and a metallic target $T$ are fixed in an evacuated glass chamber $C$. The filament is heated electrically and emits electrons by thermionic emission. A constant potential difference of several kilovolts is maintained between the filament and the target using a DC power supply so that the target is at a higher potential than the filament. The electrons emitted by the filament are, therefore, accelerated by the electric field set up between the filament and the target and hit the target with a very high speed. These electrons are stopped by the target and in the process X-rays are emitted. These X-rays are brought out of the tube through a window $W$ made of thin mica or mylar or some such material which does not absorb X-rays appreciably.

In the process, large amount of heat is developed, and thus an arrangement is provided to cool down the tube continuously by running water.

The exact design of the X-ray tube depends on the type of use for which these X-rays are required.

## 44.2 CONTINUOUS AND CHARACTERISTIC X-RAYS
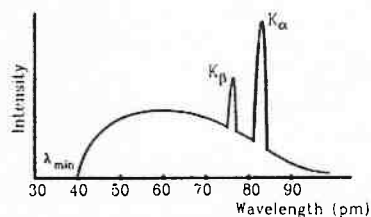


Figure 44.2

If the X-rays coming from a Coolidge tube are examined for the wavelengths present, and the intensity of different wavelength components are measured, we obtain a plot of the nature shown in figure (44.2). We see that there is a minimum wavelength below which no X-ray is emitted. This is called the *cutoff wavelength* or the *threshold wavelength*. The X-rays emitted can be clearly divided in two categories. At certain sharply defined wavelengths, the intensity of X-rays is very large as marked $K_\alpha$, $K_\beta$ in figure (44.2). These X-rays are known as *characteristic X-rays*. At other wavelengths the intensity varies gradually and these X-rays are called *continuous X-rays*. Let us examine the origin of these two types of X-rays.

Suppose, the potential difference applied between the target and the filament is $V$ and electrons are

5. *The $K_\alpha$ X-ray of molybdenum has wavelength 71 pm. If the energy of a molybdenum atom with a K electron knocked out is 23·32 keV, what will be the energy of this atom when an L electron is knocked out?*

*Solution :* $K_\alpha$ X-ray results from the transition of an electron from L shell to K shell. If the energy of the atom with a vacancy in the K shell is $E_K$ and the energy with a vacancy in the L shell is $E_L$, the energy of the photon emitted is $E_K - E_L$. The energy of the 71 pm photon is

$$E = \frac{hc}{\lambda}$$

$$= \frac{1242 \text{ eV-nm}}{71 \times 10^{-3} \text{ nm}} = 17.5 \text{ keV}.$$

Thus, $\quad E_K - E_L = 17.5 \text{ keV}$

or, $\qquad E_L = E_K - 17.5 \text{ keV}$

$$= 23.32 \text{ keV} - 17.5 \text{ keV} = 5.82 \text{ keV}.$$

6. *Show that the frequency of $K_\beta$ X-ray of a material equals the sum of the frequencies of $K_\alpha$ and $L_\alpha$ X-rays of the same material.*

*Solution :*
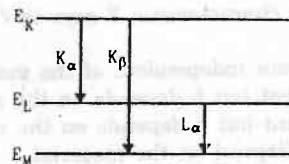


Figure 44-W1

The energy level diagram of an atom with one electron knocked out is shown in figure (44-W1).

Energy of $K_\alpha$ X-ray is $E_{K_\alpha} = E_K - E_L$

of $K_\beta$ X-ray is $E_{K_\beta} = E_K - E_M$,

and of $L_\alpha$ X-ray is $E_{L_\alpha} = E_L - E_M$.

Thus, $\qquad E_{K_\beta} = E_{K_\alpha} + E_{L_\alpha}$

or, $\qquad h\nu_{K_\beta} = h\nu_{K_\alpha} + h\nu_{L_\alpha}$

or, $\qquad \nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$.

## QUESTIONS FOR SHORT ANSWER

1. When a Coolidge tube is operated for some time it becomes hot. Where does the heat come from?

2. In a Coolidge tube, electrons strike the target and stop inside it. Does the target get more and more negatively charged as time passes?

3. Can X-rays be used for photoelectric effect?

4. Can X-rays be polarized?

5. X-ray and visible light travel at the same speed in vacuum. Do they travel at the same speed in glass?

6. Characteristic X-rays may be used to identify the element from which they are coming. Can continuous X-rays be used for this purpose?

7. Is it possible that in a Coolidge tube characterstic $L_\alpha$ X-rays are emitted but not $K_\alpha$ X-rays?

8. Can $L_\alpha$ X-ray of one material have shorter wavelength than $K_\alpha$ X-ray of another?

9. Can a hydrogen atom emit characteristic X-ray?

10. Why is exposure to X-ray injurious to health but exposure to visible light is not, when both are electromagnetic waves?

## OBJECTIVE I

1. X-ray beam can be deflected
   (a) by an electric field
   (b) by a magnetic field
   (c) by an electric field as well as by a magnetic field
   (d) neither by an electric field nor by a magnetic field.

2. Consider a photon of continuous X-ray coming from a Coolidge tube. Its energy comes from
   (a) the kinetic energy of the striking electron
   (b) the kinetic energy of the free electrons of the target
   (c) the kinetic energy of the ions of the target
   (d) an atomic transition in the target.

3. The energy of a photon of characteristic X-ray from a Coolidge tube comes from
   (a) the kinetic energy of the striking electron
   (b) the kinetic energy of the free electrons of the target
   (c) the kinetic energy of the ions of the target
   (d) an atomic transition in the target.

4. If the potential difference applied to the tube is doubled and the separation between the filament and the target is also doubled, the cutoff wavelength
   (a) will remain unchanged
   (b) will be doubled

(c) will be halved

(d) will become four times the original.

5. If the current in the circuit for heating the filament is increased, the cutoff wavelength
   (a) will increase       (b) will decrease
   (c) will remain unchanged    (d) will change.

6. Moseley's law for characteristic X-rays is $\sqrt{v} = a(Z - b)$. In this,
   (a) both $a$ and $b$ are independent of the material
   (b) $a$ is independent but $b$ depends on the material
   (c) $b$ is independent but $a$ depends on the material
   (d) both $a$ and $b$ depend on the material.

7. Frequencies of $K_\alpha$ X-rays of different materials are measured. Which one of the graphs in figure (44-Q1) may represent the relation between the frequency $v$ and the atomic number $Z$.
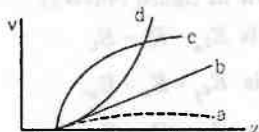


Figure 44-Q1

8. The X-ray beam coming from an X-ray tube
   (a) is monochromatic
   (b) has all wavelengths smaller than a certain maximum wavelength
   (c) has all wavelengths greater than a certain minimum wavelength
   (d) has all wavelengths lying between a minimum and a maximum wavelength.

9. One of the following wavelengths is absent and the rest are present in the X-rays coming from a Coolidge tube. Which one is the absent wavelength ?
   (a) 25 pm    (b) 50 pm    (c) 75 pm    (d) 100 pm.

10. Figure (44-Q2) shows the intensity–wavelength relations of X-rays coming from two different Coolidge tubes. The solid curve represents the relation for the tube $A$ in which the potential difference between the target and

the filament is $V_A$ and the atomic number of the target material is $Z_A$. These quantities are $V_B$ and $Z_B$ for the other tube. Then,
   (a) $V_A > V_B$, $Z_A > Z_B$     (b) $V_A > V_B$, $Z_A < Z_B$
   (c) $V_A < V_B$, $Z_A > Z_B$     (d) $V_A < V_B$, $Z_A < Z_B$.
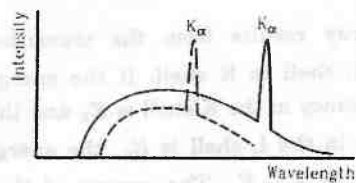


Figure 44-Q2

11. 50% of the X-ray coming from a Coolidge tube is able to pass through a 0·1 mm thick aluminium foil. If the potential difference between the target and the filament is increased, the fraction of the X-ray passing through the same foil will be
    (a) 0%    (b) < 50%    (c) 50%    (d) > 50%.

12. 50% of the X-ray coming from a Coolidge tube is able to pass through a 0·1 mm thick aluminium foil. The potential difference between the target and the filament is increased. The thickness of aluminium foil, which will allow 50% of the X-ray to pass through, will be
    (a) zero    (b) < 0·1 mm    (c) 0·1 mm    (d) > 0·1 mm.

13. X-ray from a Coolidge tube is incident on a thin aluminium foil. The intensity of the X-ray transmitted by the foil is found to be $I_0$. The heating current is increased so as to increase the temperature of the filament. The intensity of the X-ray transmitted by the foil will be.
    (a) zero    (b) < $I_0$    (c) $I_0$    (d) > $I_0$.

14. Visible light passing through a circular hole forms a diffraction disc of radius 0·1 mm on a screen. If X-ray is passed through the same set-up, the radius of the diffraction disc will be
    (a) zero    (b) < 0·1 mm    (c) 0·1 mm    (d) > 0·1 mm.

## OBJECTIVE II

1. For harder X-rays,
   (a) the wavelength is higher
   (b) the intensity is higher
   (c) the frequency is higher
   (d) the photon energy is higher.

2. Cutoff wavelength of X-rays coming from a Coolidge tube depends on the
   (a) target material      (b) accelerating voltage
   (c) separation between the target and the filament
   (d) temperature of the filament.

3. Mark the correct options.
   (a) An atom with a vacancy has smaller energy than a neutral atom.
   (b) K X-ray is emitted when a hole makes a jump from the K shell to some other shell.
   (c) The wavelength of K X-ray is smaller than the

wavelength of L X-ray of the same material.
   (d) The wavelength of $K_\alpha$ X-ray is smaller than the wavelength of $K_\beta$ X-ray of the same material.

4. For a given material, the energy and wavelength of characterstic X-rays satisfy
   (a) $E(K_\alpha) > E(K_\beta) > E(K_\gamma)$     (b) $E(M_\alpha) > E(L_\alpha) > E(K_\alpha)$
   (c) $\lambda(K_\alpha) > \lambda(K_\beta) > \lambda(K_\gamma)$     (d) $\lambda(M_\alpha) > \lambda(L_\alpha) > \lambda(K_\alpha)$.

5. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,
   (a) the intensity increases
   (b) the minimum wavelength increases
   (c) the intensity remains unchanged
   (d) the minimum wavelength decreases.

6. When an electron strikes the target in a Coolidge tube, its entire kinetic energy

(a) is converted into a photon
(b) may be converted into a photon
(c) is converted into heat
(d) may be converted into heat.

7. X-ray incident on a material
(a) exerts a force on it    (b) transfers energy to it

(c) transfers momentum to it
(d) transfers impulse to it.

8. Consider a photon of continuous X-ray and a photon of characteristic X-ray of the same wavelength. Which of the following is/are different for the two photons ?
(a) frequency    (b) energy
(c) penetrating power    (d) method of creation.

## EXERCISES

Planck constant $h = 4.14 \times 10^{-15}$ eV-s, speed of light $c = 3 \times 10^8$ m/s.

1. Find the energy, the frequency and the momentum of an X-ray photon of wavelength 0.10 nm.

2. Iron emits $K_\alpha$ X-ray of energy 6.4 keV and calcium emits $K_\alpha$ X-ray of energy 3.69 keV. Calculate the times taken by an iron $K_\alpha$ photon and a calcium $K_\alpha$ photon to cross through a distance of 3 km.

3. Find the cutoff wavelength for the continuous X-rays coming from an X-ray tube operating at 30 kV.

4. What potential difference should be applied across an X-ray tube to get X-ray of wavelength not less than 0.10 nm ? What is the maximum energy of a photon of this X-ray in joule ?

5. The X-ray coming from a Coolidge tube has a cutoff wavelength of 80 pm. Find the kinetic energy of the electrons hitting the target.

6. If the operating potential in an X-ray tube is increased by 1%, by what percentage does the cutoff wavelength decrease ?

7. The distance between the cathode (filament) and the target in an X-ray tube is 1.5 m. If the cutoff wavelength is 30 pm, find the electric field between the cathode and the target.

8. The short-wavelength limit shifts by 26 pm when the operating voltage in an X-ray tube is increased to 1.5 times the original value. What was the original value of the operating voltage ?

9. The electron beam in a colour TV is accelerated through 32 kV and then strikes the screen. What is the wavelength of the most energetic X-ray photon ?

10. When 40 kV is applied across an X-ray tube, X-ray is obtained with a maximum frequency of $9.7 \times 10^{18}$ Hz. Calculate the value of Planck constant from these data.

11. An X-ray tube operates at 40 kV. Suppose the electron converts 70% of its energy into a photon at each collision. Find the lowest three wavelengths emitted from the tube. Neglect the energy imparted to the atom with which the electron collides.

12. The wavelength of $K_\alpha$ X-ray of tungsten is 21.3 pm. It takes 11.3 keV to knock out an electron from the L shell of a tungsten atom. What should be the minimum accelerating voltage across an X-ray tube having tungsten target which allows production of $K_\alpha$ X-ray ?

13. The $K_\beta$ X-ray of argon has a wavelength of 0.36 nm. The minimum energy needed to ionize an argon atom is 16 eV. Find the energy needed to knock out an electron from the K shell of an argon atom.

14. The $K_\alpha$ X-rays of aluminium ($Z = 13$) and zinc ($Z = 30$) have wavelengths 887 pm and 146 pm respectively. Use Moseley's law $\sqrt{v} = a(Z - b)$ to find the wavelength of the $K_\alpha$ X-ray of iron ($Z = 26$).

15. A certain element emits $K_\alpha$ X-ray of energy 3.69 keV. Use the data from the previous problem to identify the element.

16. The $K_\beta$ X-rays from certain elements are given below. Draw a Moseley-type plot of $\sqrt{v}$ versus $Z$ for $K_\beta$ radiation.

| Element | Ne | P | Ca | Mn | Zn | Br |
|---|---|---|---|---|---|---|
| Energy (keV) | 0.858 | 2.14 | 4.02 | 6.51 | 9.57 | 13.3 |

17. Use Moseley's law with $b = 1$ to find the frequency of the $K_\alpha$ X-ray of La($Z = 57$) if the frequency of the $K_\alpha$ X-ray of Cu($Z = 29$) is known to be $1.88 \times 10^{18}$ Hz.

18. The $K_\alpha$ and $K_\beta$ X-rays of molybdenum have wavelengths 0.71 Å and 0.63 Å respectively. Find the wavelength of $L_\alpha$ X-ray of molybdenum.

19. The wavelengths of $K_\alpha$ and $L_\alpha$ X-rays of a material are 21.3 pm and 141 pm respectively. Find the wavelength of $K_\beta$ X-ray of the material.

20. The energy of a silver atom with a vacancy in K shell is 25.31 keV, in L shell is 3.56 keV and in M shell is 0.530 keV higher than the energy of the atom with no vacancy. Find the frequency of $K_\alpha$, $K_\beta$ and $L_\alpha$ X-rays of silver.

21. Find the maximum potential difference which may be applied across an X-ray tube with tungsten target without emitting any characteristic K or L X-ray. The energy levels of the tungsten atom with an electron knocked out are as follows.

| Cell containing vacancy | K | L | M |
|---|---|---|---|
| Energy in keV | 69.5 | 11.3 | 2.3 |

22. The electric current in an X-ray tube (from the target to the filament) operating at 40 kV is 10 mA. Assume that on an average, 1% of the total kinetic energy of the electrons hitting the target are converted into X-rays. (a) What is the total power emitted as X-rays and (b) how much heat is produced in the target every second ?

23. Heat at the rate of 200 W is produced in an X-ray tube operating at 20 kV. Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays.

24. Continuous X-rays are made to strike a tissue paper soaked with polluted water. The incoming X-rays excite the atoms of the sample by knocking out the electrons from the inner shells. Characteristic X-rays are subsequently emitted. The emitted X-rays are analysed and the intensity is plotted against the wavelength (figure 44-E1). Assuming that only $K_\alpha$ intensities are detected, list the elements present in the sample from the plot. Use Moseley's equation

$$v = (25 \times 10^{14} \text{ Hz})(Z - 1)^2.$$
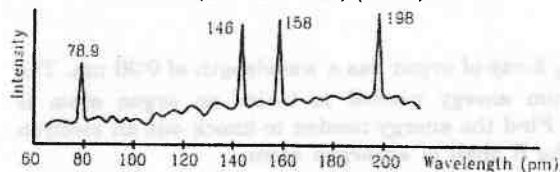


Figure 44-E1

25. A free atom of iron emits $K_\alpha$ X-rays of energy 6.4 keV. Calculate the recoil kinetic energy of the atom. Mass of an iron atom $= 9.3 \times 10^{-26}$ kg.

26. The stopping potential in a photoelectric experiment is linearly related to the inverse of the wavelength $(1/\lambda)$ of the light falling on the cathode. The potential difference applied across an X-ray tube is linearly related to the inverse of the cutoff wavelength $(1/\lambda)$ of the X-ray emitted. Show that the slopes of the lines in the two cases are equal and find its value.

27. Suppose a monochromatic X-ray beam of wavelength 100 pm is sent through a Young's double slit and the interference pattern is observed on a photographic plate placed 40 cm away from the slit. What should be the separation between the slits so that the successive maxima on the screen are separated by a distance of 0.1 mm ?

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (d) | 2. (a) | 3. (d) | 4. (c) | 5. (c) | 6. (a) |
| 7. (d) | 8. (c) | 9. (a) | 10. (b) | 11. (d) | 12. (d) |
| 13. (d) | 14. (b) | | | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (c), (d) | 2. (b) | 3. (b), (c) |
| 4. (c), (d) | 5. (c), (d) | 6. (b), (d) |
| 7. all | 8. (d) | |

### EXERCISES

1. 12.4 keV, $3 \times 10^{18}$ Hz, $6.62 \times 10^{-24}$ kg–m/s
2. 10 µs by both
3. 41.4 pm
4. 12.4 kV, $2.0 \times 10^{-15}$ J
5. 15.5 keV
6. approximately 1%
7. 27.7 kV/m
8. 15.9 kV

9. 38.8 pm
10. $4.12 \times 10^{-15}$ eV-s
11. 44.3 pm, 148 pm, 493 pm
12. 69.5 kV
13. 3.47 keV
14. 198 pm
15. calcium
17. $7.52 \times 10^{18}$ Hz
18. 5.64 Å
19. 18.5 pm
20. $5.25 \times 10^{18}$ Hz, $5.98 \times 10^{18}$ Hz, $7.32 \times 10^{17}$ Hz
21. less than 11.3 kV
22. (a) 4 W (b) 396 J
23. 10 mA
24. Zr, Zn, Cu, Fe
25. $3.9 \times 10^{-4}$ eV
26. $\dfrac{hc}{e} = 1.242 \times 10^{-6}$ V–m
27. $4 \times 10^{-7}$ m

# CHAPTER 45

# SEMICONDUCTORS
# AND SEMICONDUCTOR DEVICES

## 45.1 INTRODUCTION

We have discussed some of the properties of conductors and insulators in earlier chapters. We assumed that there is a large number of almost free electrons in a conductor which wander randomly in the whole of the body, whereas, all the electrons in an insulator are tightly bound to some nucleus or the other. If an electric field $\vec{E}$ is established inside a conductor, the free electrons experience force due to the field and acquire a drift speed. This results in an electric current. The conductivity σ is defined in terms of the electric field $\vec{E}$ existing in the conductor and the resulting current density $\vec{j}$. The relation between these quantities is

$$\vec{j} = \sigma \vec{E}.$$

Larger the conductivity σ, better is the material as a conductor.

The conductivity σ of a conductor such as copper, is fairly independent of the electric field applied and decreases as the temperature is increased. This is because as the temperature is increased, the random collisions of the free electrons with the particles in the conductor become more frequent. The electrons get less time to gain energy from the applied electric field. This results in a decrease in the drift speed and hence the conductivity decreases. The resistivity ρ = 1/σ of a conductor increases as the temperature increases.

Almost zero electric current is obtained in insulators unless a very high electric field is applied.

We now introduce another kind of solid known as *semiconductor*. These solids do conduct electricity when an electric field is applied, but the conductivity is very small as compared to the usual metallic conductors. Silicon is an example of a semiconductor, its conductivity is about $10^{11}$ times smaller than that of copper and is about $10^{11}$ times larger than that of

fused quartz. Another distinguishing feature about a semiconductor is that its conductivity increases as the temperature is increased. To understand the mechanism of conduction in solids, let us discuss qualitatively, formation of energy bands in solids.

## 45.2 ENERGY BANDS IN SOLIDS

The electrons of an isolated atom can have certain definite energies labelled as 1s, 2s, 2p, 3s etc. Pauli exclusion principle determines the maximum number of electrons which can be accommodated in each energy level. An energy level consists of several quantum states and no quantum state can contain more than one electron. Consider a sodium atom in its lowest energy state. It has 11 electrons. The electronic configuration is $(1s)^2 (2s)^2 (2p)^6 (3s)^1$. The levels 1s, 2s and 2p are completely filled and the level 3s contains only one electron although it has a capacity to accommodate 2. The next allowed energy level is 3p which can contain 6 electrons but is empty. All the energy levels above 3s are empty.

Now consider a group of $N$ sodium atoms separated from each other by large distances such as in sodium vapour. There are altogether $11N$ electrons. Assuming that each atom is in its ground state, what are the energies of these $11N$ electrons ? For each atom, there are two states in energy level 1s. There are $2N$ such states which have identical energy and are filled by $2N$ electrons. Similarly, there are $2N$ states having identical energy labelled 2s, $6N$ states having identical energy labelled 2p and $2N$ states having identical energy labelled 3s. The $2N$ states of 1s, $2N$ states of 2s and $6N$ states of 2p are completely filled whereas only $N$ of the $2N$ states of 3s are filled by the electrons and the remaining $N$ states are empty. These ideas are shown in table (45.1) and figure (45.1).
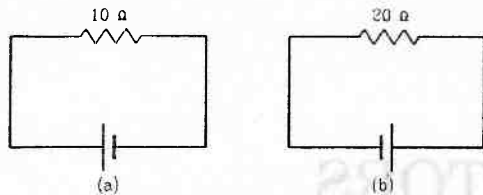
Figure 45-W3

(b) When the positive terminal of the battery is connected to the point $B$, the diode $D_2$ is forward-biased and $D_1$ is reverse biased. The equivalent circuit is shown in figure (45-W3b). The current through the battery is $2\ V/20\ \Omega = 0.1\ A$.

9. *A change of 8.0 mA in the emitter current brings a change of 7.9 mA in the collector current. How much change in the base current is required to have the same change 7.9 mA in the collector current? Find the values of $\alpha$ and $\beta$.*

**Solution :** We have,

$$I_E = I_B + I_C$$

or,                $$\Delta I_E = \Delta I_B + \Delta I_C.$$

From the question, when $\Delta I_E = 8.0$ mA, $\Delta I_C = 7.9$ mA. Thus,

$$\Delta I_B = 8.0\ mA - 7.9\ mA = 0.1\ mA.$$

So a change of $0.1$ mA in the base current is required to have a change of $7.9$ mA in the collector current.

$$\alpha = \frac{I_C}{I_E} = \frac{\Delta I_C}{\Delta I_E}$$

$$= \frac{7.9\ mA}{8.0\ mA} = 0.99.$$

$$\beta = \frac{I_C}{I_B} = \frac{\Delta I_C}{\Delta I_B}$$

$$= \frac{7.9\ mA}{0.1\ mA} = 79.$$

Check if these values of $\alpha$ and $\beta$ satisfy the equation

$$\beta = \frac{\alpha}{1 - \alpha}.$$

10. *A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 20 mV is added to the base–emitter voltage, the base current changes by 20 μA*

and the collector current changes by 2 mA. The load resistance is 5 kΩ. Calculate (a) the factor $\beta$, (b) the input resistance $R_{BE}$, (c) the transconductance and (d) the voltage gain.

**Solution :** (a) $\beta = \dfrac{\Delta I_C}{\Delta I_B} = \dfrac{2\ mA}{20\ \mu A} = 100.$

(b) The input resistance $R_{BE} = \dfrac{\Delta V_{BE}}{\Delta I_B}$

$$= \frac{20\ mV}{20\ \mu A} = 1\ k\Omega.$$

(c) Transconductance $= \dfrac{\Delta I_C}{\Delta V_{BE}}$

$$= \frac{2\ mA}{20\ mV} = 0.1\ mho.$$

(d) The change in output voltage is $R_L\,\Delta I_C$

$$= (5\ k\Omega)\,(2\ mA) = 10\ V.$$

The applied signal voltage $= 20$ mV.
Thus, the voltage gain is

$$\frac{10\ V}{20\ mV} = 500.$$

11. *Construct the truth table for the function X of A and B represented by figure (45-W4).*
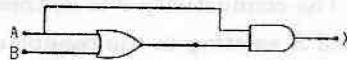


Figure 45-W4

**Solution :** Here an AND gate and an OR gate are used. Let the output of the OR gate be $Y$. Clearly, $Y = A + B$. The AND gate receives $A$ and $A + B$ as input. The output of this gate is $X$. So $X = A(A + B)$. The following table evaluates $X$ for all combinations of $A$ and $B$. The last three columns give the truth table.

| $A$ | $B$ | $Y = A + B$ | $X = A(A + B)$ | $A$ | $B$ | $X$ |
|-----|-----|-------------|----------------|-----|-----|-----|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

# QUESTIONS FOR SHORT ANSWER

1. How many 1s energy states are present in one mole of sodium vapour ? Are they all filled in normal conditions ? How many 3s energy states are present in one mole of sodium vapour ? Are they all filled in normal conditions ?

2. There are energy bands in a solid. Do we have really continuous energy variation in a band or do we have very closely spaced but still discrete energy levels ?

3. The conduction band of a solid is partially filled at 0 K. Will it be a conductor, a semiconductor or an insulator ?

4. In semiconductors, thermal collisions are responsible for taking a valence electron to the conduction band. Why does the number of conduction electrons not go on increasing with time as thermal collisions continuously take place ?

5. When an electron goes from the valence band to the conduction band in silicon, its energy is increased by 1·1 eV. The average energy exchanged in a thermal collision is of the order of $kT$ which is only 0·026 eV at room temperature. How is a thermal collision able to take some of the electrons from the valence band to the conduction band?

6. What is the resistance of an intrinsic semiconductor at 0 K?

7. We have valence electrons and conduction electrons in a semiconductor. Do we also have 'valence holes' and 'conduction holes'?

8. When a $p$-type impurity is doped in a semiconductor, a large number of holes are created. This does not make the semiconductor charged. But when holes diffuse from the $p$-side to the $n$-side in a $p$-$n$ junction, the $n$-side gets positively charged. Explain.

9. The drift current in a reverse-biased $p$-$n$ junction increases in magnitude if the temperature of the junction is increased. Explain this on the basis of creation of hole–electron pairs.

10. An ideal diode should pass a current freely in one direction and should stop it completely in the opposite direction. Which is closer to ideal—vacuum diode or a $p$-$n$ junction diode?

11. Consider an amplifier circuit using a transistor. The output power is several times greater than the input power. Where does the extra power come from?

## OBJECTIVE I

1. Electric conduction in a semiconductor takes place due to
   (a) electrons only    (b) holes only
   (c) both electrons and holes
   (d) neither electrons nor holes.

2. An electric field is applied to a semiconductor. Let the number of charge carriers be $n$ and the average drift speed be $v$. If the temperature is increased,
   (a) both $n$ and $v$ will increase
   (b) $n$ will increase but $v$ will decrease
   (c) $v$ will increase but $n$ will decrease
   (d) both $n$ and $v$ will decrease.

3. Let $n_p$ and $n_e$ be the numbers of holes and conduction electrons in an intrinsic semiconductor.
   (a) $n_p > n_e$.   (b) $n_p = n_e$.   (c) $n_p < n_e$.   (d) $n_p \neq n_e$.

4. Let $n_p$ and $n_e$ be the numbers of holes and conduction electrons in an extrinsic semiconductor.
   (a) $n_p > n_e$.   (b) $n_p = n_e$.   (c) $n_p < n_e$.   (d) $n_p \neq n_e$.

5. A $p$-type semiconductor is
   (a) positively charged    (b) negatively charged
   (c) uncharged
   (d) uncharged at 0 K but charged at higher temperatures.

6. When an impurity is doped into an intrinsic semiconductor, the conductivity of the semiconductor
   (a) increases    (b) decreases
   (c) remains the same    (d) becomes zero.

7. If the two ends of a $p$-$n$ junction are joined by a wire,
   (a) there will not be a steady current in the circuit
   (b) there will be a steady current from the $n$-side to the $p$-side
   (c) there will a steady current from the $p$-side to the $n$-side
   (d) there may or may not be a current depending upon the resistance of the connecting wire.

8. The drift current in a $p$-$n$ junction is
   (a) from the $n$-side to the $p$-side
   (b) from the $p$-side to the $n$-side
   (c) from the $n$-side to the $p$-side if the junction is forward-biased and in the opposite direction if it is reverse-biased
   (d) from the $p$-side to the $n$-side if the junction is forward-biased and in the opposite direction if it is reverse-biased.

9. The diffusion current in a $p$-$n$ junction is
   (a) from the $n$-side to the $p$-side
   (b) from the $p$-side to the $n$-side
   (c) from the $n$-side to the $p$-side if the junction is forward-biased and in the opposite direction if it is reverse-biased
   (d) from the $p$-side to the $n$-side if the junction is forward-biased and in the opposite direction if it is reverse-biased.

10. Diffusion current in a $p$-$n$ junction is greater than the drift current in magnitude
    (a) if the junction is forward-biased
    (b) if the junction is reverse-biased
    (c) if the junction is unbiased
    (d) in no case.

11. Two identical $p$-$n$ junctions may be connected in series with a battery in three ways (figure 45-Q1). The potential difference across the two $p$-$n$ junctions are equal in
    (a) circuit 1 and circuit 2    (b) circuit 2 and circuit 3
    (c) circuit 3 and circuit 1    (d) circuit 1 only.
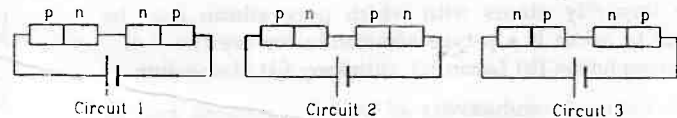


Figure 45-Q1

12. Two identical capacitors $A$ and $B$ are charged to the same potential $V$ and are connected in two circuits at $t = 0$ as shown in figure (45-Q2). The charges on the
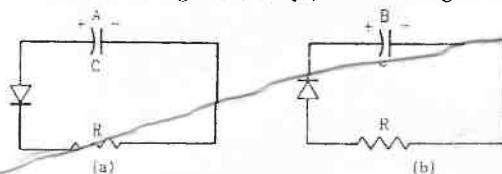


Figure 45-Q2

capacitors at a time $t = CR$ are, respectively,
(a) $VC, VC$  (b) $VC/e, VC$  (c) $VC, VC/e$  (d) $VC/e, VC/e$.

13. A hole diffuses from the $p$-side to the $n$-side in a $p$-$n$ junction. This means that
(a) a bond is broken on the $n$-side and the electron freed from the bond jumps to the conduction band
(b) a conduction electron on the $p$-side jumps to a broken bond to complete it
(c) a bond is broken on the $n$-side and the electron freed from the bond jumps to a broken bond on the $p$-side to complete it
(d) a bond is broken on the $p$-side and the electron freed from the bond jumps to a broken bond on the $n$-side to complete it.

14. In a transistor,
(a) the emitter has the least concentration of impurity
(b) the collector has the least concentration of impurity
(c) the base has the least concentration of impurity
(d) all the three regions have equal concentrations of impurity.

15. An incomplete sentence about transistors is given below:
The emitter–...... junction is ___ and the collector– ...... junction is ___. The appropriate words for the dotted empty positions are, respectively,
(a) 'collector' and 'base'      (b) 'base' and 'emitter'
(a) 'collector' and 'emitter'   (b) 'base' and 'base'

## OBJECTIVE II

1. In a semiconductor,
(a) there are no free electrons at 0 K
(b) there are no free electrons at any temperature
(c) the number of free electrons increases with temperature
(d) the number of free electrons is less than that in a conductor.

2. In a $p$-$n$ junction with open ends,
(a) there is no systematic motion of charge carriers
(b) holes and conduction electrons systematically go from the $p$-side to the $n$-side and from the $n$-side to the $p$-side respectively
(c) there is no net charge transfer between the two sides
(d) there is a constant electric field near the junction.

3. In a $p$-$n$ junction,
(a) new holes and conduction electrons are produced continuously throughout the material
(b) new holes and conduction electrons are produced continuously throughout the material except in the depletion region
(c) holes and conduction electrons recombine continuously throughout the material
(d) holes and conduction electrons recombine continuously throughout the material except in the depletion region.

4. The impurity atoms with which pure silicon may be doped to make it a $p$-type semiconductor are those of
(a) phosphorus (b) boron (c) antimony (d) aluminium.

5. The electrical conductivity of pure germanium can be increased by
(a) increasing the temperature
(b) doping acceptor impurities

(c) doping donor impurities
(d) irradiating ultraviolet light on it.

6. A semiconducting device is connected in a series circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, the current drops to almost zero. The device may be
(a) an intrinsic semiconductor
(b) a $p$-type semiconductor
(c) an $n$-type semiconductor
(d) a $p$-$n$ junction.

7. A semiconductor is doped with a donor impurity.
(a) The hole concentration increases.
(b) The hole concentration decreases.
(c) The electron concentration increases.
(d) The electron concentration decreases.

8. Let $i_e$, $i_c$ and $i_B$ represent the emitter current, the collector current and the base current respectively in a transistor. Then
(a) $i_c$ is slightly smaller than $i_e$
(b) $i_c$ is slightly greater than $i_e$
(c) $i_B$ is much smaller than $i_e$
(d) $i_B$ is much greater than $i_e$.

9. In a normal operation of a transistor,
(a) the base–emitter junction is forward-biased
(b) the base–collector junction is forward-biased
(c) the base–emitter junction is reverse-biased
(d) the base–collector junction is reverse-biased.

10. An AND gate can be prepared by repetitive use of
(a) NOT gate          (b) OR gate
(c) NAND gate         (d) NOR gate.

# EXERCISES

Planck constant = $4.14 \times 10^{-15}$ eV–s,
Boltzmann constant = $8.62 \times 10^{-5}$ eV/K.

1. Calculate the number of states per cubic metre of sodium in 3s band. The density of sodium is 1013 kg/m³. How many of them are empty ?

2. In a pure semiconductor, the number of conduction electrons is $6 \times 10^{19}$ per cubic metre. How many holes are there in a sample of size 1 cm × 1 cm × 1 mm ?

3. Indium antimonide has a band gap of 0.23 eV between the valence and the conduction band. Find the temperature at which $kT$ equals the band gap.

4. The band gap for silicon is 1.1 eV. (a) Find the ratio of the band gap to $kT$ for silicon at room temperature 300 K. (b) At what temperature does this ratio become one tenth of the value at 300 K ? (Silicon will not retain its structure at these high temperatures.)

5. When a semiconducting material is doped with an impurity, new acceptor levels are created. In a particular thermal collision, a valence electron receives an energy equal to $2kT$ and just reaches one of the acceptor levels. Assuming that the energy of the electron was at the top edge of the valence band and that the temperature $T$ is equal to 300 K, find the energy of the acceptor levels above the valence band.

6. The band gap between the valence and the conduction bands in zinc oxide (ZnO) is 3.2 eV. Suppose an electron in the conduction band combines with a hole in the valence band and the excess energy is released in the form of electromagnetic radiation. Find the maximum wavelength that can be emitted in this process.

7. Suppose the energy liberated in the recombination of a hole–electron pair is converted into electromagnetic radiation. If the maximum wavelength emitted is 820 nm, what is the band gap ?

8. Find the maximum wavelength of electromagnetic radiation which can create a hole–electron pair in germanium. The band gap in germanium is 0.65 eV.

9. In a photodiode, the conductivity increases when the material is exposed to light. It is found that the conductivity changes only if the wavelength is less than 620 nm. What is the band gap ?

10. Let $\Delta E$ denote the energy gap between the valence band and the conduction band. The population of conduction electrons (and of the holes) is roughly proportional to $e^{-\Delta E/2kT}$. Find the ratio of the concentration of conduction electrons in diamond to that in silicon at room temperature 300 K. $\Delta E$ for silicon is 1.1 eV and for diamond is 6.0 eV. How many conduction electrons are likely to be in one cubic metre of diamond ?

11. The conductivity of a pure semiconductor is roughly proportional to $T^{3/2} e^{-\Delta E/2kT}$ where $\Delta E$ is the band gap. The band gap for germanium is 0.74 eV at 4 K and 0.67 eV at 300 K. By what factor does the conductivity of pure germinium increase as the temperature is raised from 4 K to 300 K ?

12. Estimate the proportion of boron impurity which will increase the conductivity of a pure silicon sample by a factor of 100. Assume that each boron atom creates a hole and the concentration of holes in pure silicon at the same temperature is $7 \times 10^{15}$ holes per cubic metre. Density of silicon is $5 \times 10^{28}$ atoms per cubic metre.

13. The product of the hole concentration and the conduction electron concentration turns out to be independent of the amount of any impurity doped. The concentration of conduction electrons in germanium is $6 \times 10^{19}$ per cubic metre. When some phosphorus impurity is doped into a germanium sample, the concentration of conduction electrons increases to $2 \times 10^{23}$ per cubic metre. Find the concentration of the holes in the doped germanium.

14. The conductivity of an intrinsic semiconductor depends on temperature as $\sigma = \sigma_0 e^{-\Delta E/2kT}$ where $\sigma_0$ is a constant. Find the temperature at which the conductivity of an intrinsic germanium semiconductor will be double of its value at $T = 300$ K. Assume that the gap for germanium is 0.650 eV and remains constant as the temperature is increased.

15. A semiconducting material has a band gap of 1 eV. Acceptor impurities are doped into it which create acceptor levels 1 meV above the valence band. Assume that the transition from one energy level to the other is almost forbidden if $kT$ is less than 1/50 of the energy gap. Also, if $kT$ is more than twice the gap, the upper levels have maximum population. The temperature of the semiconductor is increased from 0 K. The concentration of the holes increases with temperature and after a certain temperature it becomes approximately constant. As the temperature is further increased, the hole concentration again starts increasing at a certain temperature. Find the order of the temperature range in which the hole concentration remains approximately constant.

16. In a *p-n* junction, the depletion region is 400 nm wide and an electric field of $5 \times 10^5$ V/m exists in it. (a) Find the height of the potential barrier. (b) What should be the minimum kinetic energy of a conduction electron which can diffuse from the *n*-side to the *p*-side ?

17. The potential barrier existing across an unbiased *p-n* junction is 0.2 volt. What minimum kinetic energy a hole should have to diffuse from the *p*-side to the *n*-side if (a) the junction is unbiased, (b) the junction is forward-biased at 0.1 volt and (c) the junction is reverse-biased at 0.1 volt ?

18. In a *p-n* junction, a potential barrier of 250 meV exists across the junction. A hole with a kinetic energy of 300 meV approaches the junction. Find the kinetic energy of the hole when it crosses the junction if the hole approached the junction (a) from the *p*-side and (b) from the *n*-side.

19. When a *p-n* junction is reverse-biased, the current becomes almost constant at 25 μA. When it is forward-biased at 200 mV, a current of 75 μA is obtained. Find the magnitude of diffusion current when the diode is

(a) unbiased, (b) reverse-biased at 200 mV and (c) forward-biased at 200 mV.

20. The drift current in a $p$-$n$ junction is 20·0 μA. Estimate the number of electrons crossing a cross-section per second in the depletion region.

21. The current–voltage characteristic of an ideal $p$-$n$ junction diode is given by

$$i = i_0 (e^{eV/kT} - 1)$$

where the drift current $i_0$ equals 10 μA. Take the temperature $T$ to be 300 K. (a) Find the voltage $V_0$ for which $e^{eV/kT} = 100$. One can neglect the term 1 for voltages greater than this value. (b) Find an expression for the dynamic resistance of the diode as a function of $V$ for $V > V_0$. (c) Find the voltage for which the dynamic resistance is 0·2 Ω.

22. Consider a $p$-$n$ junction diode having the characteristic $i = i_0(e^{eV/kT} - 1)$ where $i_0 = 20$ μA. The diode is operated at $T = 300$ K. (a) Find the current through the diode when a voltage of 300 mV is applied across it in forward bias. (b) At what voltage does the current double ?

23. Calculate the current through the circuit and the potential difference across the diode shown in figure (45-E1). The drift current for the diode is 20 μA.
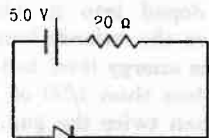


Figure 45-E1

24. Each of the resistances shown in figure (45-E2) has a value of 20 Ω. Find the equivalent resistance between $A$ and $B$. Does it depend on whether the point $A$ or $B$ is at higher potential ?
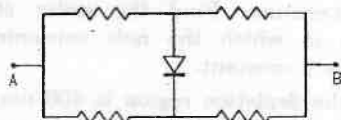


Figure 45-E2

*In problems 25 to 30, assume that the resistance of each diode is zero in forward bias and is infinity in reverse bias.*

25. Find the currents through the resistances in the circuits shown in figure (45-E3).
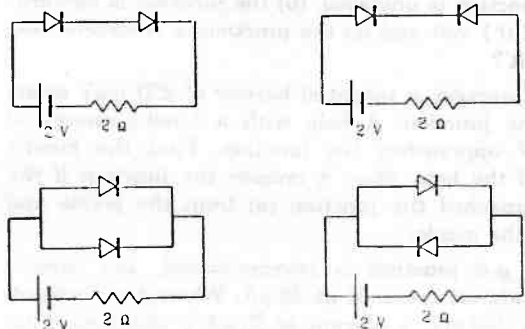


Figure 45-E3

26. What are the readings of the ammeters $A_1$ and $A_2$ shown in figure (45-E4). Neglect the resistances of the meters.
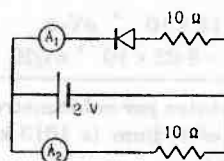


Figure 45-E4

27. Find the current through the battery in each of the circuits shown in figure (45-E5).
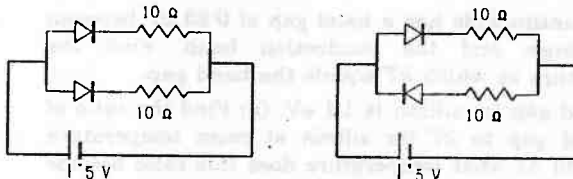


Figure 45-E5

28. Find the current through the resistance $R$ in figure (45-E6) if (a) $R = 12$ Ω (b) $R = 48$ Ω.
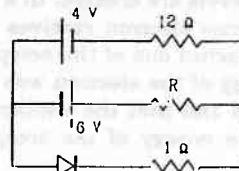


Figure 45-E6

29. Draw the current–voltage characteristics for the device shown in figure (45-E7) between the terminals $A$ and $B$.
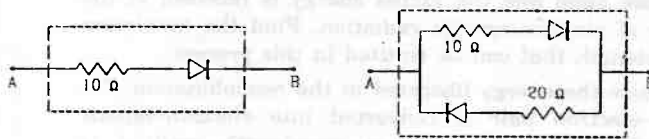


Figure 45-E7

30. Find the equivalent resistance of the network shown in figure (45-E8) between the points $A$ and $B$.
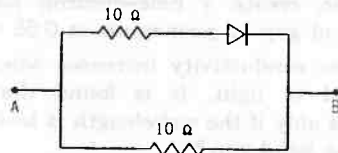


Figure 45-E8

31. When the base current in a transistor is changed from 30 μA to 80 μA, the collector current is changed from 1·0 mA to 3·5 mA. Find the current gain β.

32. A load resistor of 2 kΩ is connected in the collector branch of an amplifier circuit using a transistor in common-emitter mode. The current gain β = 50. The input resistance of the transistor is 0·50 kΩ. If the input current is changed by 50 μA, (a) by what amount does the output voltage change, (b) by what amount does the input voltage change and (c) what is the power gain ?

33. Let $X = A\overline{BC} + B\overline{CA} + C\overline{AB}$. Evaluate $X$ for
(a) $A = 1, B = 0, C = 1$,      (b) $A = B = C = 1$, and
(c) $A = B = C = 0$.

34. Design a logical circuit using AND, OR and NOT gates to evaluate $ABC + BCA$.

35. Show that $AB + AB$ is always 1.

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (b) | 3. (b) | 4. (d) | 5. (c) | 6. (a) |
| 7. (a) | 8. (a) | 9. (b) | 10. (a) | 11. (b) | 12. (b) |
| 13. (c) | 14. (c) | 15. (d) | | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (c), (d) | 2. (b), (c), (d) | 3. (a), (d) |
| 4. (b), (d) | 5. all | 6. (d) |
| 7. (b), (c) | 8. (a), (c) | 9. (a), (d) |
| 10. (c), (d) | | |

### EXERCISES

1. $5.3 \times 10^{-10}$, $2.65 \times 10$

2. $6 \times 10^{17}$

3. 2670 K

4. (a) 43 (b) 3000 K

5. 50 meV

6. 390 nm

7. 1.5 eV

8. $1.9 \times 10^{-5}$ m

9. 2.0 eV

10. $2.3 \times 10^{-}$, almost zero

11. approximately $10^{403}$

12. 1 in about $3.5 \times 10$

13. $1.8 \times 10^{16}$ per cubic metre

14. 318 K

15. 20 to 230 K

16. (a) 0.2 V (b) 0.2 eV

17. (a) 0.2 eV (b) 0.1 eV (c) 0.3 eV

18. (a) 50 meV (b) 550 meV

19. (a) 25 μA (b) zero (c) 100 μA

20. $3.1 \times 10^{13}$

21. (a) 0.12 V (b) $\dfrac{}{ei_n} e^{\cdots}$ (c) 0.25 V

22. (a) 2 A (b) 318 mV

23. 20 μA, 4.996 V = 5 V

24. 20 Ω

25. (a) 1 A (b) zero (c) 1 A (d) 1 A

26. zero, 0.2 A

27. (a) 1A (b) 0.5 A

28. (a) 0.42 A, 0.13 A

30. 5 Ω if $V_A > V_B$ and 10 Ω if $V_1 < V_B$

31. 50

32. (a) 5.0 V (b) 25 mV (c) $10^4$

33. (a) 1 (b) 0 (c) 0

# CHAPTER 46

# THE NUCLEUS

At the centre of an atom exists the *nucleus* which contains protons and neutrons. The electrons surround this nucleus to form the atom. As discussed earlier, this structure of atom was revealed by the experiments of Rutherford in which a beam of alpha particles was made to strike a thin gold foil. Most of the alpha particles crossed the foil without being appreciably deviated, but there were some alpha particles which suffered large deviation from their original lines of motion. The data suggested that positive charges in an atom are concentrated in a small volume which we call nucleus and this nucleus is responsible for the large deviation of alpha particles. Later on, the existence of protons and neutrons in the nucleus was established. In this chapter, we shall discuss the physics of the nucleus.

## 46.1 PROPERTIES OF A NUCLEUS

### Nuclear Constituents

A nucleus is made of protons and neutrons. A proton has a positive charge of magnitude equal to that of an electron and has a mass of about 1840 times the mass of an electron. A neutron has a mass slightly greater than that of a proton. The masses of a proton and a neutron are

$$m_p = 1 \cdot 6726231 \times 10^{-27} \text{ kg}$$

$$\text{and} \quad m_n = 1 \cdot 6749286 \times 10^{-27} \text{ kg}.$$

It is customary in nuclear physics and high energy physics to represent mass in energy units according to the conversion formula $E = mc^2$. (Matter can be viewed as a condensed form of energy. Theory of relativity reveals that a mass $m$ is equivalent to an energy $E$ where $E = mc^2$.) For example, the mass of an electron is $m_e = 9 \cdot 1093897 \times 10^{-31}$ kg and the equivalent energy is

$$m_e c^2 = 510 \cdot 99 \text{ keV}.$$

Thus, the mass of an electron is $510 \cdot 99$ keV/$c^2$. Similarly, the mass of a proton is $938 \cdot 27231$ MeV/$c^2$

and the mass of a neutron is $939 \cdot 56563$ MeV/$c^2$. The energy corresponding to the mass of a particle when it is at rest is called its *rest mass energy*.

Another unit which is widely used in describing mass in nuclear physics as well as in atomic physics is *unified atomic mass unit* denoted by the symbol u. It is 1/12 of the mass of a neutral carbon atom in its lowest energy state which contains six protons, six neutrons and six electrons. We have

$$1 \text{ u} = 1 \cdot 6605402 \times 10^{-27} \text{ kg} = 931 \cdot 478 \text{ MeV/}c^2.$$

Protons and neutrons are fermions and obey the Pauli exclusion principle like electrons. No two protons or two neutrons can have the same quantum state. But one proton and one neutron can exist in the same quantum state. Protons and neutrons are collectively called *nucleons*.

The number of protons in a nucleus is denoted by $Z$, the number of neutrons by $N$ and the total number of nucleons by $A$. Thus, $A = Z + N$. The total number of nucleons $A$ is also called the *mass number* of the nucleus. The number of protons $Z$ is called the *atomic number*. A nucleus is symbolically expressed as $^A_Z X$ in which X is the chemical symbol of the element. Thus, $^4_2 He$ represents helium nucleus which contains 2 protons and a total of 4 nucleons. So it contains 2 neutrons. Similarly, $^{238}_{92} U$ represents a uranium nucleus which contains 92 protons and 146 neutrons.

The distribution of electrons around the nucleus is determined by the number of protons $Z$ and hence the chemical properties of an element are also determined by $Z$. The nuclei having the same number of protons but different number of neutrons are called *isotopes*. Nuclei with the same neutron number $N$ but different atomic number $Z$ are called *isotones* and the nuclei with the same mass number $A$ are called *isobars*. All nuclei with a given $Z$ and $N$ are collectively called a *nuclide*. Thus, all the $^{56}_{26} Fe$ nuclei taken together is one nuclide and all the $^{32}_{16} S$ nuclei taken together is another nuclide.

or,                 $\dfrac{1}{t} = \dfrac{1}{t_1} + \dfrac{1}{t_2}$.

**15.** *Calculate the energy released when three alpha particles combine to form a $^{12}C$ nucleus. The atomic mass of $_2He$ is 4·002603 u.*

*Solution :* The mass of a $^{12}C$ atom is exactly 12 u. The energy released in the reaction $3\left(_2^4He\right) \rightarrow \, _6^{12}C$ is

$$[3\,m(_2He) - m(_6^{12}C)]c^{\,2}$$

$$= [3 \times 4{\cdot}002603\ u - 12\ u]\,(931\ MeV/u) = 7{\cdot}27\ MeV.$$

□

## QUESTIONS FOR SHORT ANSWER

1. If neutrons exert only attractive force, why don't we have a nucleus containing neutrons alone ?

2. Consider two pairs of neutrons. In each pair, the separation between the neutrons is the same. Can the force between the neutrons have different magnitudes for the two pairs ?

3. A molecule of hydrogen contains two protons and two electrons. The nuclear force between these two protons is always neglected while discussing the behaviour of a hydrogen molecule. Why ?

4. Is it easier to take out a nucleon from carbon or from iron ? From iron or from lead ?

5. Suppose we have 12 protons and 12 neutrons. We can assemble them to form either a $^{24}Mg$ nucleus or two $^{12}C$ nuclei. In which of the two cases more energy will be liberated ?

6. What is the difference between cathode rays and beta rays ? When the two are travelling in space, can you make out which is the cathode ray and which is the beta ray ?

7. If the nucleons of a nucleus are separated from each other, the total mass is increased. Where does this mass come from ?

8. In beta decay, an electron (or a positron) is emitted by a nucleus. Does the remaining atom get oppositely charged ?

9. When a boron nucleus ($^{10}_5B$) is bombarded by a neutron, an α-particle is emitted. Which nucleus will be formed as a result ?

10. Does a nucleus lose mass when it suffers gamma decay ?

11. In a typical fission reaction, the nucleus is split into two middle-weight nuclei of unequal masses. Which of the two (heavier or lighter) has greater kinetic energy ? Greater linear momentum ?

12. If three helium nuclei combine to form a carbon nucleus, energy is liberated. Why can't helium nuclei combine on their own and minimise the energy ?

## OBJECTIVE I

1. The mass of a neutral carbon atom in ground state is
   (a) exact 12 u          (b) less than 12 u
   (c) more than 12 u
   (d) depends on the form of carbon such as graphite or charcoal.

2. The mass number of a nucleus is equal to
   (a) the number of neutrons in the nucleus
   (b) the number of protons in the nucleus
   (c) the number of nucleons in the nucleus
   (d) none of them.

3. As compared to $^{12}C$ atom, $^{14}C$ atom has
   (a) two extra protons and two extra electrons
   (b) two extra protons but no extra electron
   (c) two extra neutrons and no extra electron
   (d) two extra neutrons and two extra electrons.

4. The mass number of a nucleus is
   (a) always less than its atomic number
   (b) always more than its atomic number
   (c) equal to its atomic number
   (d) sometimes more than and sometimes equal to its atomic number.

5. The graph of $\ln(R/R_0)$ versus $\ln A$ ($R$ = radius of a nucleus and $A$ = its mass number) is
   (a) a straight line      (b) a parabola
   (c) an ellipse           (d) none of them.

6. Let $F_{pp}$, $F_{pn}$ and $F_{nn}$ denote the magnitudes of the nuclear force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. When the separation is 1 fm,
   (a) $F_{pp} > F_{pn} = F_{nn}$          (b) $F_{pp} = F_{pn} = F_{nn}$
   (c) $F_{pp} > F_{pn} > F_{nn}$          (d) $F_{pp} < F_{pn} = F_{nn}$.

7. Let $F_{pp}$, $F_{pn}$ and $F_{nn}$ denote the magnitudes of the net force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. Neglect gravitational force. When the separation is 1 fm,
   (a) $F_{pp} > F_{pn} = F_{nn}$          (b) $F_{pp} = F_{pn} = F_{nn}$
   (c) $F_{pp} > F_{pn} > F_{nn}$          (d) $F_{pp} < F_{pn} = F_{nn}$.

8. Two protons are kept at a separation of 10 nm. Let $F_n$ and $F_e$ be the nuclear force and the electromagnetic force between them.
   (a) $F_e = F_n$.        (b) $F_e \gg F_n$.        (c) $F_e \ll F_n$.
   (d) $F_e$ and $F_n$ differ only slightly.

9. As the mass number $A$ increases, the binding energy per nucleon in a nucleus
(a) increases   (b) decreases   (c) remains the same
(d) varies in a way that depends on the actual value of $A$.

10. Which of the following is a wrong description of binding energy of a nucleus ?
(a) It is the energy required to break a nucleus into its constituent nucleons.
(b) It is the energy made available when free nucleons combine to form a nucleus.
(c) It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus.
(d) It is the sum of the kinetic energy of all the nucleons in the nucleus.

11. In one average-life,
(a) half the active nuclei decay
(b) less than half the active nuclei decay
(c) more than half the active nuclei decay
(d) all the nuclei decay.

12. In a radioactive decay, neither the atomic number nor the mass number changes. Which of the following particles is emitted in the decay ?
(a) proton   (b) neutron   (c) electron   (d) photon.

13. During a negative beta decay,
(a) an atomic electron is ejected
(b) an electron which is already present within the nucleus is ejected
(c) a neutron in the nucleus decays emitting an electron
(d) a proton in the nucleus decays emitting an electron.

14. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is
(a) 6 h   (b) 12 h   (c) 24 h   (d) 128 h.

15. The decay constant of a radioactive sample is $\lambda$. The half-life and the average-life of the sample are respectively
(a) $1/\lambda$ and $(\ln 2/\lambda)$   (b) $(\ln 2/\lambda)$ and $1/\lambda$
(c) $\lambda(\ln 2)$ and $1/\lambda$   (d) $\lambda/(\ln 2)$ and $1/\lambda$.

16. An $\alpha$-particle is bombarded on $^{14}N$. As a result, a $^{17}O$ nucleus is formed and a particle is emitted. This particle is a
(a) neutron   (b) proton   (c) electron   (d) positron.

17. Ten grams of $^{57}Co$ kept in an open container beta-decays with a half-life of 270 days. The weight of the material inside the container after 540 days will be very nearly
(a) 10 g   (b) 5 g   (c) 2·5 g   (d) 1·25 g.

18. Free $^{238}U$ nuclei kept in a train emit alpha particles. When the train is stationary and a uranium nucleus decays, a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes $x$ in time $t$ after the decay. If a decay takes place when the train is moving at a uniform speed $v$, the distance between the alpha particle and the recoiling nucleus at a time $t$ after the decay, as measured by the passenger will be
(a) $x + vt$   (b) $x - vt$   (c) $x$
(d) depends on the direction of the train.

19. During a nuclear fission reaction,
(a) a heavy nucleus breaks into two fragments by itself
(b) a light nucleus bombarded by thermal neutrons breaks up
(c) a heavy nucleus bombarded by thermal neutrons breaks up
(d) two light nuclei combine to give a heavier nucleus and possibly other products.

## OBJECTIVE II

1. As the mass number $A$ increases, which of the following quantities related to a nucleus do not change ?
(a) mass   (b) volume   (c) density   (d) binding energy.

2. The heavier nuclei tend to have larger $N/Z$ ratio because
(a) a neutron is heavier than a proton
(b) a neutron is an unstable particle
(c) a neutron does not exert electric repulsion
(d) Coulomb forces have longer range compared to the nuclear forces.

3. A free neutron decays to a proton but a free proton does not decay to a neutron. This is because
(a) neutron is a composite particle made of a proton and an electron whereas proton is a fundamental particle
(b) neutron is an uncharged particle whereas proton is a charged particle
(c) neutron has larger rest mass than the proton
(d) weak forces can operate in a neutron but not in a proton.

4. Consider a sample of a pure beta-active material.
(a) All the beta particles emitted have the same energy.
(b) The beta particles originally exist inside the nucleus and are ejected at the time of beta decay.
(c) The antineutrino emitted in a beta decay has zero mass and hence zero momentum.
(d) The active nucleus changes to one of its isobars after the beta decay.

5. In which of the following decays the element does not change ?
(a) $\alpha$-decay   (b) $\beta^+$-decay   (c) $\beta^-$-decay   (d) $\gamma$-decay.

6. In which of the following decays the atomic number decreases ?
(a) $\alpha$-decay   (b) $\beta^+$-decay   (c) $\beta^-$-decay   (d) $\gamma$-decay.

7. Magnetic field does not cause deflection in
(a) $\alpha$-rays   (b) beta-plus rays
(c) beta-minus rays   (d) gamma rays.

8. Which of the following are electromagnetic waves ?
(a) $\alpha$-rays   (b) beta-plus rays
(c) beta-minus rays   (d) gamma rays.

9. Two lithium nuclei in a lithium vapour at room temperature do not combine to form a carbon nucleus because
(a) a lithium nucleus is more tightly bound than a

carbon nucleus
(b) carbon nucleus is an unstable particle
(c) it is not energetically favourable
(d) Coulomb repulsion does not allow the nuclei to come very close.

10. For nuclei with $A > 100$,
(a) the binding energy of the nucleus decreases on an average as $A$ increases
(b) the binding energy per nucleon decreases on an average as $A$ increases
(c) if the nucleus breaks into two roughly equal parts, energy is released
(d) if two nuclei fuse to form a bigger nucleus, energy is released.

## EXERCISES

Mass of proton $m_p = 1.007276$ u, Mass of $_1^1H$ atom $= 1.007825$ u, Mass of neutron $m_n = 1.008665$ u, Mass of electron $= 0.0005486$ u $\approx 511$ keV/c$^2$, 1 u $= 931$ MeV/c$^2$.

1. Assume that the mass of a nucleus is approximately given by $M = Am_p$ where $A$ is the mass number. Estimate the density of matter in kg/m$^3$ inside a nucleus. What is the specific gravity of nuclear matter ?

2. A neutron star has a density equal to that of the nuclear matter. Assuming the star to be spherical, find the radius of a neutron star whose mass is $4.0 \times 10^{\sim}$ kg (twice the mass of the sun).

3. Calculate the mass of an $\alpha$-particle. Its binding energy is 28.2 MeV.

4. How much energy is released in the following reaction?
$$^7Li + p \rightarrow \alpha + \alpha$$
Atomic mass of $^7Li = 7.0160$ u and that of $^4He = 4.0026$ u.

5. Find the binding energy per nucleon of $_{79}^{197}Au$ if its atomic mass is 196.96 u.

6. (a) Calculate the energy released if $^{\sim\sim}U$ emits an $\alpha$-particle. (b) Calculate the energy to be supplied to $^{238}U$ if two protons and two neutrons are to be emitted one by one. The atomic masses of $^{238}U$, $^{234}Th$ and $^4He$ are 238.0508 u, 234.04363 u and 4.00260 u respectively.

7. Find the energy liberated in the reaction
$$^{223}Ra \rightarrow ^{209}Pb + ^{14}C.$$
The atomic masses needed are as follows:

| $^{223}Ra$ | $^{209}Pb$ | $^{14}C$ |
|---|---|---|
| 223.018 u | 208.981 u | 14.003 u |

8. Show that the minimum energy needed to separate a proton from a nucleus with $Z$ protons and $N$ neutrons is
$$\Delta E = (M_{Z-1,N} + M_H - M_{Z,N})c^2$$
where $M_{Z,N}$ = mass of an atom with $Z$ protons and $N$ neutrons in the nucleus and $M_H$ = mass of a hydrogen atom. This energy is known as *proton-separation energy*.

9. Calculate the minimum energy needed to separate a neutron from a nucleus with $Z$ protons and $N$ neutrons in terms of the masses $M_{Z,N}$, $M_{Z,N-1}$ and the mass of the neutron.

10. $^{32}P$ beta-decays to $^{32}S$. Find the sum of the energy of the antineutrino and the kinetic energy of the $\beta$-particle. Neglect the recoil of the daughter nucleus. Atomic mass of $^{32}P = 31.974$ u and that of $^{32}S = 31.972$ u.

11. A free neutron beta-decays to a proton with a half-life of 14 minutes. (a) What is the decay constant ? (b) Find the energy liberated in the process.

12. Complete the following decay schemes.
(a) $_{88}^{228}Ra \rightarrow \alpha +$
(b) $_8^{19}O \rightarrow _9^{19}F +$
(c) $_{13}^{25}Al \rightarrow _{12}^{25}Mg +$

13. In the decay $^{64}Cu \rightarrow ^{64}Ni + e^+ + \nu$, the maximum kinetic energy carried by the positron is found to be 0.650 MeV. (a) What is the energy of the neutrino which was emitted together with a positron of kinetic energy 0.150 MeV ? (b) What is the momentum of this neutrino in kg–m/s ? Use the formula applicable to a photon.

14. Potassium-40 can decay in three modes. It can decay by $\beta^-$-emission, $\beta^+$-emission or electron capture. (a) Write the equations showing the end products. (b) Find the $Q$-values in each of the three cases. Atomic masses of $_{18}^{40}Ar$, $_9^{40}K$ and $_{20}^{40}Ca$ are 39.9624 u, 39.9640 u and 39.9626 u respectively.

15. Lithium ($Z = 3$) has two stable isotopes $^6Li$ and $^7Li$. When neutrons are bombarded on lithium sample, electrons and $\alpha$-particles are ejected. Write down the nuclear processes taking place.

16. The masses of $^{11}C$ and $^{11}B$ are respectively 11.0114 u and 11.0093 u. Find the maximum energy a positron can have in the $\beta^+$-decay of $^{11}C$ to $^{11}B$.

17. $^{228}Th$ emits an alpha particle to reduce to $^{224}Ra$. Calculate the kinetic energy of the alpha particle emitted in the following decay:
$$^{228}Th \rightarrow ^{224}Ra^* + \alpha$$
$$^{224}Ra^* \rightarrow ^{224}Ra + \gamma (217 \text{ keV}).$$
Atomic mass of $^{228}Th$ is 228.028726 u, that of $^{224}Ra$ is 224.020196 u and that of $_2^4He$ is 4.00260 u.

18. Calculate the maximum kinetic energy of the beta particle emitted in the following decay scheme:
$$^{12}N \rightarrow ^{12}C^* + e^+ + \nu$$
$$^{12}C^* \rightarrow ^{12}C + \gamma (4.43 \text{ MeV}).$$
The atomic mass of $^{12}N$ is 12.018613 u.

19. The decay constant of $_{80}^{197}Hg$ (electron capture to $_{79}^{197}Au$) is $1.8 \times 10^{-4}$ s$^{-1}$. (a) What is the half-life ? (b) What is the average-life ? (c) How much time will it take to convert 25% of this isotope of mercury into gold ?

20. The half-life of $^{198}$Au is 2·7 days. (a) Find the activity of a sample containing 1·00 μg of $^{198}$Au. (b) What will be the activity after 7 days ? Take the atomic weight of $^{198}$Au to be 198 g/mol.

21. Radioactive $^{131}$I has a half-life of 8·0 days. A sample containing $^{131}$I has activity 20 μCi at $t = 0$. (a) What is its activity at $t = 4·0$ days ? (b) What is its decay constant at $t = 4·0$ days ?

22. The decay constant of $^{238}$U is $4·9 \times 10^{-18}$ s$^{-1}$. (a) What is the average-life of $^{238}$U ? (b) What is the half-life of $^{238}$U ? (c) By what factor does the activity of a $^{238}$U sample decrease in $9 \times 10^9$ years ?

23. A certain sample of a radioactive material decays at the rate of 500 per second at a certain time. The count rate falls to 200 per second after 50 minutes. (a) What is the decay constant of the sample ? (b) What is its half-life ?

24. The count rate from a radioactive sample falls from $4·0 \times 10^6$ per second to $1·0 \times 10^6$ per second in 20 hours. What will be the count rate 100 hours after the beginning ?

25. The half-life of $^{226}$Ra is 1602 y. Calculate the activity of 0·1 g of RaCl$_2$ in which all the radium is in the form of $^{226}$Ra. Taken atomic weight of Ra to be 226 g/mol and that of Cl to be 35·5 g/mol.

26. The half-life of a radioisotope is 10 h. Find the total number of disintegrations in the tenth hour measured from a time when the activity was 1 Ci.

27. The selling rate of a radioactive isotope is decided by its activity. What will be the second-hand rate of a one month old $^{32}$P($t_{1/2} = 14·3$ days) source if it was originally purchased for 800 rupees ?

28. $^{57}$Co decays to $^{57}$Fe by β$^+$-emission. The resulting $^{57}$Fe is in its excited state and comes to the ground state by emitting γ-rays. The half-life of β$^+$-decay is 270 days and that of the γ-emission is $10^{-8}$ s. A sample of $^{57}$Co gives $5·0 \times 10^9$ gamma rays per second. How much time will elapse before the emission rate of gamma rays drops to $2·5 \times 10^9$ per second ?

29. Carbon ($Z = 6$) with mass number 11 decays to boron ($Z = 5$). (a) Is it a β$^+$-decay or a β$^-$-decay ? (b) The half-life of the decay scheme is 20·3 minutes. How much time will elapse before a mixture of 90% carbon-11 and 10% boron-11 (by the number of atoms) converts itself into a mixture of 10% carbon-11 and 90% boron-11 ?

30. $4 \times 10^{23}$ tritium atoms are contained in a vessel. The half-life of decay of tritium nuclei is 12·3 y. Find (a) the activity of the sample, (b) the number of decays in the next 10 hours (c) the number of decays in the next 6·15 y.

31. A point source emitting alpha particles is placed at a distance of 1 m from a counter which records any alpha particle falling on its 1 cm$^2$ window. If the source contains $6·0 \times 10^{16}$ active nuclei and the counter records a rate of 50000 counts/second, find the decay constant. Assume that the source emits alpha particles uniformly in all directions and the alpha particles fall nearly normally on the window.

32. $^{238}$U decays to $^{206}$Pb with a half-life of $4·47 \times 10^9$ y. This happens in a number of steps. Can you justify a single half-life for this chain of processes ? A sample of rock is found to contain 2·00 mg of $^{238}$U and 0·600 mg of $^{206}$Pb. Assuming that all the lead has come from uranium, find the life of the rock.

33. When charcoal is prepared from a living tree, it shows a disintegration rate of 15·3 disintegrations of $^{14}$C per gram per minute. A sample from an ancient piece of charcoal shows $^{14}$C activity to be 12·3 disintegrations per gram per minute. How old is this sample ? Half-life of $^{14}$C is 5730 y.

34. Natural water contains a small amount of tritium ($^3_1$H). This isotope beta-decays with a half-life of 12·5 years. A mountaineer while climbing towards a difficult peak finds debris of some earlier unsuccessful attempt. Among other things he finds a sealed bottle of whisky. On return he analyses the whisky and finds that it contains only 1·5 per cent of the $^3$H radioactivity as compared to a recently purchased bottle marked '8 years old'. Estimate the time of that unsuccessful attempt.

35. The count rate of nuclear radiation coming from a radioactive sample containing $^{128}$I varies with time as follows.

| Time $t$ (minute): | 0 | 25 | 50 | 75 | 100 |
|---|---|---|---|---|---|
| Count rate $R(10^9$ s$^{-1})$: | 30 | 16 | 8·0 | 3·8 | 2·0 |

(a) Plot $\ln(R_0/R)$ against $t$. (b) From the slope of the best straight line through the points, find the decay constant λ. (c) Calculate the half-life $t_{1/2}$.

36. The half-life of $^{40}$K is $1·30 \times 10^9$ y. A sample of 1·00 g of pure KCl gives 160 counts/s. Calculate the relative abundance of $^{40}$K (fraction of $^{40}$K present) in natural potassium.

37. $^{197}_{80}$Hg decays to $^{197}$Au through electron capture with a decay constant of 0·257 per day. (a) What other particle or particles are emitted in the decay ? (b) Assume that the electron is captured from the K shell. Use Moseley's law $\sqrt{v} = a(Z - b)$ with $a = 4·95 \times 10^7$ s$^{-1/2}$ and $b = 1$ to find the wavelength of the $K_\alpha$ X-ray emitted following the electron capture.

38. A radioactive isotope is being produced at a constant rate $dN/dt = R$ in an experiment. The isotope has a half-life $t_{1/2}$. Show that after a time $t \gg t_{1/2}$, the number of active nuclei will become constant. Find the value of this constant.

39. Consider the situation of the previous problem. Suppose the production of the radioactive isotope starts at $t = 0$. Find the number of active nuclei at time $t$.

40. In an agricultural experiment, a solution containing 1 mole of a radioactive material ($t_{1/2} = 14·3$ days) was injected into the roots of a plant. The plant was allowed 70 hours to settle down and then activity was measured in its fruit. If the activity measured was 1 μCi, what per cent of activity is transmitted from the root to the fruit in steady state ?

41. A vessel of volume 125 cm$^3$ contains tritium ($^3$H, $t_{1/2} = 12·3$ y) at 500 kPa and 300 K. Calculate the activity of the gas.

**42.** $^{212}_{83}$Bi can disintegrate either by emitting an α-particle or by emitting a β⁻-particle. (a) Write the two equations showing the products of the decays. (b) The probabilities of disintegration by α- and β-decays are in the ratio 7/13. The overall half-life of $^{212}$Bi is one hour. If 1 g of pure $^{212}$Bi is taken at 12·00 noon, what will be the composition of this sample at 1 p.m. the same day?

**43.** A sample contains a mixture of $^{108}$Ag and $^{110}$Ag isotopes each having an activity of $8·0 \times 10^8$ disintegrations per second. $^{110}$Ag is known to have larger half-life than $^{108}$Ag. The activity $A$ is measured as a function of time and the following data are obtained.

| Time (s) | Activity ($A$) ($10^8$ disintegrations/s) | Time (s) | Activity ($A$) ($10^8$ disintegrations/s) |
|---|---|---|---|
| 20 | 11·799 | 200 | 3·0828 |
| 40 | 9·1680 | 300 | 1·8899 |
| 60 | 7·4492 | 400 | 1·1671 |
| 80 | 6·2684 | 500 | 0·7212 |
| 100 | 5·4115 | | |

(a) Plot $\ln(A/A_0)$ versus time. (b) See that for large values of time, the plot is nearly linear. Deduce the half-life of $^{110}$Ag from this portion of the plot. (c) Use the half-life of $^{110}$Ag to calculate the activity corresponding to $^{108}$Ag in the first 50 s. (d) Plot $\ln(A/A_0)$ versus time for $^{108}$Ag for the first 50 s. (e) Find the half-life of $^{108}$Ag.

**44.** A human body excretes (removes by waste discharge, sweating etc.) certain materials by a law similar to radioactivity. If technitium is injected in some form in a human body, the body excretes half the amount in 24 hours. A patient is given an injection containing $^{99}$Tc. This isotope is radioactive with a half-life of 6 hours. The activity from the body just after the injection is 6 μCi. How much time will elapse before the activity falls to 3 μCi?

**45.** A charged capacitor of capacitance $C$ is discharged through a resistance $R$. A radioactive sample decays with an average-life τ. Find the value of $R$ for which the ratio of the electrostatic field energy stored in the capacitor to the activity of the radioactive sample remains constant in time.

**46.** Radioactive isotopes are produced in a nuclear physics experiment at a constant rate $dN/dt = R$. An inductor of inductance 100 mH, a resistor of resistance 100 Ω and a battery are connected to form a series circuit. The circuit is switched on at the instant the production of radioactive isotope starts. It is found that $i/N$ remains constant in time where $i$ is the current in the circuit at time $t$ and $N$ is the number of active nuclei at time $t$. Find the half-life of the isotope.

**47.** Calculate the energy released by 1 g of natural uranium assuming 200 MeV is released in each fission event and that the fissionable isotope $^{235}$U has an abundance of 0·7% by weight in natural uranium.

**48.** A uranium reactor develops thermal energy at a rate of 300 MW. Calculate the amount of $^{235}$U being consumed every second. Average energy released per fission is 200 MeV.

**49.** A town has a population of 1 million. The average electric power needed per person is 300 W. A reactor is to be designed to supply power to this town. The efficiency with which thermal power is converted into electric power is aimed at 25%. (a) Assuming 200 MeV of thermal energy to come from each fission event on an average, find the number of events that should take place every day. (b) Assuming the fission to take place largely through $^{235}$U, at what rate will the amount of $^{235}$U decrease? Express your answer in kg/day. (c) Assuming that uranium enriched to 3% in $^{235}$U will be used, how much uranium is needed per month (30 days)?

**50.** Calculate the $Q$-values of the following fusion reactions:
(a) $^2_1\text{H} + ^2_1\text{H} \rightarrow ^1_1\text{H} + ^1_1\text{H}$
(b) $^2_1\text{H} + ^2_1\text{H} \rightarrow ^3_2\text{He} + n$
(c) $^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + n.$

Atomic masses are $m(^2_1\text{H}) = 2·014102$ u, $m(^3_1\text{H}) = 3·016049$ u, $m(^3_2\text{He}) = 3·016029$ u, $m(^4_2\text{He}) = 4·002603$ u.

**51.** Consider the fusion in helium plasma. Find the temperature at which the average thermal energy $1·5\,kT$ equals the Coulomb potential energy at 2 fm.

**52.** Calculate the $Q$-value of the fusion reaction
$$^4\text{He} + ^4\text{He} = ^8\text{Be}.$$
Is such a fusion energetically favourable? Atomic mass of $^8$Be is 8·0053 u and that of $^4$He is 4·0026 u.

**53.** Calculate the energy that can be obtained from 1 kg of water through the fusion reaction
$$^2\text{H} + ^2\text{H} \rightarrow ^3\text{H} + p.$$
Assume that $1·5 \times 10^{-2}$ % of natural water is heavy water $\text{D}_2\text{O}$ (by number of molecules) and all the deuterium is used for fusion.

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (a) | 2. (c) | 3. (c) | 4. (d) | 5. (a) | 6. (b) |
| 7. (d) | 8. (b) | 9. (d) | 10. (d) | 11. (c) | 12. (d) |
| 13. (c) | 14. (b) | 15. (b) | 16. (b) | 17. (a) | 18. (c) |
| 19. (c) | | | | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (c) | 2. (c), (d) | 3. (c) |
| 4. (d) | 5. (d) | 6. (a), (b) |
| 7. (d) | 8. (d) | 9. (d) |
| 10. (b), (c) | | |

# EXERCISES

1. $3 \times 10^{17}$ kg/m$^3$, $3 \times 10^{14}$

2. 15 km

3. 4·0016 u

4. 17·34 MeV

5. 7·94 MeV

6. (a) 4·255 MeV (b) 24·03 MeV

7. 31·65 MeV

9. $(M_{Z, N-1} + m_n - M_{Z, N})c^2$

10. 1·86 MeV

11. (a) $8·25 \times 10^{-4}$ s$^{-1}$ (b) 782 keV

12. (a) $^{222}_{86}$Rn (b) $\bar{e} + \bar{v}$ (c) $e^+ + v$

13. (a) 500 keV (b) $2·67 \times 10^{-22}$ kg–m/s

14. (a) $^{40}_{19}$K $\rightarrow$ $^{40}_{20}$Ca $+ e^- + \bar{v}$, $^{40}_{19}$K $\rightarrow$ $^{40}_{18}$Ar $+ e^+ + v$,
    $^{40}_{19}$K $+ e^- \rightarrow$ $^{40}_{18}$Ar $+ v$
    (b) 1·3034 MeV, 0·4676 MeV, 1·490 MeV

15. $^6_3$Li $+ n \rightarrow$ $^7_3$Li, $^7_3$Li $+ n \rightarrow$ $^8_3$Li $\rightarrow$ $^8_4$Be $+ e^- + \bar{v}$,
    $^8_4$Be $\rightarrow$ $^4$He $+ ^4$He

16. 933·6 keV

17. 5·304 MeV

18. 11·88 MeV

19. (a) 64 min (b) 92 min (c) 1600 s

20. (a) 0·244 Ci (b) 0·040 Ci

21. (a) 14 µCi (b) $1·4 \times 10^{-6}$ s$^{-1}$

22. $6·49 \times 10^9$ y (b) $4·5 \times 10^9$ y (c) 4

23. $3·05 \times 10^{-4}$ s (b) 38 min

24. $3·9 \times 10^3$ per second

25. $2·8 \times 10^9$ disintegrations/s

26. $6·91 \times 10^{13}$

27. 187 rupees

28. 270 days

29. (a) $\beta^+$ (b) 64 min

30. (a) $7·146 \times 10^{14}$ disintegrations/s
    (b) $2·57 \times 10^{13}$ (c) $1·17 \times 10^{23}$

31. $1·05 \times 10^{-7}$ s$^{-1}$

32. $1·92 \times 10^9$ y

33. 1800 y

34. about 83 years ago

35. (b) 0·028 min$^{-1}$ approx. (c) 25 min approx.

36. 0·12%

37. (a) neutrino (b) 20 pm

38. $\dfrac{Rt_{1/2}}{0·693}$

39. $\dfrac{R}{\lambda}(1 - e^{-\lambda t})$

40. $1·26 \times 10^{-11}$ %

41. 724 Ci

42. (a) $^{212}_{83}$Bi $\rightarrow$ $^{208}_{81}$Tl $+ \alpha$, $^{212}_{83}$Bi $\rightarrow$ $^{212}_{84}$Bi $\rightarrow$ $^{212}_{84}$Po $+ e^- + \bar{v}$
    (b) 0·50 g–Bi, 0·175 g–Tl, 0·325 g–Po

43. the half-life of $^{110}$Ag = 24·4 s and of $^{108}$Ag = 144 s

44. 4·8 hours

45. $2\tau/C$

46. $6·93 \times 10^{-4}$ s

47. $5·7 \times 10^8$ J

48. 3·7 mg

49. (a) $3·24 \times 10^{24}$ (b) 1·264 kg/day (c) 1263 kg

50. (a) 4·05 MeV (b) 3·25 MeV (c) 17·57 MeV

51. $2·23 \times 10^{10}$ K

52. – 93·1 keV, no

53. 3200 MJ

□

or,                $\frac{(t - 5080 \text{ s})}{5080 \text{ s}} = -3.48 \times 10^{-10}$

or,                $(t - 5080 \text{ s}) = -1.77 \times 10^{-6}$ s.

The satellite's clock falls behind by $1.77 \times 10^{-6}$ s in one revolution.

**5.** *The radius of our galaxy is about $3 \times 10^{20}$ m. With what speed should a person travel so that he can reach from the centre of the galaxy to its edge in 20 years of his lifetime ?*

**Solution :** Let the speed of the person be $v$. As seen by the person, the edge of the galaxy is coming towards him at a speed $v$. In 20 years (as measured by the person), the edge moves $(20 \text{ y}) \, v$ and reaches the person. The radius of the galaxy as measured by the person is, therefore, $(20 \text{ y}) v$. The rest length of the radius of the galaxy is $3 \times 10^{20}$ m. Thus,

$$(20 \text{ y})v = (3 \times 10^{20} \text{ m}) \sqrt{1 - v^2/c^2}$$

or,   $(6.312 \times 10^{8} \text{ s})^2 \, v^2 = (9 \times 10^{40} \text{ m}^2)(1 - v^2/c^2).$

Solving this,

$$v = 0.9999996 \, c.$$

**6.** *Find the speed at which the mass of an electron is double of its rest mass.*

**Solution :** The mass of an electron at speed $v$ is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where $m_0$ is its rest mass. If $m = 2 \, m_0$,

$$2 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

or,                $1 - \frac{v^2}{c^2} = \frac{1}{4}$

or,                $v = \frac{\sqrt{3}}{2} \, c = 2.598 \times 10^{8} \text{ m/s.}$

**7.** *Calculate the increase in mass when a body of rest mass 1 kg is lifted up through 1 m near the earth's surface.*

**Solution :** The increase in energy = $mgh$

= $(1 \text{ kg}) (9.8 \text{ m/s}^2) (1 \text{ m}) = 9.8$ J.

The increase in mass = $\dfrac{9.8 \text{ J}}{c^2}$

= $1.11 \times 10^{-16}$ kg.

**8.** *A body of rest mass $m_0$ collides perfectly inelastically at a speed of $0.8c$ with another body of equal rest mass kept at rest. Calculate the common speed of the bodies after the collision and the rest mass of the combined body.*

**Solution :** The linear momentum of the first body

$$= \frac{m_0 \, v}{\sqrt{1 - v^2/c^2}} = \frac{m_0 \times 0.8c}{0.6}$$

$$= \frac{4}{3} \, m_0 c.$$

This should be the total linear momentum after the collision. If the rest mass of the combined body is $M_0$ and it moves at speed $v'$,

$$\frac{M_0 \, v'}{\sqrt{1 - v'^2/c^2}} = \frac{4}{3} \, m_0 c. \qquad \text{.... (i)}$$

The energy before the collison is

$$\frac{m_0}{\sqrt{1 - v^2/c^2}} c^2 + m_0 c^2 = m_0 c^2 \left( \frac{1}{0.6} + 1 \right)$$

$$= \frac{8}{3} \, m_0 c^2.$$

The energy after the collision is

$$\frac{M_0 c^2}{\sqrt{1 - v'^2/c^2}}$$

Thus,          $\dfrac{M_0 c^2}{\sqrt{1 - v'^2/c^2}} = \dfrac{8}{3} \, m_0 c^2.$   .... (ii)

Dividing (i) by (ii),

$$\frac{v'}{c^2} = \frac{1}{2c} \quad \text{or,} \quad v' = \frac{c}{2}.$$

Putting this value of $v'$ in (ii),

$$M_0 = \frac{8}{3} \, m_0 \sqrt{1 - \frac{1}{4}}$$

or,                $M_0 = 2.309 \, m_0.$

The rest mass of the combined body is greater than the sum of the rest masses of the individual bodies.

□

## QUESTIONS FOR SHORT ANSWER

**1.** The speed of light in glass is $2.0 \times 10^{8}$ m/s. Does it violate the second postulate of special relativity ?

**2.** A uniformly moving train passes by a long platform. Consider the events 'engine crossing the beginning of the platform' and 'engine crossing the end of the platform'. Which frame (train frame or the platform frame) is the proper frame for the pair of events ?

3. An object may be regarded to be at rest or in motion depending on the frame of reference chosen to view the object. Because of length contraction it would mean that the same rod may have two different lengths depending on the state of the observer. Is this true ?

4. Mass of a particle depends on its speed. Does the attraction of the earth on the particle also depend on the particle's speed ?

5. A person travelling in a fast spaceship measures the distance between the earth and the moon. Is it the same, smaller or larger than the value quoted in this book ?

## OBJECTIVE I

1. The magnitude of linear momentum of a particle moving at a relativistic speed $v$ is proportional to
   (a) $v$
   (b) $1 - v^2/c^2$
   (c) $\sqrt{1 - v^2/c^2}$
   (d) none of these.

2. As the speed of a particle increases, its rest mass
   (a) increases
   (b) decreases
   (c) remains the same
   (d) changes.

3. An experimenter measures the length of a rod. Initially the experimenter and the rod are at rest with respect to the lab. Consider the following statements.
   (A) If the rod starts moving parallel to its length but the observer stays at rest, the measured length will be reduced.
   (B) If the rod stays at rest but the observer starts moving parallel to the measured length of the rod, the length will be reduced.
   (a) A is true but B is false.
   (b) B is true but A is false.
   (c) Both A and B are true.
   (d) Both A and B are false.

4. An experimenter measures the length of a rod. In the cases listed, all motions are with respect to the lab and parallel to the length of the rod. In which of the cases the measured length will be minimum ?
   (a) The rod and the experimenter move with the same speed $v$ in the same direction.
   (b) The rod and the experimenter move with the same speed $v$ in opposite directions.
   (c) The rod moves at speed $v$ but the experimenter stays at rest.
   (d) The rod stays at rest but the experimenter moves with the speed $v$.

5. If the speed of a particle moving at a relativistic speed is doubled, its linear momentum will
   (a) become double
   (b) become more than double
   (c) remain equal
   (d) become less than double.

6. If a constant force acts on a particle, its acceleration will
   (a) remain constant
   (b) gradually decrease
   (c) gradually increase
   (d) be undefined.

7. A charged particle is projected at a very high speed perpendicular to a uniform magnetic field. The particle will
   (a) move along a circle
   (b) move along a curve with increasing radius of curvature
   (c) move along a curve with decreasing radius of curvature
   (d) move along a straight line.

## OBJECTIVE II

1. Mark the correct statements:
   (a) Equations of special relativity are not applicable for small speeds.
   (b) Equations of special relativity are applicable for all speeds.
   (c) Nonrelativistic equations give exact result for small speeds.
   (d) Nonrelativistic equations never give exact result.

2. If the speed of a rod moving at a relativistic speed parallel to its length is doubled,
   (a) the length will become half of the original value
   (b) the mass will become double of the original value
   (c) the length will decrease
   (d) the mass will increase.

3. Two events take place simultaneously at points $A$ and $B$ as seen in the lab frame. They also occur simultaneously in a frame moving with respect to the lab in a direction
   (a) parallel to $AB$
   (b) perpendicular to $AB$
   (c) making an angle of 45° with $AB$
   (d) making an angle of 135° with $AB$.

4. Which of the following quantities related to an electron has a finite upper limit ?
   (a) mass
   (b) momentum
   (c) speed
   (d) kinetic energy.

5. A rod of rest length $L$ moves at a relativistic speed. Let $L' = L/\gamma$. Its length
   (a) must be equal to $L'$
   (b) may be equal to $L$
   (c) may be more than $L'$ but less than $L$
   (d) may be more than $L$.

6. When a rod moves at a relativistic speed $v$, its mass
   (a) must incrase by a factor of $\gamma$
   (b) may remain unchanged
   (c) may increase by a factor other than $\gamma$
   (d) may decrease.

## EXERCISES

1. The *guru* of a *yogi* lives in a Himalyan cave, 1000 km away from the house of the yogi. The yogi claims that whenever he thinks about his guru, the guru immediately knows about it. Calculate the minimum possible time interval between the yogi thinking about the guru and the guru knowing about it.

2. A suitcase kept on a shop's rack is measured 50 cm × 25 cm × 10 cm by the shop's owner. A traveller takes this suitcase in a train moving with velocity 0·6c. If the suitcase is placed with its length along the train's velocity, find the dimensions measured by (a) the traveller and (b) a ground observer.

3. The length of a rod is exactly 1 m when measured at rest. What will be its length when it moves at a speed of (a) $3 \times 10^5$ m/s, (b) $3 \times 10^6$ m/s and (c) $3 \times 10^7$ m/s ?

4. A person standing on a platform finds that a train moving with velocity 0·6c takes one second to pass by him. Find (a) the length of the train as seen by the person and (b) the rest length of the train.

5. An aeroplane travels over a rectangular field 100 m × 50 m, parallel to its length. What should be the speed of the plane so that the field becomes square in the plane frame ?

6. The rest distance between Patna and Delhi is 1000 km. A nonstop train travels at 360 km/h. (a) What is the distance between Patna and Delhi in the train frame ? (b) How much time elapses in the train frame between Patna and Delhi ?

7. A person travels by a car at a speed of 180 km/h. It takes exactly 10 hours by his wristwatch to go from the station A to the station B. (a) What is the rest distance between the two stations ? (b) How much time is taken in the road frame by the car to go from the station A to the station B ?

8. A person travels on a spaceship moving at a speed of 5c/13. (a) Find the time interval calculated by him between the consecutive birthday celebrations of his friend on the earth. (b) Find the time interval calculated by the friend on the earth between the consecutive birthday celebrations of the traveller.

9. According to the station clocks, two babies are born at the same instant, one in Howrah and other in Delhi. (a) Who is elder in the frame of 2301 Up Rajdhani Express going from Howrah to Delhi ? (b) Who is elder in the frame of 2302 Dn Rajdhani Express going from Delhi to Howrah.

10. Two babies are born in a moving train, one in the compartment adjacent to the engine and other in the compartment adjacent to the guard. According to the train frame, the babies are born at the same instant of time. Who is elder according to the ground frame ?

11. Suppose Swarglok (heaven) is in constant motion at a speed of 0·9999c with respect to the earth. According to the earth's frame, how much time passes on the earth before one day passes on Swarglok ?

12. If a person lives on the average 100 years in his rest frame, how long does he live in the earth frame if he spends all his life on a spaceship going at 60% of the speed of light.

13. An electric bulb, connected to a make and break power supply, switches off and on every second in its rest frame. What is the frequency of its switching off and on as seen from a spaceship travelling at a speed 0·8c ?

14. A person travelling by a car moving at 100 km/h finds that his wristwatch agrees with the clock on a tower A. By what amount will his wristwatch lag or lead the clock on another tower B, 1000 km (in the earth's frame) from the tower A when the car reaches there ?

15. At what speed the volume of an object shrinks to half its rest value ?

16. A particular particle created in a nuclear reactor leaves a 1 cm track before decaying. Assuming that the particle moved at 0·995c, calculate the life of the particle (a) in the lab frame and (b) in the frame of the particle.

17. By what fraction does the mass of a spring change when it is compressed by 1 cm ? The mass of the spring is 200 g at its natural length and the spring constant is 500 N/m.

18. Find the increase in mass when 1 kg of water is heated from 0°C to 100°C. Specific heat capacity of water = 4200 J/kg–K.

19. Find the loss in the mass of 1 mole of an ideal monatomic gas kept in a rigid container as it cools down by 10°C. The gas constant $R = 8·3$ J/mol–K.

20. By what fraction does the mass of a boy increase when he starts running at a speed of 12 km/h ?

21. A 100 W bulb together with its power supply is suspended from a sensitive balance. Find the change in the mass recorded after the bulb remains on for 1 year.

22. The energy from the sun reaches just outside the earth's atmosphere at a rate of 1400 W/m$^2$. The distance between the sun and the earth is $1·5 \times 10^{11}$ m. (a) Calculate the rate at which the sun is losing its mass. (b) How long will the sun last assuming a constant decay at this rate ? The present mass of the sun is $2 \times 10^{30}$ kg.

23. An electron and a positron moving at small speeds collide and annihilate each other. Find the energy of the resulting gamma photon.

24. Find the mass, the kinetic energy and the momentum of an electron moving at 0·8c.

25. Through what potential difference should an electron be accelerated to give it a speed of (a) 0·6c, (b) 0·9c and (c) 0·99c ?

26. Find the speed of an electron with kinetic energy (a) 1 eV, (b) 10 keV and (c) 10 MeV.

27. What is the kinetic energy of an electron in electronvolts with mass equal to double its rest mass ?

28. Find the speed at which the kinetic energy of a particle will differ by 1% from its nonrelativistic value $\frac{1}{2} m_0 v^2$.

# ANSWERS

## OBJECTIVE I

1. (d)    2. (c)    3. (c)    4. (b)    5. (b)    6. (b)
7. (b)

## OBJECTIVE II

1. (b), (d)       2. (c), (d)       3. (b)
4. (c)            5. (b), (c)       6. (a)

## EXERCISES

1. 1/300 s
2. (a) 50 cm × 25 cm × 10 cm (b) 40 cm × 25 cm × 10 cm
3. (a) 0·9999995 m (b) 0·99995 m (c) 0·995 m
4. (a) $1·8 × 10^8$ m (b) $2·25 × 10^8$ m
5. 0·866c
6. (a) 56 nm less than 1000 km
   (b) 0·56 ns less than $\dfrac{500}{3}$ min
7. (a) 25 nm more than 1800 km
   (b) 0·5 ns more than 10 hours
8. $\dfrac{13}{12}$ y in both cases
9. (a) Delhi baby is elder (b) Howrah baby is elder

10. the baby adjacent to the guard is elder
11. 70·7 days
12. 125 y
13. $0·6\ \text{s}^{-1}$
14. will lag by 0·154 ns
15. $\dfrac{\sqrt{3}c}{2}$
16. (a) 33·5 ps (b) 3·35 ps
17. $1·4 × 10^{-18}$
18. $4·7 × 10^{-12}$ kg
19. $1·38 × 10^{-15}$ kg
20. $6·17 × 10^{-17}$
21. $3·5 × 10^{-8}$ kg
22. (a) $4·4 × 10^9$ kg/s (b) $1·44 × 10^{13}$ y
23. 1·02 MeV
24. $15·2 × 10^{-31}$ kg, $5·5 × 10^{-14}$ J, $3·65 × 10^{-22}$ kg-m/s
25. (a) 128 kV (b) 661 kV (c) 3·1 MV
26. (a) $5·92 × 10^5$ m/s (b) $5·85 × 10^7$ m/s (c) $2·996 × 10^8$ m/s
27. 511 keV
28. $3·46 × 10^7$ m/s

**6.** *Test dimensionally if the equation $v^2 = u^2 + 2ax$ may be correct.*

**Solution :** There are three terms in this equation $v^2$, $u^2$ and $2ax$. The equation may be correct if the dimensions of these three terms are equal.

$$[v^2] = \left(\frac{L}{T}\right)^2 = L^2 T^{-2}$$

$$[u^2] = \left(\frac{L}{T}\right)^2 = L^2 T^{-2}$$

$$[2ax] = [a][x] = \left(\frac{L}{T^2}\right)L = L^2 T^{-2}$$

Thus, the equation may be correct.

**7.** *The distance covered by a particle in time $t$ is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of $a$, $b$, $c$ and $d$.*

**Solution :** The equation contains five terms. All of them should have the same dimensions. Since $[x]$ = *length*, each of the remaining four must have the dimension of length.

Thus,   $[a]$ = *length* = L

$[bt] = L$,              or, $[b] = LT^{-1}$.

$[ct^2] = L$,            or, $[c] = LT^{-2}$.

and   $[dt^3] = L$,       or, $[d] = LT^{-3}$.

**8.** *If the centripetal force is of the form $m^a v^b r^c$, find the values of $a$, $b$ and $c$.*

**Solution :** Dimensionally,

$$Force = (Mass)^a \times (velocity)^b \times (length)^c$$

or,   $MLT^{-2} = M^a (L^b T^{-b}) L^c = M^a L^{b+c} T^{-b}$

Equating the exponents of similar quantities,

$a = 1$, $b + c = 1$, $-b = -2$

or,   $a = 1$, $b = 2$, $c = -1$    or, $F = \dfrac{mv^2}{r}$.

**9.** *When a solid sphere moves through a liquid, the liquid opposes the motion with a force $F$. The magnitude of $F$ depends on the coefficient of viscosity $\eta$ of the liquid, the radius $r$ of the sphere and the speed $v$ of the sphere.*

*Assuming that $F$ is proportional to different powers of these quantities, guess a formula for $F$ using the method of dimensions.*

**Solution :** Suppose the formula is $F = k \eta^a r^b v^c$.

Then,   $MLT^{-2} = [ML^{-1}T^{-1}]^a L^b \left(\dfrac{L}{T}\right)^c$

$$= M^a L^{-a+b+c} T^{-a-c}$$

Equating the exponents of M, L and T from both sides,

$$a = 1$$
$$-a + b + c = 1$$
$$-a - c = -2.$$

Solving these, $a = 1$, $b = 1$, and $c = 1$.
Thus, the formula for $F$ is $F = k\eta rv$.

**10.** *The heat produced in a wire carrying an electric current depends on the current, the resistance and the time. Assuming that the dependence is of the product of powers type, guess an equation between these quantities using dimensional analysis. The dimensional formula of resistance is $ML^2 I^{-2} T^{-3}$ and heat is a form of energy.*

**Solution :** Let the heat produced be $H$, the current through the wire be $I$, the resistance be $R$ and the time be $t$. Since heat is a form of energy, its dimensional formula is $ML^2 T^{-2}$.

Let us assume that the required equation is

$$H = kI^a R^b t^c,$$

where $k$ is a dimensionless constant.
Writing dimensions of both sides,

$$ML^2 T^{-2} = I^a (ML^2 I^{-2} T^{-3})^b T^c$$
$$= M^b L^{2b} T^{-3b+c} I^{a-2b}$$

Equating the exponents,

$$b = 1$$
$$2b = 2$$
$$-3b + c = -2$$
$$a - 2b = 0.$$

Solving these, we get $a = 2$, $b = 1$ and $c = 1$.

Thus, the required equation is $H = kI^2 Rt$.

□

## QUESTIONS FOR SHORT ANSWER

**1.** The metre is defined as the distance travelled by light in $\dfrac{1}{299,792,458}$ second. Why didn't people choose some easier number such as $\dfrac{1}{300,000,000}$ second ? Why not 1 second ?

**2.** What are the dimensions of :
   (a) volume of a cube of edge $a$,
   (b) volume of a sphere of radius $a$,
   (c) the ratio of the volume of a cube of edge $a$ to the volume of a sphere of radius $a$ ?

3. Suppose you are told that the linear size of everything in the universe has been doubled overnight. Can you test this statement by measuring sizes with a metre stick ? Can you test it by using the fact that the speed of light is a universal constant and has not changed ? What will happen if all the clocks in the universe also start running at half the speed ?

4. If all the terms in an equation have same units, is it necessary that they have same dimensions ? If all the terms in an equation have same dimensions, is it necessary that they have same units ?

5. If two quantities have same dimensions, do they represent same physical content ?

6. It is desirable that the standards of units be easily available, invariable, indestructible and easily reproducible. If we use foot of a person as a standard unit of length, which of the above features are present and which are not ?

7. Suggest a way to measure :
   (a) the thickness of a sheet of paper,
   (b) the distance between the sun and the moon.

## OBJECTIVE I

1. Which of the following sets cannot enter into the list of fundamental quantities in any system of units ?
   - (a) length, mass and velocity,
   - (b) length, time and velocity,
   - (c) mass, time and velocity,
   - (d) length, time and mass.

2. A physical quantity is measured and the result is expressed as $nu$ where $u$ is the unit used and $n$ is the numerical value. If the result is expressed in various units then
   (a) $n \propto$ size of $u$    (b) $n \propto u^2$
   (c) $n \propto \sqrt{u}$    (d) $n \propto \dfrac{1}{u}$.

3. Suppose a quantity $x$ can be dimensionally represented in terms of M, L and T, that is, $[x] = M^a L^b T^c$. The quantity mass
   (a) can always be dimensionally represented in terms of L, T and $x$,
   (b) can never be dimensoinally represented in terms of L, T and $x$,
   (c) may be represented in terms of L, T and $x$ if $a = 0$,
   (d) may be represented in terms of L, T and $x$ if $a \neq 0$.

4. A dimensionless quantity
   (a) never has a unit,    (b) always has a unit,
   (c) may have a unit,    (d) does not exist.

5. A unitless quantity
   (a) never has a nonzero dimension,
   (b) always has a nonzero dimension,
   (c) may have a nonzero dimension,
   (d) does not exist.

6. $\displaystyle\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right]$.
   The value of $n$ is
   (a) 0    (b) −1
   (c) 1    (d) none of these.
   You may use dimensional analysis to solve the problem.

## OBJECTIVE II

1. The dimensions $ML^{-1}T^{-2}$ may correspond to
   (a) work done by a force
   (b) linear momentum
   (c) pressure
   (d) energy per unit volume.

2. Choose the correct statement(s) :
   (a) A dimensionally correct equation may be correct.
   (b) A dimensionally correct equation may be incorrect.
   (c) A dimensionally incorrect equation may be correct.
   (d) A dimensionally incorrect equation may be incorrect.

3. Choose the correct statement(s) :
   (a) All quantities may be represented dimensionally in terms of the base quantities.
   (b) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.
   (c) The dimension of a base quantity in other base quantities is always zero.
   (d) The dimension of a derived quantity is never zero in any base quantity.

## EXERCISES

1. Find the dimensions of
   (a) linear momentum, (b) frequency and (c) pressure.

2. Find the dimensions of
   (a) angular speed $\omega$,    (b) angular acceleration $\alpha$,
   (c) torque $\Gamma$ and    (d) moment of interia $I$.
   Some of the equations involving these quantities are

$\omega = \dfrac{\theta_2 - \theta_1}{t_2 - t_1}$, $\alpha = \dfrac{\omega_2 - \omega_1}{t_2 - t_1}$, $\Gamma = F.r$ and $I = mr^2$.

The symbols have standard meanings.

3. Find the dimensions of
   (a) electric field $E$,           (b) magnetic field $B$   and
   (c) magnetic permeability $\mu_0$.
   The relevant equations are

   $F = qE$, $F = qvB$, and $B = \dfrac{\mu_0 I}{2\pi a}$

   where $F$ is force, $q$ is charge, $v$ is speed, $I$ is current,
   and $a$ is distance.

4. Find the dimensions of
   (a) electric dipole moment $p$ and
   (b) magnetic dipole moment $M$.
   The defining equations are $p = q.d$ and $M = IA$
   where $d$ is distance, $A$ is area, $q$ is charge and $I$ is
   current.

5. Find the dimensions of Planck's constant $h$ from the
   equation $E = h\nu$ where $E$ is the energy and $\nu$ is the
   frequency.

6. Find the dimensions of
   (a) the specific heat capacity $c$,
   (b) the coefficient of linear expansion $\alpha$ and
   (c) the gas constant $R$.
   Some of the equations involving these quantities are
   $Q = mc(T_2 - T_1)$, $l_t = l_0[1 + \alpha(T_2 - T_1)]$ and $PV = nRT$.

7. Taking force, length and time to be the fundamental
   quantities find the dimensions of
   (a) density,            (b) pressure,
   (c) momentum and      (d) energy.

8. Suppose the acceleration due to gravity at a place is
   10 m/s$^2$. Find its value in cm/(minute)$^2$.

9. The average speed of a snail is 0·020 miles/hour and
   that of a leopard is 70 miles/hour. Convert these speeds
   in SI units.

10. The height of mercury column in a barometer in a
    Calcutta laboratory was recorded to be 75 cm. Calculate
    this pressure in SI and CGS units using the following
    data : Specific gravity of mercury = 13·6, Density of
    water = 10$^3$ kg/m$^3$, $g$ = 9·8 m/s$^2$ at Calcutta. Pressure
    = $h\rho g$ in usual symbols.

11. Express the power of a 100 watt bulb in CGS unit.

12. The normal duration of I.Sc. Physics practical period in
    Indian colleges is 100 minutes. Express this period in
    microcenturies. 1 microcentury = $10^{-6} \times 100$ years. How
    many microcenturies did you sleep yesterday ?

13. The surface tension of water is 72 dyne/cm. Convert it
    in SI unit.

14. The kinetic energy $K$ of a rotating body depends on its
    moment of inertia $I$ and its angular speed $\omega$. Assuming
    the relation to be $K = kI^a\omega^b$ where $k$ is a dimensionless
    constant, find $a$ and $b$. Moment of inertia of a sphere
    about its diameter is $\dfrac{2}{5}Mr^2$.

15. Theory of relativity reveals that mass can be converted
    into energy. The energy $E$ so obtained is proportional to
    certain powers of mass $m$ and the speed $c$ of light. Guess
    a relation among the quantities using the method of
    dimensions.

16. Let $I$ = current through a conductor, $R$ = its resistance
    and $V$ = potential difference across its ends. According
    to Ohm's law, product of two of these quantities equals
    the third. Obtain Ohm's law from dimensional analysis.
    Dimensional formulae for $R$ and $V$ are $ML^2I^{-2}T^{-3}$ and
    $ML^2T^{-3}I^{-1}$ respectively.

17. The frequency of vibration of a string depends on the
    length $L$ between the nodes, the tension $F$ in the string
    and its mass per unit length $m$. Guess the expression
    for its frequency from dimensional analysis.

18. Test if the following equations are dimensionally
    correct :

    (a) $h = \dfrac{2S\cos\theta}{\rho rg}$,            (b) $v = \sqrt{\dfrac{P}{\rho}}$,

    (c) $V = \dfrac{\pi P r^4 t}{8 \eta l}$,            (d) $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{mgl}{I}}$;

    where $h$ = height, $S$ = surface tension, $\rho$ = density, $P$ =
    pressure, $V$ = volume, $\eta$ = coefficient of viscosity, $\nu$ =
    frequency and $I$ = moment of inertia.

19. Let $x$ and $a$ stand for distance. Is $\displaystyle\int \dfrac{dx}{\sqrt{a^2 - x^2}}$
    $= \dfrac{1}{a}\sin^{-1}\dfrac{a}{x}$ dimensionally correct ?

□

## ANSWERS

### OBJECTIVE I

1. (b)    2. (d)    3. (d)    4. (c)    5. (a)    6. (a)

### OBJECTIVE II

1. (c), (d)         2. (a), (b), (d)      3. (a), (b), (c)

### EXERCISES

1. (a) $MLT^{-1}$    (b) $T^{-1}$           (c) $ML^{-1}T^{-2}$
2. (a) $T^{-1}$      (b) $T^{-2}$  (c) $ML^2T^{-2}$       (d) $ML^2$
3. (a) $MLT^{-3}I^{-1}$  (b) $MT^{-2}I^{-1}$     (c) $MLT^{-2}I^{-2}$
4. (a) $LTI$         (b) $L^2I$
5. $ML^2T^{-1}$
6. (a) $L^2T^{-2}K^{-1}$  (b) $K^{-1}$  (c) $ML^2T^{-2}K^{-1}(mol)^{-1}$

7. (a) $FL^{-4}T^2$    (b) $FL^{-2}$   (c) $FT$   (d) $FL$

8. $36 \times 10^{6}$ cm/(minute)$^2$

9. 0·0089 m/s, 31 m/s

10. $10 \times 10^4$ N/m$^2$,   $10 \times 10^5$ dyne/cm$^2$

11. $10^9$ erg/s

12. 1·9 microcenturies

13. 0·072 N/m

14. $a = 1,\ b = 2$

15. $E = kmc^2$

16. $V = IR$

17. $\dfrac{k}{L}\sqrt{\dfrac{F}{m}}$

18. all are dimensionally correct

19. no

*Solution :* The area can be divided into strips by drawing ordinates between $x = 0$ and $x = 6$ at a regular interval of $dx$. Consider the strip between the ordinates at $x$ and $x + dx$. The height of this strip is $y = x^2$. The area of this strip is $dA = y \, dx = x^2 \, dx$.

The total area of the shaded part is obtained by summing up these strip-areas with $x$ varying from 0 to 6. Thus,

$$A = \int_0^6 x^2 \, dx$$

$$= \left[ \frac{x^3}{3} \right]_0^6 = \frac{216 - 0}{3} = 72.$$

15. *Evaluate $\int_0^t A \sin \omega t \, dt$ where $A$ and $\omega$ are constants.*

*Solution :*  $\int_0^t A \sin \omega t \, dt$

$$= A \left[ \frac{-\cos \omega t}{\omega} \right]_0^t = \frac{A}{\omega} (1 - \cos \omega t).$$

16. *The velocity $v$ and displacement $x$ of a particle executing simple harmonic motion are related as*

$$v \frac{dv}{dx} = -\omega^2 x.$$

*At $x = 0$, $v = v_0$. Find the velocity $v$ when the displacement becomes $x$.*

*Solution :* We have

$$v \frac{dv}{dx} = -\omega^2 x$$

or,                $$v \, dv = -\omega^2 x \, dx$$

or,          $$\int_{v_0}^{v} v \, dv = \int_0^x -\omega^2 x \, dx \qquad \ldots \text{(i)}$$

When summation is made on $-\omega^2 x \, dx$ the quantity to be varied is $x$. When summation is made on $v \, dv$ the quantity to be varied is $v$. As $x$ varies from 0 to $x$ the velocity varies from $v_0$ to $v$. Therefore, on the left the

limits of integration are from $v_0$ to $v$ and on the right they are from 0 to $x$. Simplifying (i),

$$\left[ \frac{1}{2} v^2 \right]_{v_0}^{v} = -\omega^2 \left[ \frac{x^2}{2} \right]_0^x$$

or,          $$\frac{1}{2} (v^2 - v_0^2) = -\omega^2 \frac{x^2}{2}$$

or,                $$v^2 = v_0^2 - \omega^2 x^2$$

or,                $$v = \sqrt{v_0^2 - \omega^2 x^2} \, .$$

17. *The charge flown through a circuit in the time interval between $t$ and $t + dt$ is given by $dq = e^{-t/\tau} \, dt$ where $\tau$ is a constant. Find the total charge flown through the circuit between $t - 0$ to $t = \tau$.*

*Solution :* The total charge flown is the sum of all the $dq$'s for $t$ varying from $t = 0$ to $t = t_0$. Thus, the total charge flown is

$$Q = \int_0^\tau e^{-t/\tau} \, dt$$

$$= \left[ \frac{e^{-t/\tau}}{-1/\tau} \right]_0^\tau = \tau \left( 1 - \frac{1}{e} \right).$$

18. *Evaluate $(21 \cdot 6002 + 234 + 2732 \cdot 10) \times 13$.*

*Solution :*

| 21·6002 | | 22 |
| 234 | $\Rightarrow$ | 234 |
| 2732·10 | | 2732 |
| | | 2988 |

The three numbers are arranged with their decimal points aligned (shown on the left part above). The column just left to the decimals has 4 as the doubtful digit. Thus, all the numbers are rounded to this column. The rounded numbers are shown on the right part above. The required expression is $2988 \times 13 = 38844$. As 13 has only two significant digits the product should be rounded off after two significant digits. Thus the result is 39000.

□

## QUESTIONS FOR SHORT ANSWER

1. Is a vector necessarily changed if it is rotated through an angle ?

2. Is it possible to add two vectors of unequal magnitudes and get zero ? Is it possible to add three vectors of equal magnitudes and get zero ?

3. Does the phrase "direction of zero vector" have physical significance ? Discuss in terms of velocity, force etc.

4. Can you add three unit vectors to get a unit vector ? Does your answer change if two unit vectors are along the co-ordinate axes ?

5. Can we have physical quantities having magnitude and direction which are not vectors ?

6. Which of the following two statements is more appropriate ?

   (a) Two forces are added using triangle rule because force is a vector quantity.

   (b) Force is a vector quantity because two forces are added using triangle rule.

7. Can you add two vectors representing physical quantities having different dimensions ? Can you multiply two vectors representing physical quantities having different dimensions ?

8. Can a vector have zero component along a line and still have nonzero magnitude ?

9. Let $\varepsilon_1$ and $\varepsilon_2$ be the angles made by $\vec{A}$ and $-\vec{A}$ with the positive $X$-axis. Show that $\tan \varepsilon_1 = \tan \varepsilon_2$. Thus, giving $\tan \varepsilon$ does not uniquely determine the direction of $\vec{A}$.

10. Is the vector sum of the unit vectors $\vec{i}$ and $\vec{j}$ a unit vector ? If no, can you multiply this sum by a scalar number to get a unit vector ?

11. Let $\vec{A} = 3\,\vec{i} + 4\,\vec{j}$. Write four vectors $\vec{B}$ such that $\vec{A} \neq \vec{B}$ but $A = B$.

12. Can you have $\vec{A} \times \vec{B} = \vec{A} \cdot \vec{B}$ with $A \neq 0$ and $B \neq 0$ ? What if one of the two vectors is zero ?

13. If $\vec{A} \times \vec{B} = 0$, can you say that (a) $\vec{A} = \vec{B}$, (b) $\vec{A} \neq \vec{B}$ ?

14. Let $\vec{A} = 5\,\vec{i} - 4\,\vec{j}$ and $\vec{B} = -7.5\,\vec{i} + 6\,\vec{j}$. Do we have $\vec{B} = k\,\vec{A}$ ? Can we say $\dfrac{\vec{B}}{\vec{A}} = k$ ?

## OBJECTIVE I

1. A vector is not changed if
   (a) it is rotated through an arbitrary angle
   (b) it is multiplied by an arbitrary scalar
   (c) it is cross multiplied by a unit vector
   (d) it is slid parallel to itself.

2. Which of the sets given below may represent the magnitudes of three vectors adding to zero ?
   (a) 2, 4, 8    (b) 4, 8, 16    (c) 1, 2, 1    (d) 0·5, 1, 2.

3. The resultant of $\vec{A}$ and $\vec{B}$ makes an angle $\alpha$ with $\vec{A}$ and $\beta$ with $\vec{B}$,
   (a) $\alpha < \beta$                      (b) $\alpha < \beta$ if $A < B$
   (c) $\alpha < \beta$ if $A > B$          (d) $\alpha < \beta$ if $A = B$.

4. The component of a vector is
   (a) always less than its magnitude
   (b) always greater than its magnitude
   (c) always equal to its magnitude
   (d) none of these.

5. A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is
   (a) along west            (b) along east
   (c) zero                  (d) vertically downward.

6. The radius of a circle is stated as 2·12 cm. Its area should be written as
   (a) 14 cm$^2$  (b) 14·1 cm$^2$  (c) 14·11 cm$^2$  (d) 14·1124 cm$^2$

## OBJECTIVE II

1. A situation may be described by using different sets of co-ordinate axes having different orientations. Which of the following do not depend on the orientation of the axes ?
   (a) the value of a scalar    (b) component of a vector
   (c) a vector                 (d) the magnitude of a vector.

2. Let $\vec{C} = \vec{A} + \vec{B}$.
   (a) $|\vec{C}|$ is always greater than $|\vec{A}|$
   (b) It is possible to have $|\vec{C}| < |\vec{A}|$ and $|\vec{C}| < |\vec{B}|$
   (c) $C$ is always equal to $A + B$
   (d) $C$ is never equal to $A + B$.

3. Let the angle between two nonzero vectors $\vec{A}$ and $\vec{B}$ be 120° and its resultant be $\vec{C}$.
   (a) $C$ must be equal to $|A - B|$

   (b) $C$ must be less than $|A - B|$
   (c) $C$ must be greater than $|A - B|$
   (d) $C$ may be equal to $|A - B|$.

4. The $x$-component of the resultant of several vectors
   (a) is equal to the sum of the $x$-components of the vectors
   (b) may be smaller than the sum of the magnitudes of the vectors
   (c) may be greater than the sum of the magnitudes of the vectors
   (d) may be equal to the sum of the magnitudes of the vectors.

5. The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ may be
   (a) greater than $AB$        (b) equal to $AB$
   (c) less than $AB$           (d) equal to zero.

# EXERCISES

1. A vector $\vec{A}$ makes an angle of 20° and $\vec{B}$ makes an angle of 110° with the X-axis. The magnitudes of these vectors are 3 m and 4 m respectively. Find the resultant.

2. Let $\vec{A}$ and $\vec{B}$ be the two vectors of magnitude 10 unit each. If they are inclined to the X-axis at angles 30° and 60° respectively, find the resultant.

3. Add vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ each having magnitude of 100 unit and inclined to the X-axis at angles 45°, 135° and 315° respectively.

4. Let $\vec{a} = 4\vec{i} + 3\vec{j}$ and $\vec{b} = 3\vec{i} + 4\vec{j}$. (a) Find the magnitudes of (a) $\vec{a}$, (b) $\vec{b}$, (c) $\vec{a} + \vec{b}$ and (d) $\vec{a} - \vec{b}$.

5. Refer to figure (2-E1). Find (a) the magnitude, (b) x and y components and (c) the angle with the X-axis of the resultant of $\overrightarrow{OA}$, $\overrightarrow{BC}$ and $\overrightarrow{DE}$.



Figure 2-E1

6. Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is (a) 1 unit, (b) 5 unit and (c) 7 unit.

7. A spy report about a suspected car reads as follows. "The car moved 2·00 km towards east, made a perpendicular left turn, ran for 500 m, made a perpendicular right turn, ran for 4·00 km and stopped". Find the displacement of the car.

8. A carrom board (4 ft × 4 ft square) has the queen at the centre. The queen, hit by the striker moves to the front edge, rebounds and goes in the hole behind the striking line. Find the magnitude of displacement of the queen (a) from the centre to the front edge, (b) from the front edge to the hole and (c) from the centre to the hole.

9. A mosquito net over a 7 ft × 4 ft bed is 3 ft high. The net has a hole at one corner of the bed through which a mosquito enters the net. It flies and sits at the diagonally opposite upper corner of the net. (a) Find the magnitude of the displacement of the mosquito. (b) Taking the hole as the origin, the length of the bed as the X-axis, its width as the Y-axis, and vertically up as the Z-axis, write the components of the displacement vector.

10. Suppose $\vec{a}$ is a vector of magnitude 4·5 unit due north. What is the vector (a) $3\vec{a}$, (b) $-4\vec{a}$?

11. Two vectors have magnitudes 2 m and 3 m. The angle between them is 60°. Find (a) the scalar product of the two vectors, (b) the magnitude of their vector product.
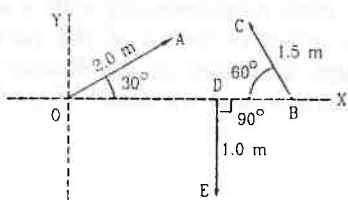
12. Let $A_1 A_2 A_3 A_4 A_5 A_6 A_1$ be a regular hexagon. Write the x-components of the vectors represented by the six sides taken in order. Use the fact that the resultant of these six vectors is zero, to prove that

$\cos 0 + \cos \pi/3 + \cos 2\pi/3 + \cos 3\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0$.

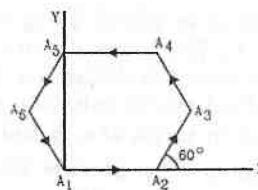Use the known cosine values to verify the result.



Figure 2-E2

13. Let $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$. Find the angle between them.

14. Prove that $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$.

15. If $\vec{A} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{B} = 4\vec{i} + 3\vec{j} + 2\vec{k}$, find $\vec{A} \times \vec{B}$.

16. If $\vec{A}$, $\vec{B}$, $\vec{C}$ are mutually perpendicular, show that $\vec{C} \times (\vec{A} \times \vec{B}) = 0$. Is the converse true?

17. A particle moves on a given straight line with a constant speed $v$. At a certain time it is at a point P on its straight line path. O is a fixed point. Show that $\overrightarrow{OP} \times \vec{v}$ is independent of the position P.

18. The force on a charged particle due to electric and magnetic fields is given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. Suppose $\vec{E}$ is along the X-axis and $\vec{B}$ along the Y-axis. In what direction and with what minimum speed $v$ should a positively charged particle be sent so that the net force on it is zero?

19. Give an example for which $\vec{A} \cdot \vec{B} = \vec{C} \cdot \vec{B}$ but $\vec{A} \neq \vec{C}$.

20. Draw a graph from the following data. Draw tangents at x = 2, 4, 6 and 8. Find the slopes of these tangents. Verify that the curve drawn is $y = 2x^2$ and the slope of tangent is $\tan\theta = \dfrac{dy}{dx} = 4x$.

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|----|
| y | 2 | 8 | 18 | 32 | 50 | 72 | 98 | 128 | 162 | 200 |

21. A curve is represented by $y = \sin x$. If x is changed from $\dfrac{\pi}{3}$ to $\dfrac{\pi}{3} + \dfrac{\pi}{100}$, find approximately the change in y.

22. The electric current in a charging R–C circuit is given by $i = i_0 e^{-t/RC}$ where $i_0$, R and C are constant parameters of the circuit and t is time. Find the rate of change of current at (a) t = 0, (b) t = RC, (c) t = 10 RC.

23. The electric current in a discharging R–C circuit is given by $i = i_0 e^{-t/RC}$ where $i_0$, R and C are constant parameters and t is time. Let $i_0 = 2·00$ A, R = 6·00 ×

$10^5$ Ω and $C = 0.500$ µF. (a) Find the current at $t = 0.3$ s. (b) Find the rate of change of current at $t = 0.3$ s. (c) Find approximately the current at $t = 0.31$ s.

24. Find the area bounded under the curve $y = 3x^2 + 6x + 7$ and the $X$-axis with the ordinates at $x = 5$ and $x = 10$.

25. Find the area enclosed by the curve $y = \sin x$ and the $X$-axis between $x = 0$ and $x = \pi$.

26. Find the area bounded by the curve $y = e^{-x}$, the $X$-axis and the $Y$-axis.

27. A rod of length $L$ is placed along the $X$-axis between $x = 0$ and $x = L$. The linear density (mass/length) $\rho$ of the rod varies with the distance $x$ from the origin as $\rho = a + bx$. (a) Find the SI units of $a$ and $b$. (b) Find the mass of the rod in terms of $a$, $b$ and $L$.

28. The momentum $p$ of a particle changes with time $t$ according to the relation $\frac{dp}{dt} = (10\ N) + (2\ N/s)t$. If the momentum is zero at $t = 0$, what will the momentum be at $t = 10$ s ?

29. The changes in a function $y$ and the independent variable $x$ are related as $\frac{dy}{dx} = x^2$. Find $y$ as a function of $x$.

30. Write the number of significant digits in (a) 100, (b) 100.1, (c) 100.10, (d) 0.001001.

31. A metre scale is graduated at every millimetre. How many significant digits will be there in a length measurement with this scale ?

32. Round the following numbers to 2 significant digits (a) 3472, (b) 84.16, (c) 2.55 and (d) 28.5.

33. The length and the radius of a cylinder measured with a slide callipers are found to be 4.54 cm and 1.75 cm respectively. Calculate the volume of the cylinder.

34. The thickness of a glass plate is measured to be 2.17 mm, 2.17 mm and 2.18 mm at three different places. Find the average thickness of the plate from the data.

35. The length of the string of a simple pendulum is measured with a metre scale to be 90.0 cm. The radius of the bob plus the length of the hook is calculated to be 2.13 cm using measurements with a slide callipers. What is the effective length of the pendulum ? (The effective length is defined as the distance between the point of suspension and the centre of the bob.)

## ANSWER

### OBJECTIVE I

1. (d)    2. (c)    3. (c)    4. (d)    5. (a)    6. (b)

### OBJECTIVE II

1. (a), (c), (d)    2. (b)    3. (c)    4. (a), (b), (d)
5. (b), (c), (d)

### EXERCISES

1. 5 m at 73° with $X$-axis
2. 20 cos15° unit at 45° with $X$-axis
3. 100 unit at 45° with $X$-axis
4. (a) 5        (b) 5        (c) 7√2        (d) √2
5. (a) 1.6 m    (b) 0.98 m and 1.3 m respectively
   (c) $\tan^{-1}(1.32)$
6. (a) 180°        (b) 90°        (c) 0
7. 6.02 km, $\tan^{-1}\dfrac{1}{12}$.
8. (a) $\dfrac{2}{3}\sqrt{10}$ ft    (b) $\dfrac{4}{3}\sqrt{10}$ ft    (c) 2√2 ft
9. (a) √74 ft        (b) 7 ft, 4 ft, 3 ft
10. (a) 13.5 unit due north    (b) 18 unit due south

11. (a) 3 m$^2$            (b) 3√3 m$^2$
13. $\cos^{-1}(38/\sqrt{1450})$
15. $-6\vec{i} + 12\vec{j} - 6\vec{k}$
16. no
18. along $Z$-axis with speed $E/B$
21. 0.0157
22. (a) $\dfrac{-i_0}{RC}$      (b) $\dfrac{-i_0}{RCe}$      (c) $\dfrac{-i_0}{RCe^{10}}$
23. (a) $\dfrac{2.00}{e}$ A    (b) $\dfrac{20}{3e}$ A/s    (c) $\dfrac{5.0}{3e}$ A
24. 1135
25. 2
26. 1
27. (a) kg/m, kg/m$^2$        (b) $aL + bL^2/2$
28. 200 kg–m/s
29. $y = \dfrac{x^3}{3} + C$
30. (a) 4        (b) 4        (c) 5        (d) 4
31. 1, 2, 3 or 4
32. (a) 3500    (b) 84    (c) 2.6    (d) 28
33. 43.7 cm$^3$
34. 2.17 mm
35. 92.1 cm

*the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particles meet each other?*

**Solution :** The motion of the particles is roughly sketched in figure (3-W15). By symmetry they will meet at the centroid O of the triangle. At any instant the particles will form an equilateral triangle ABC with the same
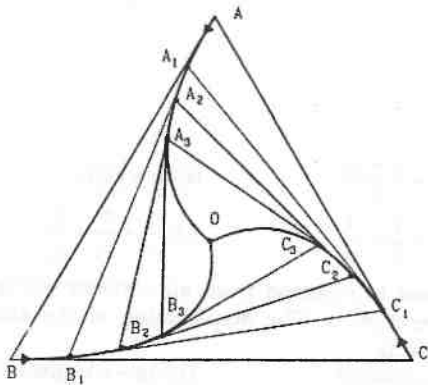


Figure 3-W15

centriod O. Concentrate on the motion of any one

particle, say A. At any instant its velocity makes angle 30° with AO.

The component of this velocity along AO is $v \cos 30°$. This component is the rate of decrease of the distance AO. Initially,

$$AO = \frac{2}{3}\sqrt{d^2 - \left(\frac{d}{2}\right)^2} = \frac{d}{\sqrt{3}}.$$

Therefore, the time taken for AO to become zero

$$= \frac{d/\sqrt{3}}{v \cos 30°} = \frac{2d}{\sqrt{3}\,v \times \sqrt{3}} = \frac{2d}{3v}.$$

**Alternative :** Velocity of A is v along AB. The velocity of B is along BC. Its component along BA is $v \cos 60°$ $= v/2$. Thus, the separation AB decreases at the rate

$$v + \frac{v}{2} = \frac{3v}{2}.$$

Since this rate is constant, the time taken in reducing the separation AB from d to zero is

$$t = \frac{d}{\frac{3v}{2}} = \frac{2d}{3v}.$$

□

# QUESTIONS FOR SHORT ANSWER

1. Galileo was punished by the Church for teaching that the sun is stationary and the earth moves around it. His opponents held the view that the earth is stationary and the sun moves around it. If the absolute motion has no meaning, are the two viewpoints not equally correct or equally wrong?

2. When a particle moves with constant velocity, its average velocity, its instantaneous velocity and its speed are all equal. Comment on this statement.

3. A car travels at a speed of 60 km/hr due north and the other at a speed of 60 km/hr due east. Are the velocities equal? If no, which one is greater? If you find any of the questions irrelevant, explain.

4. A ball is thrown vertically upward with a speed of 20 m/s. Draw a graph showing the velocity of the ball as a function of time as it goes up and then comes back.

5. The velocity of a particle is towards west at an instant. Its acceleration is not towards west, not towards east, not towards north and not towards south. Give an example of this type of motion.

6. At which point on its path a projectile has the smallest speed?

7. Two particles A and B start from rest and move for equal time on a straight line. The particle A has an acceleration a for the first half of the total time and 2a for the second half. The particle B has an acceleration

2a for the first half and a for the second half. Which particle has covered larger distance?

8. If a particle is accelerating, it is either speeding up or speeding down. Do you agree with this statement?

9. A food packet is dropped from a plane going at an altitude of 100 m. What is the path of the packet as seen from the plane? What is the path as seen from the ground? If someone asks "what is the actual path", what will you answer?

10. Give examples where (a) the velocity of a particle is zero but its acceleration is not zero, (b) the velocity is opposite in direction to the acceleration, (c) the velocity is perpendicular to the acceleration.

11. Figure (3-Q1) shows the x coordinate of a particle as a function of time. Find the signs of $v_x$ and $a_x$ at $t = t_1$, $t = t_2$ and $t = t_3$.
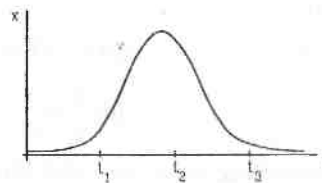


Figure 3-Q1

12. A player hits a baseball at some angle. The ball goes high up in space. The player runs and catches the ball before it hits the ground. Which of the two (the player or the ball) has greater displacement?

13. The increase in the speed of a car is proportional to the additional petrol put into the engine. Is it possible to accelerate a car without putting more petrol or less petrol into the engine?

14. Rain is falling vertically. A man running on the road keeps his umbrella tilted but a man standing on the street keeps his umbrella vertical to protect himself from the rain. But both of them keep their umbrella vertical to avoid the vertical sun-rays. Explain.

## OBJECTIVE I

1. A motor car is going due north at a speed of 50 km/h. It makes a 90° left turn without changing the speed. The change in the velocity of the car is about
   (a) 50 km/h towards west
   (b) 70 km/h towards south-west
   (c) 70 km/h towards north-west
   (d) zero.

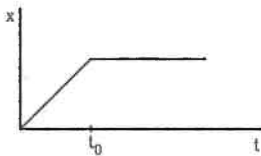2. Figure (3-Q2) shows the displacement-time graph of a particle moving on the X-axis.



Figure 3-Q2

(a) the particle is continuously going in positive $x$ direction
(b) the particle is at rest
(c) the velocity increases up to a time $t_0$, and then becomes constant
(d) the particle moves at a constant velocity up to a time $t_0$, and then stops.

3. A particle has a velocity $u$ towards east at $t = 0$. Its acceleration is towards west and is constant. Let $x_A$ and $x_B$ be the magnitude of displacements in the first 10 seconds and the next 10 seconds
   (a) $x_A < x_B$     (b) $x_A = x_B$     (c) $x_A > x_B$
   (d) the information is insufficient to decide the relation of $x_A$ with $x_B$.

4. A person travelling on a straight line moves with a uniform velocity $v_1$ for some time and with uniform velocity $v_2$ for the next equal time. The average velocity $v$ is given by
   (a) $v = \dfrac{v_1 + v_2}{2}$     (b) $v = \sqrt{v_1 v_2}$
   (c) $\dfrac{2}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$     (d) $\dfrac{1}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$.

5. A person travelling on a straight line moves with a uniform velocity $v_1$ for a distance $x$ and with a uniform velocity $v_2$ for the next equal distance. The average velocity $v$ is given by

(a) $v = \dfrac{v_1 + v_2}{2}$     (b) $v = \sqrt{v_1 v_2}$
(c) $\dfrac{2}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$     (d) $\dfrac{1}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$.

6. A stone is released from an elevator going up with an acceleration $a$. The acceleration of the stone after the release is
   (a) $a$ upward     (b) $(g - a)$ upward
   (c) $(g - a)$ downward     (d) $g$ downward.

7. A person standing near the edge of the top of a building throws two balls $A$ and $B$. The ball $A$ is thrown vertically upward and $B$ is thrown vertically downward with the same speed. The ball $A$ hits the ground with a speed $v_A$ and the ball $B$ hits the ground with a speed $v_B$. We have
   (a) $v_A > v_B$  (b). $v_A < v_B$ (c) $v_A = v_B$
   (d) the relation between $v_A$ and $v_B$ depends on height of the building above the ground.

8. In a projectile motion the velocity
   (a) is always perpendicular to the acceleration
   (b) is never perpendicular to the acceleration
   (c) is perpendicular to the acceleration for one instant only
   (d) is perpendicular to the acceleration for two instants.

9. Two bullets are fired simultaneously, horizontally and with different speeds from the same place. Which bullet will hit the ground first?
   (a) the faster one       (b) the slower one
   (c) both will reach simultaneously
   (d) depends on the masses.

10. The range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be
    (a) 25 m     (b) 37 m     (c) 50 m     (d) 100 m.

11. Two projectiles $A$ and $B$ are projected with angle of projection 15° for the projectile $A$ and 45° for the projectile $B$. If $R_A$ and $R_B$ be the horizontal range for the two projectiles, then
    (a) $R_A < R_B$ (b) $R_A = R_B$ (c) $R_A > R_B$
    (d) the information is insufficient to decide the relation of $R_A$ with $R_B$.

12. A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction

(a) due north     (b) 30° east of north
(c) 30° north of west    (d) 60° east of north.

13. In the arrangement shown in figure (3-Q3), the ends $P$ and $Q$ of an inextensible string move downwards with uniform speed $u$. Pulleys $A$ and $B$ are fixed. The mass $M$ moves upwards with a speed
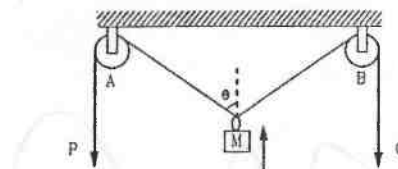(a) $2u \cos\theta$    (b) $u/\cos\theta$    (c) $2u/\cos\theta$    (d) $u\cos\theta$.



Figure 3-Q3

# OBJECTIVE II

1. Consider the motion of the tip of the minute hand of a clock. In one hour
(a) the displacement is zero
(b) the distance covered is zero
(c) the average speed is zero
(d) the average velocity is zero

2. A particle moves along the $X$-axis as
$$x = u(t - 2 \text{ s}) + a(t - 2 \text{ s})^2.$$
(a) the initial velocity of the particle is $u$
(b) the acceleration of the particle is $a$
(c) the acceleration of the particle is $2a$
(d) at $t = 2$ s particle is at the origin.

3. Pick the correct statements :
(a) Average speed of a particle in a given time is never less than the magnitude of the average velocity.

(b) It is possible to have a situation in which $\left|\dfrac{d\vec{v}}{dt}\right| \neq 0$

but $\dfrac{d}{dt}\left|\vec{v}\right| = 0$.

(c) The average velocity of a particle is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval.

(d) The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval. (Infinite accelerations are not allowed.)

4. An object may have
(a) varying speed without having varying velocity
(b) varying velocity without having varying speed
(c) nonzero acceleration without having varying velocity
(d) nonzero acceleration without having varying speed.

5. Mark the correct statements for a particle going on a straight line :
(a) If the velocity and acceleration have opposite sign, the object is slowing down.
(b) If the position and velocity have opposite sign, the particle is moving towards the origin.
(c) If the velocity is zero at an instant, the acceleration should also be zero at that instant.
(d) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.

6. The velocity of a particle is zero at $t = 0$.
(a) The acceleration at $t = 0$ must be zero.
(b) The acceleration at $t = 0$ may be zero.
(c) If the acceleration is zero from $t = 0$ to $t - 10$ s, the speed is also zero in this interval.
(d) If the speed is zero from $t = 0$ to $t = 10$ s the acceleration is also zero in this interval.

7. Mark the correct statements :
(a) The magnitude of the velocity of a particle is equal to its speed.
(b) The magnitude of average velocity in an interval is equal to its average speed in that interval.
(c) It is possible to have a situation in which the speed of a particle is always zero but the average speed is not zero.
(d) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.

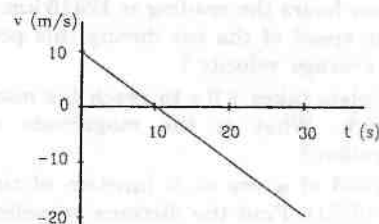8. The velocity-time plot for a particle moving on a straight line is shown in the figure (3-Q4).



Figure 3-Q4

(a) The particle has a constant acceleration.
(b) The particle has never turned around.
(c) The particle has zero displacement.
(d) The average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s.

9. Figure (3-Q5) shows the position of a particle moving on the $X$-axis as a function of time.

(a) The particle has come to rest 6 times.
(b) The maximum speed is at $t = 6$ s.
(c) The velocity remains positive for $t = 0$ to $t = 6$ s.
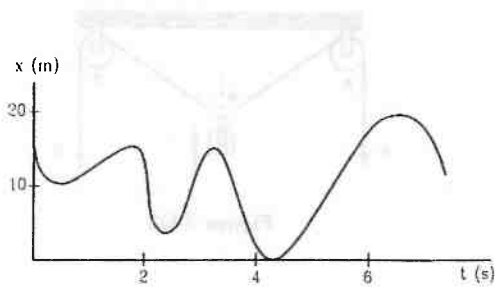(d) The average velocity for the total period shown is negative.

Figure 3-Q5

10. The accelerations of a particle as seen from two frames $S_1$ and $S_2$ have equal magnitude 4 m/s$^2$.

(a) The frames must be at rest with respect to each other.

(b) The frames may be moving with respect to each other but neither should be accelerated with respect to the other.

(c) The acceleration of $S_2$ with respect to $S_1$ may either be zero or 8 m/s$^2$.

(d) The acceleration of $S_2$ with respect to $S_1$ may be anything between zero and 8 m/s$^2$.

## EXERCISES

1. A man has to go 50 m due north, 40 m due east and 20 m due south to reach a field. (a) What distance he has to walk to reach the field ? (b) What is his displacement from his house to the field ?

2. A particle starts from the origin, goes along the X-axis to the point (20 m, 0) and then returns along the same line to the point (−20 m, 0). Find the distance and displacement of the particle during the trip.

3. It is 260 km from Patna to Ranchi by air and 320 km by road. An aeroplane takes 30 minutes to go from Patna to Ranchi whereas a delux bus takes 8 hours. (a) Find the average speed of the plane. (b) Find the average speed of the bus. (c) Find the average velocity of the plane. (d) Find the average velocity of the bus.

4. When a person leaves his home for sightseeing by his car, the meter reads 12352 km. When he returns home after two hours the reading is 12416 km. (a) What is the average speed of the car during this period ? (b) What is the average velocity ?

5. An athelete takes 2·0 s to reach his maximum speed of 18·0 km/h. What is the magnitude of his average acceleration ?

6. The speed of a car as a function of time is shown in figure (3-E1). Find the distance travelled by the car in 8 seconds and its acceleration.
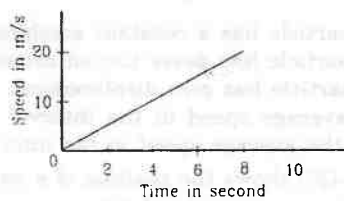


Figure 3-E1

7. The acceleration of a cart started at $t = 0$, varies with time as shown in figure (3-E2). Find the distance travelled in 30 seconds and draw the position-time graph.

8. Figure (3-E3) shows the graph of velocity versus time for a particle going along the X-axis. Find (a) the
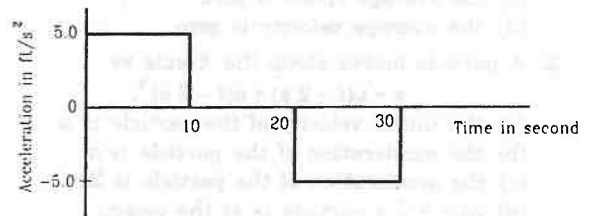


Figure 3-E2

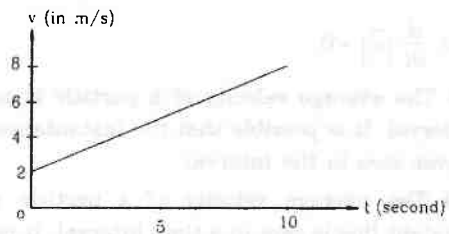acceleration, (b) the distance travelled in 0 to 10 s and (c) the displacement in 0 to 10 s.



Figure 3-E3

9. Figure (3-E4) shows the graph of the x-coordinate of a particle going along the X-axis as a function of time. Find (a) the average velocity during 0 to 10 s, (b) instantaneous velocity at 2, 5, 8 and 12s.
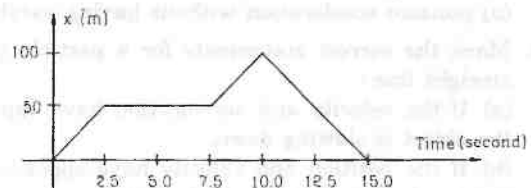


Figure 3-E4

10. From the velocity–time plot shown in figure (3-E5), find the distance travelled by the particle during the first 40

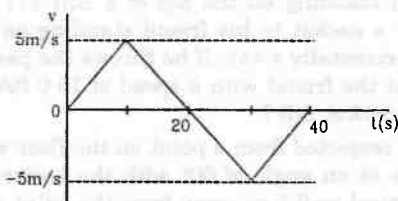seconds. Also find the average velocity during this period.



Figure 3-E5

11. Figure (3-E6) shows $x$-$t$ graph of a particle. Find the time $t$ such that the average velocity of the particle during the period 0 to $t$ is zero.
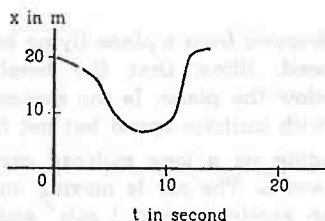


Figure 3-E6

12. A particle starts from a point $A$ and travels along the solid curve shown in figure (3-E7). Find approximately the position $B$ of the particle such that the average velocity between the positions $A$ and $B$ has the same direction as the instantaneous velocity at $B$.
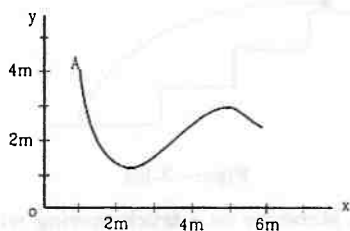


Figure 3-E7

13. An object having a velocity $4.0$ m/s is accelerated at the rate of $1.2$ m/s² for $5.0$ s. Find the distance travelled during the period of acceleration.

14. A person travelling at $43.2$ km/h applies the brake giving a deceleration of $6.0$ m/s² to his scooter. How far will it travel before stopping?

15. A train starts from rest and moves with a constant acceleration of $2.0$ m/s² for half a minute. The brakes are then applied and the train comes to rest in one minute. Find (a) the total distance moved by the train, (b) the maximum speed attained by the train and (c) the position(s) of the train at half the maximum speed.

16. A bullet travelling with a velocity of 16 m/s penetrates a tree trunk and comes to rest in $0.4$ m. Find the time taken during the retardation.

17. A bullet going with speed 350 m/s enters a concrete wall and penetrates a distance of $5.0$ cm before coming to rest. Find the deceleration.

18. A particle starting from rest moves with constant acceleration. If it takes $5.0$ s to reach the speed 18 km/h find (a) the average velocity during this period and (b) the distance travelled by the particle during this period.

19. A driver takes $0.20$ s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of $6.0$ m/s², find the distance travelled by the car after he sees the need to put the brakes on.

20. Complete the following table :

| Car Model | Driver $X$ Reaction time 0·20 s | Driver $Y$ Reaction time 0·30 s |
|---|---|---|
| A (deceleration on hard braking = $6.0$ m/s²) | Speed = 54 km/h Braking distance $a$ = ......................... Total stopping distance $b$ = ......................... | Speed = 72 km/h Braking distance $c$ = ......................... Total stopping distance $d$ = ......................... |
| B (deceleration on hard braking = $7.5$ m/s²) | Speed = 54 km/h Braking distance $e$ = ......................... Total stopping distance $f$ = ......................... | Speed = 72km/h Braking distance $g$ – ......................... Total stopping distance $h$ = ......................... |

21. A police jeep is chasing a culprit going on a motorbike. The motorbike crosses a turning at a speed of 72 km/h. The jeep follows it at a speed of 90 km/h, crossing the turning ten seconds later than the bike. Assuming that they travel at constant speeds, how far from the turning will the jeep catch up with the bike?

22. A car travelling at 60 km/h overtakes another car travelling at 42 km/h. Assuming each car to be $5.0$ m long, find the time taken during the overtake and the total road distance used for the overtake.

23. A ball is projected vertically upward with a speed of 50 m/s. Find (a) the maximum height, (b) the time to reach the maximum height, (c) the speed at half the maximum height. Take $g = 10$ m/s².

24. A ball is dropped from a balloon going up at a speed of 7 m/s. If the balloon was at a height 60 m at the time of dropping the ball, how long will the ball take in reaching the ground?

25. A stone is thrown vertically upward with a speed of 28 m/s. (a) Find the maximum height reached by the stone. (b) Find its velocity one second before it reaches the maximum height. (c) Does the answer of part (b) change if the initial speed is more than 28 m/s such as 40 m/s or 80 m/s?

26. A person sitting on the top of a tall building is dropping balls at regular intervals of one second. Find the positions of the 3rd, 4th and 5th ball when the 6th ball is being dropped.

27. A healthy youngman standing at a distance of 7 m from a 11·8 m high building sees a kid slipping from the top floor. With what speed (assumed uniform) should he run to catch the kid at the arms height (1·8 m) ?

28. An NCC parade is going at a uniform speed of 6 km/h through a place under a berry tree on which a bird is sitting at a height of 12·1 m. At a particular instant the bird drops a berry. Which cadet (give the distance from the tree at the instant) will receive the berry on his uniform ?

29. A ball is dropped from a height. If it takes 0·200 s to cross the last 6·00 m before hitting the ground, find the height from which it was dropped. Take $g = 10$ m/s².

30. A ball is dropped from a height of 5 m onto a sandy floor and penetrates the sand up to 10 cm before coming to rest. Find the retardation of the ball in sand assuming it to be uniform.

31. An elevator is descending with uniform acceleration. To measure the acceleration, a person in the elevator drops a coin at the moment the elevator starts. The coin is 6 ft above the floor of the elevator at the time it is dropped. The person observes that the coin strikes the floor in 1 second. Calculate from these data the acceleration of the elevator.

32. A ball is thrown horizontally from a point 100 m above the ground with a speed of 20 m/s. Find (a) the time it takes to reach the ground, (b) the horizontal distance it travels before reaching the ground, (c) the velocity (direction and magnitude) with which it strikes the ground.

33. A ball is thrown at a speed of 40 m/s at an angle of 60° with the horizontal. Find (a) the maximum height reached and (b) the range of the ball. Take $g = 10$ m/s².

34. In a soccer practice session the football is kept at the centre of the field 40 yards from the 10 ft high goalposts. A goal is attempted by kicking the football at a speed of 64 ft/s at an angle of 45° to the horizontal. Will the ball reach the goal post ?

35. A popular game in Indian villages is *goli* which is played with small glass balls called golis. The goli of one player is situated at a distance of 2·0 m from the goli of the second player. This second player has to project his goli by keeping the thumb of the left hand at the place of his goli, holding the goli between his two middle fingers and making the throw. If the projected goli hits the goli of the first player, the second player wins. If the height from which the goli is projected is 19·6 cm from the ground and the goli is to be projected horizontally, with what speed should it be projected so that it directly hits the stationary goli without falling on the ground earlier ?

36. Figure (3-E8) shows a 11·7 ft wide ditch, with the approach roads at an angle of 15° with the horizontal. With what minimum speed should a motorbike be moving on the road so that it safely crosses the ditch ?



11.7 ft

Figure 3-E8

Assume that the length of the bike is 5 ft, and it leaves the road when the front part runs out of the approach road.

37. A person standing on the top of a cliff 171 ft high has to throw a packet to his friend standing on the ground 228 ft horizontally away. If he throws the packet directly aiming at the friend with a speed of 15·0 ft/s, how short will the packet fall ?

38. A ball is projected from a point on the floor with a speed of 15 m/s at an angle of 60° with the horizontal. Will it hit a vertical wall 5 m away from the point of projection and perpendicular to the plane of projection without hitting the floor ? Will the answer differ if the wall is 22 m away ?

39. Find the average velocity of a projectile between the instants it crosses half the maximum height. It is projected with a speed $u$ at an angle $\theta$ with the horizontal.

40. A bomb is dropped from a plane flying horizontally with uniform speed. Show that the bomb will explode vertically below the plane. Is the statement true if the plane flies with uniform speed but not horizontally ?

41. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s² and the projection velocity in the vertical direction is 9·8 m/s. How far behind the boy will the ball fall on the car ?

42. A staircase contains three steps each 10 cm high and 20 cm wide (figure 3-E9). What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the lowest plane ?
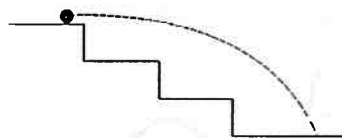


Figure 3-E9

43. A person is standing on a truck moving with a constant velocity of 14·7 m/s on a horizontal road. The man throws a ball in such a way that it returns to the truck after the truck has moved 58·8 m. Find the speed and the angle of projection (a) as seen from the truck, (b) as seen from the road.

44. The benches of a gallery in a cricket stadium are 1 m wide and 1 m high. A batsman strikes the ball at a level one metre above the ground and hits a mammoth sixer. The ball starts at 35 m/s at an angle of 53° with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110 m from the batsman. On which bench will the ball hit ?

45. A man is sitting on the shore of a river. He is in the line of a 1·0 m long boat and is 5·5 m away from the centre of the boat. He wishes to throw an apple into the boat. If he can throw the apple only with a speed of 10 m/s, find the minimum and maximum angles of projection for successful shot. Assume that the point of

projection and the edge of the boat are in the same horizontal level.

46. A river 400 m wide is flowing at a rate of 2·0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river. (a) Find the time taken by the boat to reach the opposite bank. (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank ?

47. A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h. (a) If he heads in a direction making an angle $\theta$ with the flow, find the time he takes to cross the river. (b) Find the shortest possible time to cross the river.

48. Consider the situation of the previous problem. The man has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.

49. An aeroplane has to go from a point $A$ to another point $B$, 500 km away due 30° east of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. (a) Find the direction in which the pilot should head the plane to reach the point $B$. (b) Find the time taken by the plane to go from $A$ to $B$.

50. Two friends $A$ and $B$ are standing a distance $x$ apart in an open field and wind is blowing from $A$ to $B$. $A$ beats a drum and $B$ hears the sound $t_1$ time after he sees the event. $A$ and $B$ interchange their positions and the experiment is repeated. This time $B$ hears the drum $t_2$ time after he sees the event. Calculate the velocity of sound in still air $v$ and the velocity of wind $u$. Neglect the time light takes in travelling between the friends.

51. Suppose $A$ and $B$ in the previous problem change their positions in such a way that the line joining them becomes perpendicular to the direction of wind while maintaining the separation $x$. What will be the time lag $B$ finds between seeing and hearing the drum beating by $A$ ?

52. Six particles situated at the corners of a regular hexagon of side $a$ move at a constant speed $v$. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (b) | 2. (d) | 3. (d) | 4. (a) | 5. (c) | 6. (d) |
| 7. (c) | 8. (c) | 9. (c) | 10. (d) | 11. (d) | 12. (a) |
| 13. (b) | | | | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (d) | 2. (c), (d) | 3. (a), (b), (c) |
| 4. (b), (d) | 5. (a), (b), (d) | 6. (b), (c), (d) |
| 7. (a) | 8. (a), (d) | 9. (a) |
| 10. (d) | | |

### EXERCISES

1. (a) 110 m     (b) 50 m, $\tan^{-1} 3/4$ north to east
2. 60 m, 20 m in the negative direction
3. (a) 520 km/h     (b) 40 km/h
   (c) 520 km/h Patna to Ranchi
   (d) 32·5 km/h Patna to Ranchi
4. 32 km/h     (b) zero
5. 2·5 m/s$^2$
6. 80 m, 2·5 m/s$^2$
7. 1000 ft
8. (a) 0·6 m/s$^2$     (b) 50 m     (c) 50 m
9. (a) 10 m/s     (b) 20 m/s, zero, 20 m/s, – 20 m/s
10. 100 m, zero
11. 12 s
12. $x = 5$ m, $y = 3$ m
13. 35 m
14. 12 m
15. (a) 2·7 km     (b) 60 m/s     (c) 225 m and 2·25 km
16. 0·05 s
17. $12·2 \times 10^5$ m/s$^2$
18. (a) 2·5 m/s     (b) 12·5 m
19. 22 m
20. (a) 19 m     (b) 22 m     (c) 33 m     (d) 39 m
    (e) 15 m     (f) 18 m     (g) 27 m     (h) 33 m
21. 1·0 km
22. 2 s, 38 m
23. (a) 125 m     (b) 5 s     (c) 35 m/s
24. 4·3 s
25. (a) 40 m     (b) 9·8 m/s     (c) No
26. 44·1 m, 19·6 m and 4·9 m below the top
27. 4·9 m/s
28. 2·62 m
29. 48 m
30. 490 m/s$^2$
31. 20 ft/s$^2$
32. (a) 4·5 s     (b) 90 m     (c) 49 m/s, $\theta = 66°$ with horizontal
33. (a) 60 m     (b) $80\sqrt{3}$ m
34. Yes
35. 10 m/s
36. 32 ft/s

37. 192 ft

38. Yes, Yes

39. $u \cos\theta$, horizontal in the plane of projection

41. 2 m

42. 2 m/s

43. (a) 19·6 m/s upward
    (b) 24·5 m/s at 53° with horizontal

44. Sixth

45. Minimum angle 15°, maximum angle 75° but there is an interval of 53° between 15° and 75°, which is not allowed for successful shot

46. (a) 40 s    (b) 80 m

47. (a) $\dfrac{10 \text{ minute}}{\sin\theta}$    (b) 10 minute

48. 2/3 km

49. (a) $\sin^{-1}(1/15)$ east of the line $AB$    (b) 50 min

50. $\dfrac{x}{2}\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right),\ \dfrac{x}{2}\left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right)$

51. $\dfrac{x}{\sqrt{v^2 - u^2}}$

52. $2\,a/v$.

Two neutral objects placed far away exert only negligible force on each other but when they are placed closer they may exert appreciable force.

3. *Calculate the ratio of electric to gravitational force between two electrons.*

*Solution* : The electric force $= \dfrac{e^2}{4 \pi \varepsilon_0 r^2}$

and the gravitational force $= \dfrac{G(m_e)^2}{r^2}$.

The ratio is $\dfrac{e^2}{4 \pi \varepsilon_0 G (m_e)^2}$

$= \dfrac{9 \times 10^9 \dfrac{\text{N-m}^2}{\text{C}^2} \times (1.6 \times 10^{-19} \text{ C})^2}{6.67 \times 10^{-11} \dfrac{\text{N-m}^2}{\text{kg}^2} \times (9.1 \times 10^{-31} \text{ kg})^2} = 4.17 \times 10^{42}$

□

## QUESTIONS FOR SHORT ANSWER

1. A body of mass $m$ is placed on a table. The earth is pulling the body with a force $mg$. Taking this force to be the action, what is the reaction ?

2. A boy is sitting on a chair placed on the floor of a room. Write as many action-reaction pairs of forces as you can.

3. A lawyer alleges in court that the police has forced his client to issue a statement of confession. What kind of force is this ?

4. When you hold a pen and write on your notebook, what kind of force is exerted by you on the pen ? By the pen on the notebook ? By you on the notebook ?

5. Is it true that the reaction of a gravitational force is always gravitational, of an electromagnetic force is always electromagnetic and so on ?

6. Suppose the magnitude of Nuclear force between two protons varies with the distance between them as shown in figure (4-Q1). Estimate the ratio "Nuclear force/Coulomb force" for (a) $x = 8$ fm (b) $x = 4$ fm, (c) $x = 2$ fm and (d) $x = 1$ fm (1 fm $= 10^{-15}$ m).
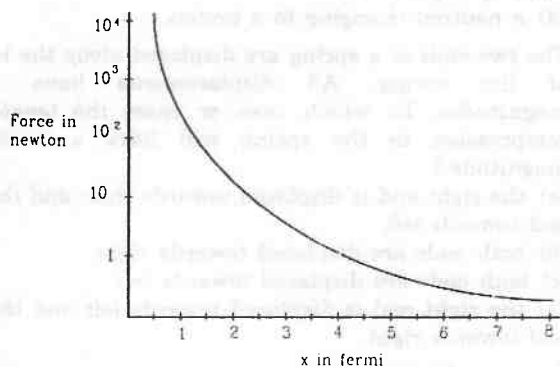


Figure 4-Q1

7. List all the forces acting on the block $B$ in figure (4-Q2).

8. List all the forces acting on (a) the pulley $A$, (b) the boy and (c) the block $C$ in figure (4-Q3).
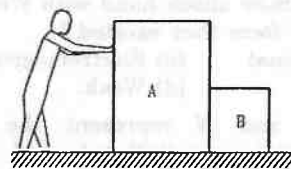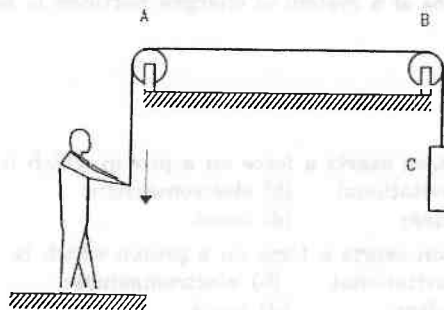


Figure 4-Q2



Figure 4-Q3

9. Figure (4-Q4) shows a boy pulling a wagon on a road. List as many forces as you can which are relevant with this figure. Find the pairs of forces connected by Newton's third law of motion.
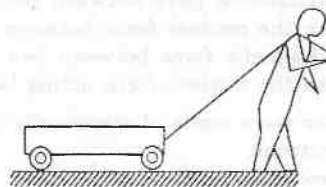


Figure 4-Q4

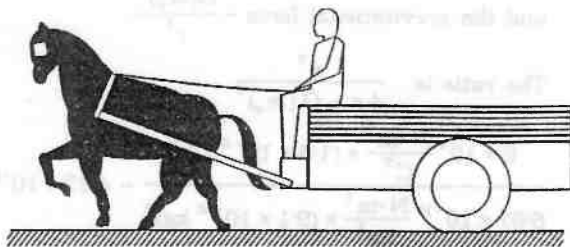10. Figure (4-Q5) shows a cart. Complete the table shown below.

Figure 4-Q5

| Force on | Force by | Nature of the force | Direction |
|---|---|---|---|
| Cart | 1<br>2<br>3<br>: | | |
| Horse | 1<br>2<br>3<br>: | | |
| Driver | 1<br>2<br>3<br>: | | |

## OBJECTIVE I

1. When Neils Bohr shook hand with Werner Heisenberg, what kind of force they exerted ?
   (a) Gravitational       (b) Electromagnetic
   (c) Nuclear             (d) Weak.

2. Let $E$, $G$ and $N$ represent the magnitudes of electromagnetic, gravitational and nuclear forces between two electrons at a given separation. Then
   (a) $N>E>G$   (b) $E>N>G$   (c) $G>N>E$   (d) $E>G>N$.

3. The sum of all electromagnetic forces between different particles of a system of charged particles is zero

   (a) only if all the particles are positively charged
   (b) only if all the particles are negatively charged
   (c) only if half the particles are positively charged and half are negatively charged
   (d) irrespective of the signs of the charges.

4. A 60 kg man pushes a 40 kg man by a force of 60 N. The 40 kg man has pushed the other man with a force of
   (a) 40 N       (b) 0      (c) 60 N       (d) 20 N.

## OBJECTIVE II

1. A neutron exerts a force on a proton which is
   (a) gravitational     (b) electromagnetic
   (c) nuclear           (d) weak.

2. A proton exerts a force on a proton which is
   (a) gravitational     (b) electromagnetic
   (c) nuclear           (d) weak.

3. Mark the correct statements :
   (a) The nuclear force between two protons is always greater than the electromagnetic force between them.
   (b) The electromagnetic force between two protons is always greater than the gravitational force between them.
   (c) The gravitational force between two protons may be greater than the nuclear force between them.
   (d) Electromagnetic force between two protons may be greater than the nuclear force acting between them.

4. If all matter were made of electrically neutral particles such as neutrons,
   (a) there would be no force of friction
   (b) there would be no tension in the string
   (c) it would not be possible to sit on a chair
   (d) the earth could not move around the sun.

5. Which of the following systems may be adequately described by classical physics ?
   (a) motion of a cricket ball
   (b) motion of a dust particle
   (c) a hydrogen atom
   (d) a neutron changing to a proton.

6. The two ends of a spring are displaced along the length of the spring. All displacements have equal magnitudes. In which case or cases the tension or compression in the spring will have a maximum magnitude ?
   (a) the right end is displaced towards right and the left end towards left
   (b) both ends are displaced towards right
   (c) both ends are displaced towards left
   (d) the right end is displaced towards left and the left end towards right.

7. Action and reaction
   (a) act on two different objects
   (b) have equal magnitude
   (c) have opposite directions
   (d) have resultant zero.

## EXERCISES

1. The gravitational force acting on a particle of 1 g due to a similar particle is equal to $6.67 \times 10^{-11}$ N. Calculate the separation between the particles.

2. Calculate the force with which you attract the earth.

3. At what distance should two charges, each equal to 1 C, be placed so that the force between them equals your weight ?

4. Two spherical bodies, each of mass 50 kg, are placed at a separation of 20 cm. Equal charges are placed on the bodies and it is found that the force of Coulomb repulsion equals the gravitational attraction in magnitude. Find the magnitude of the charge placed on either body.

5. A monkey is sitting on a tree limb. The limb exerts a normal force of 48 N and a frictional force of 20 N. Find the magnitude of the total force exerted by the limb on the monkey.

6. A body builder exerts a force of 150 N against a bullworker and compresses it by 20 cm. Calculate the spring constant of the spring in the bullworker.

7. A satellite is projected vertically upwards from an earth station. At what height above the earth's surface will the force on the satellite due to the earth be reduced to half its value at the earth station ? (Radius of the earth is 6400 km.)

8. Two charged particles placed at a separation of 20 cm exert 20 N of Coulomb force on each other. What will be the force if the separation is increased to 25 cm ?

9. The force with which the earth attracts an object is called the weight of the object. Calculate the weight of the moon from the following data : The universal constant of gravitation $G = 6.67 \times 10^{-11}$ N–m$^2$/kg$^2$, mass of the moon = $7.36 \times 10^{22}$ kg, mass of the earth = $6 \times 10^{24}$ kg and the distance between the earth and the moon = $3.8 \times 10^5$ km.

10. Find the ratio of the magnitude of the electric force to the gravitational force acting between two protons.

11. The average separation between the proton and the electron in a hydrogen atom in ground state is $5.3 \times 10^{-11}$ m. (a) Calculate the Coulomb force between them at this separation. (b) When the atom goes into its first excited state the average separation between the proton and the electron increases to four times its value in the ground state. What is the Coulomb force in this state ?

12. The geostationary orbit of the earth is at a distance of about 36000 km from the earth's surface. Find the weight of a 120-kg equipment placed in a geostationary satellite. The radius of the earth is 6400 km.

□

## ANSWERS

### OBJECTIVE I

1. (b)     2. (d)     3. (d)     4. (c)

### OBJECTIVE II

1. (a), (c)          2. (a), (b), (c)          3. (b), (c), (d)

4. (a), (b), (c)     5. (a), (b)               6. (a), (d)

7. (a), (b), (c), (d)

### EXERCISES

1. 1 m
4. $4.3 \times 10^{-9}$ C
5. 52 N
6. 750 N/m
7. 2650 km
8. 13 N
9. $2 \times 10^{20}$ N
10. $1.24 \times 10^{36}$
11. (a) $8.2 \times 10^{-8}$ N, (b) $5.1 \times 10^{-9}$ N
12. 27 N

□

11. *All the surfaces shown in figure (5-W14) are assumed to be fricitionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.*
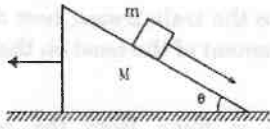


Figure 5-W14

**Solution** : Let the acceleration of the prism be $a_0$ in the backward direction. Consider the motion of the smaller block from the frame of the prism.

The forces on the block are (figure 5-W15a)

(i) $\mathcal{N}$ normal force,

(ii) $mg$ downward (gravity),

(iii) $ma_0$ forward (psuedo).

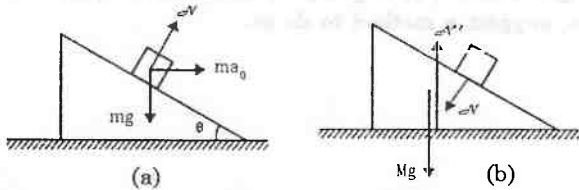

Figure 5-W15

The block slides down the plane. Components of the forces parallel to the incline give

$$ma_0 \cos\theta + mg \sin\theta = ma$$

or,                          $a = a_0 \cos\theta + g \sin\theta.$   ... (i)

Components of the force perpendicular to the incline give

$$\mathcal{N} + ma_0 \sin\theta = mg \cos\theta.$$   ... (ii)

Now consider the motion of the prism from the lab frame. No pseudo force is needed as the frame used is inertial. The forces are (figure 5-W15b)

(i) $Mg$ downward,

(ii) $\mathcal{N}$ normal to the incline (by the block),

(iii) $\mathcal{N}'$ upward (by the horizontal surface).

Horizontal components give,

$$\mathcal{N} \sin\theta = Ma_0 \quad \text{or,} \quad \mathcal{N} = Ma_0 / \sin\theta.$$   ... (iii)

Putting in (ii),

$$\frac{Ma_0}{\sin\theta} + ma_0 \sin\theta = mg \cos\theta$$

or,                          $a_0 = \dfrac{m g \sin\theta \cos\theta}{M + m \sin^2\theta}.$

From (i),          $a = \dfrac{m g \sin\theta \cos^2\theta}{M + m \sin^2\theta} + g \sin\theta$

$$= \frac{(M + m) g \sin\theta}{M + m \sin^2\theta}.$$

---

## QUESTIONS FOR SHORT ANSWER

1. The apparent weight of an object increases in an elevator while accelerating upward. A moongphaliwala sells his moongphali using a beam balance in an elevator. Will he gain more if the elevator is accelerating up ?

2. A boy puts a heavy box of mass $M$ on his head and jumps down from the top of a multistoried building to the ground. How much is the force exerted by the box on his head during his free fall ? Does the force greatly increase during the period he balances himself after striking the ground ?

3. A person drops a coin. Describe the path of the coin as seen by the person if he is in (a) a car moving at constant velocity and (b) in a freely falling elevator.

4. Is it possible for a particle to describe a curved path if no force acts on it ? Does your answer depend on the frame of reference chosen to view the particle ?

5. You are riding in a car. The driver suddenly applies the brakes and you are pushed forward. Who pushed you forward ?

6. It is sometimes heard that inertial frame of reference is only an ideal concept and no such inertial frame actually exists. Comment.

7. An object is placed far away from all the objects that can exert force on it. A frame of reference is constructed by taking the origin and axes fixed in this object. Will the frame be necessarily inertial ?

8. Figure (5-Q1) shows a light spring balance connected to two blocks of mass 20 kg each. The graduations in the balance measure the tension in the spring. (a) What is the reading of the balance? (b) Will the reading change if the balance is heavy, say 2·0 kg ? (c) What will happen if the spring is light but the blocks have unequal masses ?
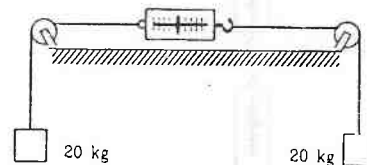


Figure 5-Q1

9. The acceleration of a particle is zero as measured from an inertial frame of reference. Can we conclude that no force acts on the particle ?

10. Suppose you are running fast in a field when you suddenly find a snake in front of you. You stop quickly. Which force is responsible for your deceleration?

11. If you jump barefooted on a hard surface, your legs get injured. But they are not injured if you jump on a soft surface like sand or pillow. Explain.

12. According to Newton's third law each team pulls the opposite team with equal force in a tug of war. Why then one team wins and the other loses?

13. A spy jumps from an airplane with his parachute. The spy accelerates downward for some time when the parachute opens. The acceleration is suddenly checked and the spy slowly falls on the ground. Explain the action of parachute in checking the acceleration.

14. Consider a book lying on a table. The weight of the book and the normal force by the table on the book are equal in magnitude and opposite in direction. Is this an example of Newton's third law?

15. Two blocks of unequal masses are tied by a spring. The blocks are pulled stretching the spring slightly and the system is released on a frictionless horizontal platform. Are the forces due to the spring on the two blocks equal and opposite? If yes, is it an example of Newton's third law?

16. When a train starts, the head of a standing passenger seems to be pushed backward. Analyse the situation from the ground frame. Does it really go backward? Coming back to the train frame, how do you explain the backward movement of the head on the basis of Newton's laws?

17. A plumb bob is hung from the ceiling of a train compartment. If the train moves with an acceleration 'a' along a straight horizontal track, the string supporting the bob makes an angle $\tan^{-1}(a/g)$ with the normal to the ceiling. Suppose the train moves on an inclined straight track with uniform velocity. If the angle of incline is $\tan^{-1}(a/g)$, the string again makes the same angle with the normal to the ceiling. Can a person sitting inside the compartment tell by looking at the plumb line whether the train is accelerated on a horizontal straight track or it is going on an incline? If yes, how? If no, suggest a method to do so.

## OBJECTIVE I

1. A body of weight $w_1$ is suspended from the ceiling of a room through a chain of weight $w_2$. The ceiling pulls the chain by a force

   (a) $w_1$  (b) $w_2$  (c) $w_1 + w_2$  (d) $\dfrac{w_1 + w_2}{2}$.

2. When a horse pulls a cart, the force that helps the horse to move forward is the force exerted by
   (a) the cart on the horse  (b) the ground on the horse
   (c) the ground on the cart  (d) the horse on the ground.

3. A car accelerates on a horizontal road due to the force exerted by
   (a) the engine of the car  (b) the driver of the car
   (c) the earth  (d) the road.

4. A block of mass 10 kg is suspended through two light spring balances as shown in figure (5-Q2).



Figure 5-Q2

   (a) Both the scales will read 10 kg.
   (b) Both the scales will read 5 kg.
   (c) The upper scale will read 10 kg and the lower zero.
   (d) The readings may be anything but their sum will be 10 kg.

5. A block of mass $m$ is placed on a smooth inclined plane of inclination $\theta$ with the horizontal. The force exerted by the plane on the block has a magnitude
   (a) $mg$  (b) $mg/\cos\theta$  (c) $mg\cos\theta$  (d) $mg\tan\theta$.

6. A block of mass $m$ is placed on a smooth wedge of inclination $\theta$. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block has a magnitude
   (a) $mg$  (b) $mg/\cos\theta$  (c) $mg\cos\theta$  (d) $mg\tan\theta$.

7. Neglect the effect of rotation of the earth. Suppose the earth suddenly stops attracting objects placed near its surface. A person standing on the surface of the earth will
   (a) fly up  (b) slip along the surface
   (c) fly along a tangent to the earth's surface
   (d) remain standing.

8. Three rigid rods are joined to form an equilateral triangle $ABC$ of side 1 m. Three particles carrying charges 20 μC each are attached to the vertices of the triangle. The whole system is at rest in an inertial frame. The resultant force on the charged particle at $A$ has the magnitude
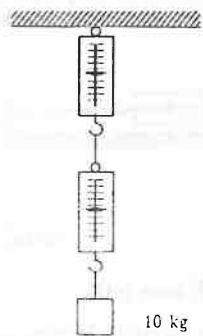   (a) zero  (b) 3·6 N  (c) 3·6√3 N  (d) 7·2 N.

9. A force $F_1$ acts on a particle so as to accelerate it from rest to a velocity $v$. The force $F_1$ is then replaced by $F_2$ which decelerates it to rest.
   (a) $F_1$ must be equal to $F_2$    (b) $F_1$ may be equal to $F_2$
   (c) $F_1$ must be unequal to $F_2$   (d) none of these.

10. Two objects $A$ and $B$ are thrown upward simultaneously with the same speed. The mass of $A$ is greater than the mass of $B$. Suppose the air exerts a constant and equal force of resistance on the two bodies.
    (a) The two bodies will reach the same height.
    (b) $A$ will go higher than $B$.
    (c) $B$ will go higher than $A$.
    (d) Any of the above three may happen depending on the speed with which the objects are thrown.

11. A smooth wedge $A$ is fitted in a chamber hanging from a fixed ceiling near the earth's surface. A block $B$ placed at the top of the wedge takes a time $T$ to slide down the length of the wedge. If the block is placed at the top of the wedge and the cable supporting the chamber is broken at the same instant, the block will
    (a) take a time longer than $T$ to slide down the wedge
    (b) take a time shorter than $T$ to slide down the wedge
    (c) remain at the top of the wedge
    (d) jump off the wedge.

12. In an imaginary atmosphere, the air exerts a small force $F$ on any particle in the direction of the particle's motion. A particle of mass $m$ projected upward takes a time $t_1$ in reaching the maximum height and $t_2$ in the return journey to the original point. Then
    (a) $t_1 < t_2$   (b) $t_1 > t_2$   (c) $t_1 = t_2$   (d) the relation between $t_1$ and $t_2$ depends on the mass of the particle.

13. A person standing on the floor of an elevator drops a coin. The coin reaches the floor of the elevator in a time $t_1$ if the elevator is stationary and in time $t_2$ if it is moving uniformly. Then
    (a) $t_1 = t_2$ (b) $t_1 < t_2$ (c) $t_1 > t_2$ (d) $t_1 < t_2$ or $t_1 > t_2$ depending on whether the lift is going up or down.

14. A free $^{238}U$ nucleus kept in a train emits an alpha particle. When the train is stationary, a nucleus decays and a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes $x$ at time $t$ after the decay. If the decay takes place while the train is moving at a uniform velocity $v$, the distance between the alpha particle and the recoiling nucleus at a time $t$ after the decay as measured by the passenger is
    (a) $x + v t$      (b) $x - v t$      (c) $x$
    (d) depends on the direction of the train.

## OBJECTIVE II

1. A reference frame attached to the earth
   (a) is an inertial frame by definition
   (b) cannot be an inertial frame because the earth is revolving around the sun
   (c) is an inertial frame because Newton's laws are applicable in this frame
   (d) cannot be an inertial frame because the earth is rotating about its axis.

2. A particle stays at rest as seen in a frame. We can conclude that
   (a) the frame is inertial
   (b) resultant force on the particle is zero
   (c) the frame may be inertial but the resultant force on the particle is zero
   (d) the frame may be noninertial but there is a nonzero resultant force.

3. A particle is found to be at rest when seen from a frame $S_1$ and moving with a constant velocity when seen from another frame $S_2$. Mark out the possible options.
   (a) Both the frames are inertial.
   (b) Both the frames are noninertial.
   (c) $S_1$ is inertial and $S_2$ is noninertial.
   (d) $S_1$ is noninertial and $S_1$ is inertial.

4. Figure (5-Q3) shows the displacement of a particle going along the $X$-axis as a function of time. The force acting on the particle is zero in the region
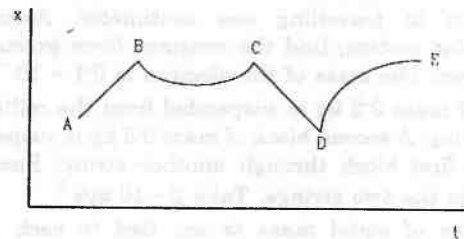   (a) $AB$     (b) $BC$     (c) $CD$     (d) $DE$.



Figure 5-Q3

5. Figure (5-Q4) shows a heavy block kept on a frictionless surface and being pulled by two ropes of equal mass $m$. At $t = 0$, the force on the left rope is withdrawn but the force on the right end continues to act. Let $F_1$ and $F_2$ be the magnitudes of the forces by the right rope and the left rope on the block respectively.



Figure 5-Q4

(a) $F_1 = F_2 = F$ for $t < 0$
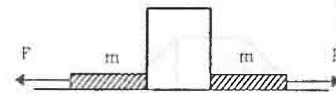(b) $F_1 = F_2 = F + mg$ for $t < 0$
(c) $F_1 = F$, $F_2 = F$ for $t > 0$
(d) $F_1 < F$, $F_2 = F$ for $t > 0$.

6. The force exerted by the floor of an elevator on the foot of a person standing there is more than the weight of the person if the elevator is
(a) going up and slowing down
(b) going up and speeding up
(c) going down and slowing down
(d) going down and speeding up.

7. If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may be
(a) going up with increasing speed
(b) going down with increasing speed
(c) going up with uniform speed
(d) going down with uniform speed.

8. A particle is observed from two frames $S_1$ and $S_2$. The frame $S_2$ moves with respect to $S_1$ with an acceleration

$a$. Let $F_1$ and $F_2$ be the pseudo forces on the particle when seen from $S_1$ and $S_2$ respectively. Which of the following are not possible ?
(a) $F_1 = 0$, $F_2 \neq 0$   (b) $F_1 \neq 0$, $F_2 = 0$
(c) $F_1 \neq 0$, $F_2 \neq 0$   (d) $F_1 = 0$, $F_2 = 0$.

9. A person says that he measured the acceleration of a particle to be nonzero while no force was acting on the particle.
(a) He is a liar.
(b) His clock might have run slow.
(c) His meter scale might have been longer than the standard.
(d) He might have used noninertial frame.

## EXERCISES

1. A block of mass 2 kg placed on a long frictionless horizontal table is pulled horizontally by a constant force $F$. It is found to move 10 m in the first two seconds. Find the magnitude of $F$.

2. A car moving at 40 km/h is to be stopped by applying brakes in the next 4·0 m. If the car weighs 2000 kg, what average force must be applied on it ?

3. In a TV picture tube electrons are ejected from the cathode with negligible speed and reach a velocity of $5 \times 10^6$ m/s in travelling one centimeter. Assuming straight line motion, find the constant force exerted on the electron. The mass of the electron is $9·1 \times 10^{-31}$ kg.

4. A block of mass 0·2 kg is suspended from the ceiling by a light string. A second block of mass 0·3 kg is suspended from the first block through another string. Find the tensions in the two strings. Take $g = 10$ m/s$^2$.

5. Two blocks of equal mass $m$ are tied to each other through a light string. One of the blocks is pulled along the line joining them with a constant force $F$. Find the tension in the string joining the blocks.

6. A particle of mass 50 g moves on a straight line. The variation of speed with time is shown in figure (5-E1). Find the force acting on the particle at $t = 2, 4$ and $6$ seconds.
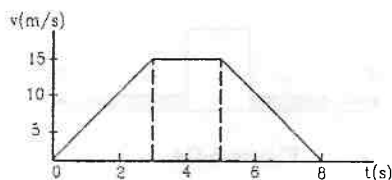


Figure 5-E1

7. Two blocks $A$ and $B$ of mass $m_A$ and $m_B$ respectively are kept in contact on a frictionless table. The experimenter pushes the block $A$ from behind so that

the blocks accelerate. If the block $A$ exerts a force $F$ on the block $B$, what is the force exerted by the experimenter on $A$ ?

8. Raindrops of radius 1 mm and mass 4 mg are falling with a speed of 30 m/s on the head of a bald person. The drops splash on the head and come to rest. Assuming equivalently that the drops cover a distance equal to their radii on the head, estimate the force exerted by each drop on the head.

9. A particle of mass 0·3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point $x = 20$ cm ?

10. Both the springs shown in figure (5-E2) are unstretched. If the block is displaced by a distance $x$ and released, what will be the initial acceleration?
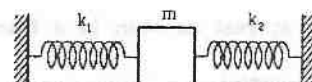


Figure 5-E2

11. A small block $B$ is placed on another block $A$ of mass 5 kg and length 20 cm. Initially the block $B$ is near the right end of block $A$ (figure 5-E3). A constant horizontal force of 10 N is applied to the block $A$. All the surfaces are assumed frictionless. Find the time elapsed before the block $B$ separates from $A$.
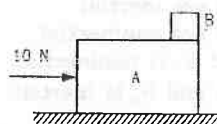


Figure 5-E3

12. A man has fallen into a ditch of width $d$ and two of his friends are slowly pulling him out using a light rope and two fixed pulleys as shown in figure (5-E4). Show that

the force (assumed equal for both the friends) exerted by each friend on the rope increases as the man moves up. Find the force when the man is at a depth $h$.
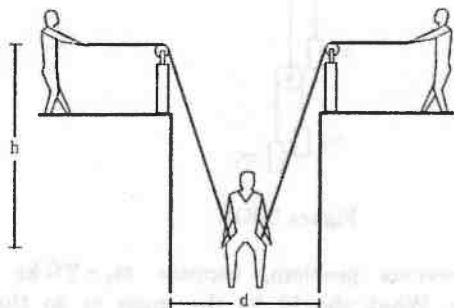


Figure 5-E4

13. The elevator shown in figure (5-E5) is descending with an acceleration of $2 \text{ m/s}^2$. The mass of the block $A$ is 0·5 kg. What force is exerted by the block $A$ on the block $B$?
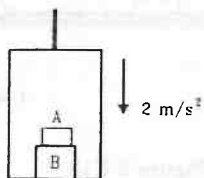


Figure 5-E5

14. A pendulum bob of mass 50 g is suspended from the ceiling of an elevator. Find the tension in the string if the elevator (a) goes up with acceleration $1·2 \text{ m/s}^2$, (b) goes up with deceleration $1·2 \text{ m/s}^2$, (c) goes up with uniform velocity, (d) goes down with acceleration $1·2 \text{ m/s}^2$, (e) goes down with deceleration $1·2 \text{ m/s}^2$ and (f) goes down with uniform velocity.

15. A person is standing on a weighing machine placed on the floor of an elevator. The elevator starts going up with some acceleration, moves with uniform velocity for a while and finally decelerates to stop. The maximum and the minimum weights recorded are 72 kg and 60 kg. Assuming that the magnitudes of the acceleration and the deceleration are the same, find (a) the true weight of the person and (b) the magnitude of the acceleration. Take $g = 9·9 \text{ m/s}^2$.
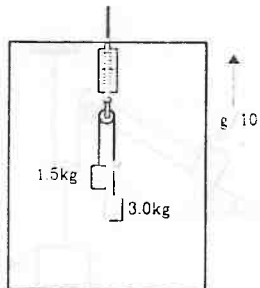


Figure 5-E6

16. Find the reading of the spring balance shown in figure (5-E6). The elevator is going up with an acceleration of $g/10$, the pulley and the string are light and the pulley is smooth.

17. A block of 2 kg is suspended from the ceiling through a massless spring of spring constant $k = 100 \text{ N/m}$. What is the elongation of the spring? If another 1 kg is added to the block, what would be the further elongation?

18. Suppose the ceiling in the previous problem is that of an elevator which is going up with an acceleration of $2·0 \text{ m/s}^2$. Find the elongations.

19. The force of buoyancy exerted by the atmosphere on a balloon is $B$ in the upward direction and remains constant. The force of air resistance on the balloon acts opposite to the direction of velocity and is proportional to it. The balloon carries a mass $M$ and is found to fall down near the earth's surface with a constant velocity $v$. How much mass should be removed from the balloon so that it may rise with a constant velocity $v$?

20. An empty plastic box of mass $m$ is found to accelerate up at the rate of $g/6$ when placed deep inside water. How much sand should be put inside the box so that it may accelerate down at the rate of $g/6$?

21. A force $\vec{F} = \vec{v} \times \vec{A}$ is exerted on a particle in addition to the force of gravity, where $\vec{v}$ is the velocity of the particle and $\vec{A}$ is a constant vector in the horizontal direction. With what minimum speed a particle of mass $m$ be projected so that it continues to move undeflected with a constant velocity?

22. In a simple Atwood machine, two unequal masses $m_1$ and $m_2$ are connected by a string going over a clamped light smooth pulley. In a typical arrangement (figure 5-E7) $m_1 = 300 \text{ g}$ and $m_2 = 600 \text{ g}$. The system is released from rest. (a) Find the distance travelled by the first block in the first two seconds. (b) Find the tension in the string. (c) Find the force exerted by the clamp on the pulley.
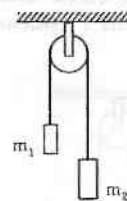


Figure 5-E7

23. Consider the Atwood machine of the previous problem. The larger mass is stopped for a moment 2·0 s after the system is set into motion. Find the time elapsed before the string is tight again.

24. Figure (5-E8) shows a uniform rod of length 30 cm having a mass of 3·0 kg. The strings shown in the figure are pulled by constant forces of 20 N and 32 N. Find the force exerted by the 20 cm part of the rod on the 10 cm part. All the surfaces are smooth and the strings and the pulleys are light.

Figure 5-E8

25. Consider the situation shown in figure (5-E9). All the surfaces are frictionless and the string and the pulley are light. Find the magnitude of the acceleration of the two blocks.



Figure 5-E9

26. A constant force $F = m_2 g/2$ is applied on the block of mass $m_1$ as shown in figure (5-E10). The string and the pulley are light and the surface of the table is smooth. Find the acceleration of $m_1$.
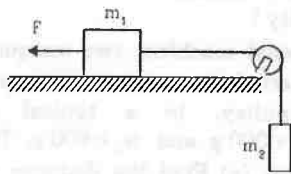


Figure 5-E10

27. In figure (5-E11) $m_1 = 5$ kg, $m_2 = 2$ kg and $F = 1$ N. Find the acceleration of either block. Describe the motion of $m_1$ if the string breaks but $F$ continues to act.
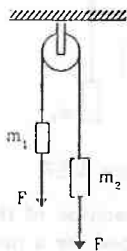


Figure 5-E11

28. Let $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg in figure (5-E12). Find the accelerations of $m_1$, $m_2$ and $m_3$. The string from the upper pulley to $m_1$ is 20 cm when the system is released from rest. How long will it take before $m_1$ strikes the pulley ?
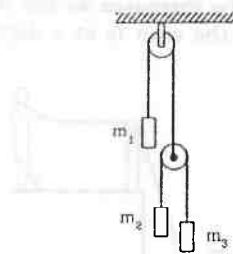


Figure 5-E12

29. In the previous problem, suppose $m_2 = 2.0$ kg and $m_3 = 3.0$ kg. What should be the mass $m$ so that it remains at rest ?

30. Calculate the tension in the string shown in figure (5-E13). The pulley and the string are light and all the surfaces are frictionless. Take $g = 10$ m/s$^2$.
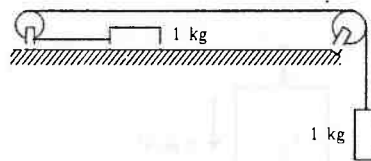


Figure 5-E13

31. Consider the situation shown in figure (5-E14). Both the pulleys and the string are light and all the surfaces are frictionless. (a) Find the acceleration of the mass $M$. (b) Find the tension in the string. (c) Calculate the force exerted by the clamp on the pulley $A$ in the figure.



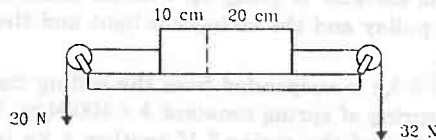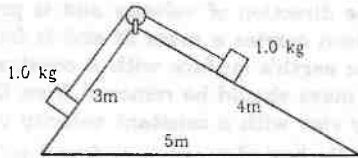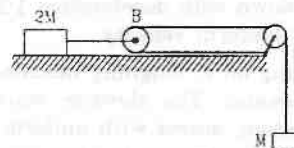Figure 5-E14

32. Find the acceleration of the block of mass $M$ in the situation shown in figure (5-E15). All the surfaces are frictionless and the pulleys and the string are light.
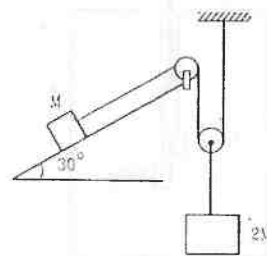


Figure 5-E15

33. Find the mass $M$ of the hanging block in figure (5-E16) which will prevent the smaller block from slipping over the triangular block. All the surfaces are frictionless and the strings and the pulleys are light.
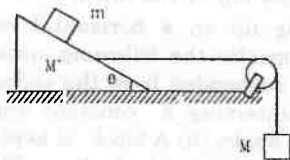


Figure 5-E16

34. Find the acceleration of the blocks $A$ and $B$ in the three situations shown in figure (5-E17).
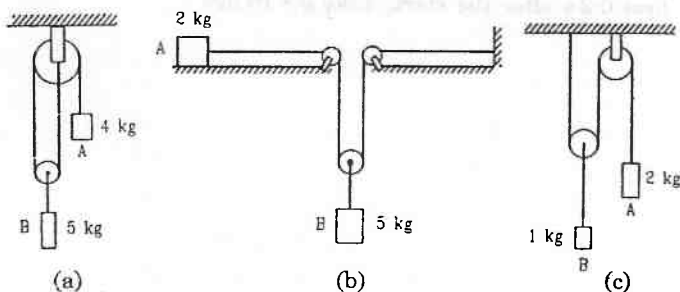


Figure 5-E17

35. Find the acceleration of the 500 g block in figure (5-E18).
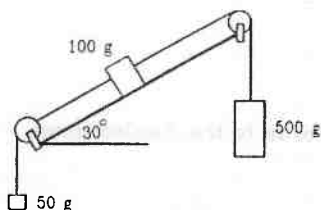


Figure 5-E18

36. A monkey of mass 15 kg is climbing on a rope with one end fixed to the ceiling. If it wishes to go up with an acceleration of 1 m/s$^2$, how much force should it apply to the rope? If the rope is 5 m long and the monkey starts from rest, how much time will it take to reach the ceiling?

37. A monkey is climbing on a rope that goes over a smooth light pulley and supports a block of equal mass at the other end (figure 5-E19). Show that whatever force the monkey exerts on the rope, the monkey and the block move in the same direction with equal acceleration. If initially both were at rest, their separation will not change as time passes.
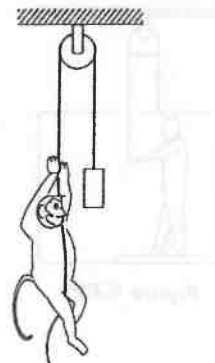


Figure 5-E19

38. The monkey $B$ shown in figure (5-E20) is holding on to the tail of the monkey $A$ which is climbing up a rope. The masses of the monkeys $A$ and $B$ are 5 kg and 2 kg respectively. If $A$ can tolerate a tension of 30 N in its tail, what force should it apply on the rope in order to carry the monkey $B$ with it? Take $g = 10$ m/s$^2$.
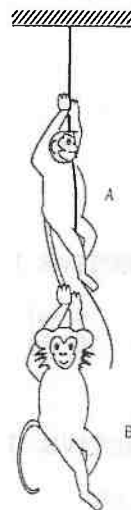


Figure 5-E20

39. Figure (5-E21) shows a man of mass 60 kg standing on a light weighing machine kept in a box of mass 30 kg. The box is hanging from a pulley fixed to the ceiling through a light rope, the other end of which is held by the man himself. If the man manages to keep the box at rest, what is the weight shown by the machine? What force should he exert on the rope to get his correct weight on the machine?
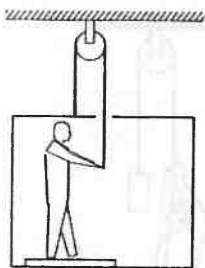
Figure 5-E21
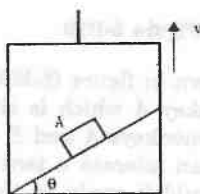


Figure 5-E22

40. A block $A$ can slide on a frictionless incline of angle $\theta$ and length $l$, kept inside an elevator going up with uniform velocity $v$ (figure 5-E22). Find the time taken by the block to slide down the length of the incline if it is released from the top of the incline.

41. A car is speeding up on a horizontal road with an acceleration $a$. Consider the following situations in the car. (i) A ball is suspended from the ceiling through a string and is maintaining a constant angle with the vertical. Find this angle. (ii) A block is kept on a smooth incline and does not slip on the incline. Find the angle of the incline with the horizontal.

42. A block is kept on the floor of an elevator at rest. The elevator starts descending with an acceleration of $12$ m/s$^2$. Find the displacement of the block during the first $0·2$ s after the start. Take $g = 10$ m/s$^2$.

□

# ANSWERS

### OBJECTIVE I

1. (c)    2. (b)    3. (d)    4. (a)    5. (c)    6. (b)
7. (d)    8. (a)    9. (b)    10. (b)    11. (c)    12. (b)
13. (a)    14. (c)

### OBJECTIVE II

1. (b), (d)    2. (c), (d)    3. (a), (b)
4. (a), (c)    5. (a)    6. (b), (c)
7. (c), (d)    8. (d)    9. (d)

### EXERCISES

1. 10 N
2. $3·1 \times 10^4$ N
3. $1·1 \times 10^{-15}$ N
4. 5 N and 3 N
5. $F/2$
6. $0·25$ N along the motion, zero and $0·25$ N opposite to the motion.
7. $F\left(1 + \dfrac{m_a}{m_B}\right)$
8. $1·8$ N

9. 10 m/s$^2$
10. $(k_1 + k_2)\dfrac{x}{m}$ opposite to the displacement.
11. $0·45$ s.
12. $\dfrac{mg}{4\,h}\sqrt{d^2 + 4\,h^2}$
13. 4 N
14. (a) $0·55$ N    (b) $0·43$ N    (c) $0·49$ N
     (d) $0·43$ N    (e) $0·55$ N    (f) $0·49$ N
15. 66 kg and $0·9$ m/s$^2$
16. $4·4$ kg
17. $0·2$ m, $0·1$ m
18. $0·24$ m, $0·12$ m
19. $2\left(M - \dfrac{B}{g}\right)$
20. $2$ m/5
21. $mg/A$
22. (a) $6·5$ m    (b) $3·9$ N    (c) $7·8$ N
23. $2/3$ s
24. 24 N
25. $g/10$

26. $\dfrac{m_2 g}{2(m_1 + m_2)}$  towards right

27. $4.3$ m/s$^2$, moves downward with acceleration
    $g + 0.2$ m/s$^2$

28. $\dfrac{19}{29} g \, (up)$, $\dfrac{17}{29} g \, (down)$, $\dfrac{21}{29} g \, (down)$, $0.25$ s

29. $4.8$ kg

30. $5$ N

31. (a) $2 g/3$          (b) $Mg/3$

    (c) $\sqrt{2} \, Mg/3$ at an angle  of $45°$ with the horizontal

32. $g/3$ up the plane

33. $\dfrac{M' + m}{\cot\theta - 1}$

34. (a) $\dfrac{2}{7} g$ downward, $\dfrac{g}{7}$ upward

    (b) $\dfrac{10}{13} g$ forward, $\dfrac{5}{13} g$ downward

    (c) $\dfrac{2}{3} g$ downward, $\dfrac{g}{3}$ upward

35. $\dfrac{8}{13} g$ downward

36. $165$ N, $\sqrt{10}$ s

38. between $70$ N and $105$ N

39. $15$ kg, $1800$ N

40. $\sqrt{\dfrac{2 l}{g \sin\theta}}$

41. $\tan^{-1}(a/g)$ in each case

42. $20$ cm

☐

*Solution* : (a) Take the 2 kg block as the system. The forces on this block are shown in figure (6-W16) with $M$ = 2 kg. It is assumed that $m$ has its minimum value so that the 2 kg block has a tendency to slip down. As the block is in equilibrium, the resultant force should be zero.



Figure 6-W16          Figure 6-W17
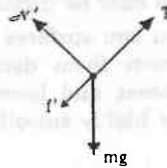
Taking components $\perp$ to the incline

$$\mathcal{N} = Mg \cos 45° = Mg/\sqrt{2}.$$

Taking components $\parallel$ to the incline

$$T + f = Mg \sin 45° = Mg/\sqrt{2}$$

or,

$$T = Mg/\sqrt{2} - f.$$

As it is a case of limiting equilibrium,

$$f = \mu_s \mathcal{N}$$

or,

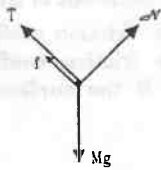$$T = \frac{Mg}{\sqrt{2}} - \mu_s \frac{Mg}{\sqrt{2}} = \frac{Mg}{\sqrt{2}} (1 - \mu_s). \quad \ldots \text{(i)}$$

Now consider the other block as the system. The forces acting on this block are shown in figure (6-W17).

Taking components $\perp$ to the incline,

$$\mathcal{N}' = mg \cos 45° = mg/\sqrt{2}.$$

Taking components $\parallel$ to the incline

$$T = mg \sin 45° + f' = \frac{mg}{\sqrt{2}} + f'.$$

As it is the case of limiting equilibrium

$$f' = \mu_s \mathcal{N}' = \mu_s \frac{mg}{\sqrt{2}}.$$

Thus,

$$T = \frac{mg}{\sqrt{2}} (1 + \mu_s). \quad \ldots \text{(ii)}$$

From (i) and (ii)

$$m(1 + \mu_s) = M (1 - \mu_s) \quad \ldots \text{(iii)}$$

or,

$$m = \frac{(1 - \mu_s)}{(1 + \mu_s)} M = \frac{1 - 0.28}{1 + 0.28} \times 2 \text{ kg}$$

$$= \frac{9}{8} \text{ kg}.$$

When maximum possible value of $m$ is supplied, the directions of friction are reversed because $m$ has the tendency to slip down and 2 kg block to slip up. Thus, the maximum value of $m$ can be obtained from (iii) by putting $\mu_s = -0.28$. Thus, the maximum value of $m$ is

$$m = \frac{1 + 0.28}{1 - 0.28} \times 2 \text{ kg}$$

$$= \frac{32}{9} \text{ kg}.$$

(b) If $m$ = 9/8 kg and the system is gently pushed, kinetic friction will operate. Thus,

$$f = \mu_k \cdot \frac{Mg}{\sqrt{2}} \quad \text{and} \quad f' = \frac{\mu_k \, mg}{\sqrt{2}},$$

where $\mu_k$ = 0.25. If the acceleration is $a$, Newton's second law for $M$ gives (figure 6-W16).

$$Mg \sin 45° - T - f = Ma$$

or,

$$\frac{Mg}{\sqrt{2}} - T - \frac{\mu_k Mg}{\sqrt{2}} = Ma. \quad \ldots \text{(iv)}$$

Applying Newton's second law $m$ (figure 6-W17),

$$T - mg \sin 45° - f' = ma$$

or,

$$T - \frac{mg}{\sqrt{2}} - \frac{\mu_k \, mg}{\sqrt{2}} = ma. \quad \ldots \text{(v)}$$

Adding (iv) and (v)

$$\frac{Mg}{\sqrt{2}} (1 - \mu_k) - \frac{mg}{\sqrt{2}} (1 + \mu_k) = (M + m) a$$

or,

$$a = \frac{M(1 - \mu_k) - m (1 + \mu_k)}{\sqrt{2} (M + m)} g$$

$$= \frac{2 \times 0.75 - 9/8 \times 1.25}{\sqrt{2} (2 + 9/8)} g$$

$$= 0.31 \text{ m/s}^2.$$

□

## QUESTIONS FOR SHORT ANSWER

1. For most of the surfaces used in daily life, the friction coefficient is less than 1. Is it always necessary that the friction coefficient is less than 1 ?

2. Why is it easier to push a heavy block from behind than to press it on the top and push ?

3. What is the average friction force when a person has a usual 1 km walk ?

4. Why is it difficult to walk on solid ice ?

5. Can you accelerate a car on a frictionless horizontal road by putting more petrol in the engine ? Can you stop a car going on a frictionless horizontal road by applying brakes ?

6. Spring fitted doors close by themselves when released. You want to keep the door open for a long time say for an hour. If you put a half kg stone in front of the open door, it does not help. The stone slides with the door and the door gets closed. However, if you sandwich a

20 g piece of wood in the small gap between the door and the floor, the door stays open. Explain why a much lighter piece of wood is able to keep the door open while the heavy stone fails.

7. A classroom demonstration of Newton's first law is as follows : A glass is covered with a plastic card and a coin is placed on the card. The card is given a quick strike and the coin falls in the glass. (a) Should the friction coefficient between the card and the coin be small or large ? (b) Should the coin be light or heavy ? (c) Why does the experiment fail if the card is gently pushed ?

8. Can a tug of war be ever won on a frictionless surface?

9. Why do tyres have a better grip of the road while going on a level road than while going on an incline ?

10. You are standing with your bag in your hands on the ice in the middle of a pond. The ice is so slippery that it can offer no friction. How can you come out of the ice?

11. When two surfaces are polished, the friction coefficient between them decreases. But the friction coefficient increases and becomes very large if the surfaces are made highly smooth. Explain.

# OBJECTIVE I

1. In a situation the contact force by a rough horizontal surface on a body placed on it has constant magnitude. If the angle between this force and the vertical is decreased, the frictional force between the surface and the body will
   (a) increase                (b) decrease
   (c) remain the same         (d) may increase or decrease.

2. While walking on ice, one should take small steps to avoid slipping. This is because smaller steps ensure
   (a) larger friction         (b) smaller friction
   (c) larger normal force      (d) smaller normal force.

3. A body of mass $M$ is kept on a rough horizontal surface (friction coefficient = $\mu$). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on $A$ is $F$ where
   (a) $F = Mg$                 (b) $F = \mu\, Mg$
   (c) $Mg < F < Mg\sqrt{1 + \mu^2}$   (d) $Mg > F > Mg\sqrt{1 - \mu^2}$.

4. A scooter starting from rest moves with a constant acceleration for a time $\Delta t_1$, then with a constant velocity for the next $\Delta t_2$ and finally with a constant deceleration for the next $\Delta t_3$ to come to rest. A 500 N man sitting on the scooter behind the driver manages to stay at rest with respect to the scooter without touching any other part. The force exerted by the seat on the man is
   (a) 500 N throughout the journey
   (b) less than 500 N throughout the journey
   (c) more than 500 N throughout the journey
   (d) > 500 N for time $\Delta t_1$ and $\Delta t_3$ and 500 N for $\Delta t_2$.

5. Consider the situation shown in figure (6-Q1). The wall is smooth but the surfaces of $A$ and $B$ in contact are rough. The friction on $B$ due to $A$ in equilibrium

   (a) is upward        (b) is downward        (c) is zero
   (d) the system cannot remain in equilibrium.

6. Suppose all the surfaces in the previous problem are rough. The direction of friction on $B$ due to $A$
   (a) is upward        (b) is downward        (c) is zero
   (d) depends on the masses of $A$ and $B$.

7. Two cars of unequal masses use similar tyres. If they are moving at the same initial speed, the minimum stopping distance
   (a) is smaller for the heavier car
   (b) is smaller for the lighter car
   (c) is same for both cars
   (d) depends on the volume of the car.

8. In order to stop a car in shortest distance on a horizontal road, one should
   (a) apply the brakes very hard so that the wheels stop rotating
   (b) apply the brakes hard enough to just prevent slipping
   (c) pump the brakes (press and release)
   (d) shut the engine off and not apply brakes.

9. A block $A$ kept on an inclined surface just begins to slide if the inclination is 30°. The block is replaced by another block $B$ and it is found that it just begins to slide if the inclination is 40°.
   (a) mass of $A$ > mass of $B$ (b) mass of $A$ < mass of $B$
   (c) mass of $A$ = mass of $B$ (d) all the three are possible.

10. A boy of mass $M$ is applying a horizontal force to slide a box of mass $M'$ on a rough horizontal surface. The coefficient of friction between the shoes of the boy and the floor is $\mu$ and that between the box and the floor is $\mu'$. In which of the following cases it is certainly not possible to slide the box ?
    (a) $\mu < \mu'$, $M < M'$        (b) $\mu > \mu'$, $M < M'$
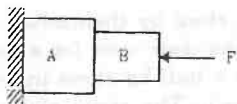    (c) $\mu < \mu'$, $M > M'$        (d) $\mu > \mu'$, $M > M'$.



Figure 6-Q1

## OBJECTIVE II

1. Let $F$, $F_N$ and $f$ denote the magnitudes of the contact force, normal force and the friction exerted by one surface on the other kept in contact. If none of these is zero,
   (a) $F > F_N$  (b) $F > f$  (c) $F_N > f$  (d) $F_N - f < F < F_N + f$.

2. The contact force exerted by a body $A$ on another body $B$ is equal to the normal force between the bodies. We conclude that
   (a) the surfaces must be frictionless
   (b) the force of friction between the bodies is zero
   (c) the magnitude of normal force equals that of friction
   (d) the bodies may be rough but they don't slip on each other.

3. Mark the correct statements about the friction between two bodies.
   (a) Static friction is always greater than the kinetic friction.
   (b) Coefficient of static friction is always greater than the coefficient of kinetic friction.
   (c) Limiting friction is always greater than the kinetic friction.
   (d) Limiting friction is never less than static friction.

4. A block is placed on a rough floor and a horizontal force $F$ is applied on it. The force of friction $f$ by the floor on the block is measured for different values of $F$ and a graph is plotted between them.
   (a) The graph is a straight line of slope 45°.
   (b) The graph is a straight line parallel to the $F$-axis.
   (c) The graph is a straight line of slope 45° for small $F$ and a straight line parallel to the $F$-axis for large $F$.
   (d) There is a small kink on the graph.

5. Consider a vehicle going on a horizontal road towards east. Neglect any force by the air. The frictional forces on the vehicle by the road
   (a) is towards east if the vehicle is accelerating
   (b) is zero if the vehicle is moving with a uniform velocity
   (c) must be towards east
   (d) must be towards west.

## EXERCISES

1. A body slipping on a rough horizontal plane moves with a deceleration of $4.0 \text{ m/s}^2$. What is the coefficient of kinetic friction between the block and the plane ?

2. A block is projected along a rough horizontal road with a speed of 10 m/s. If the coefficient of kinetic friction is 0.10, how far will it travel before coming to rest ?

3. A block of mass $m$ is kept on a horizontal table. If the static friction coefficient is $\mu$, find the frictional force acting on the block.

4. A block slides down an inclined surface of inclination 30° with the horizontal. Starting from rest it covers 8 m in the first two seconds. Find the coefficient of kinetic friction between the two.

5. Suppose the block of the previous problem is pushed down the incline with a force of 4 N. How far will the block move in the first two seconds after starting from rest ? The mass of the block is 4 kg.

6. A body of mass 2 kg is lying on a rough inclined plane of inclination 30°. Find the magnitude of the force parallel to the incline needed to make the block move (a) up the incline  (b) down the incline. Coefficient of static friction = 0.2.

7. Repeat part (a) of problem 6 if the push is applied horizontally and not parallel to the incline.

8. In a children-park an inclined plane is constructed with an angle of incline 45° in the middle part (figure 6-E1). Find the acceleration of a boy sliding on it if the friction coefficient between the cloth of the boy and the incline is 0.6 and $g = 10 \text{ m/s}^2$.
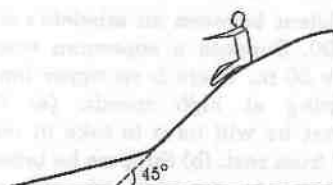


Figure 6-E1

9. A body starts slipping down an incline and moves half meter in half second. How long will it take to move the next half meter ?

10. The angle between the resultant contact force and the normal force exerted by a body on the other is called the *angle of friction*. Show that, if $\lambda$ be the angle of friction and $\mu$ the coefficient of static friction, $\lambda < \tan^{-1} \mu$.

11. Consider the situation shown in figure (6-E2). Calculate (a) the acceleration of the 1.0 kg blocks, (b) the tension in the string connecting the 1.0 kg blocks and (c) the tension in the string attached to 0.50 kg.
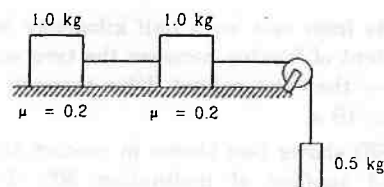


Figure 6-E2

12. If the tension in the string in figure (6-E3) is 16 N and the acceleration of each block is $0.5$ m/s$^2$, find the friction coefficients at the two contacts with the blocks.
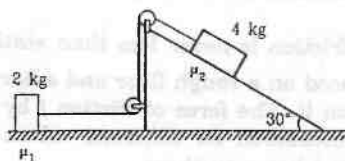


Figure 6-E3

13. The friction coefficient between the table and the block shown in figure (6-E4) is $0.2$. Find the tensions in the two strings.
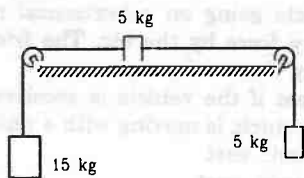


Figure 6-E4

14. The friction coefficient between a road and the tyre of a vehicle is 4/3. Find the maximum incline the road may have so that once hard brakes are applied and the wheel starts skidding, the vehicle going down at a speed of 36 km/hr is stopped within 5 m.

15. The friction coefficient between an athelete's shoes and the ground is $0.90$. Suppose a superman wears these shoes and races for 50 m. There is no upper limit on his capacity of running at high speeds. (a) Find the minimum time that he will have to take in completing the 50 m starting from rest. (b) Suppose he takes exactly this minimum time to complete the 50 m, what minimum time will he take to stop ?

16. A car is going at a speed of $21.6$ km/hr when it encounters a $12.8$ m long slope of angle 30° (figure 6-E5). The friction coefficient between the road and the tyre is $1/2\sqrt{3}$. Show that no matter how hard the driver applies the brakes, the car will reach the bottom with a speed greater than 36 km/hr. Take $g = 10$ m/s$^2$.
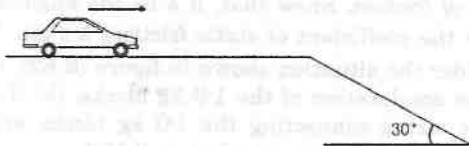


Figure 6-E5

17. A car starts from rest on a half kilometer long bridge. The coefficient of friction between the tyre and the road is $1.0$. Show that one cannot drive through the bridge in less than 10 s.

18. Figure (6-E6) shows two blocks in contact sliding down an inclined surface of inclination 30°. The friction coefficient between the block of mass $2.0$ kg and the incline is $\mu_1$, and that between the block of mass $4.0$ kg

and the incline is $\mu_2$. Calculate the acceleration of the $2.0$ kg block if (a) $\mu_1 = 0.20$ and $\mu_2 = 0.30$, (b) $\mu_1 = 0.30$ and $\mu_2 = 0.20$. Take $g = 10$ m/s$^2$.
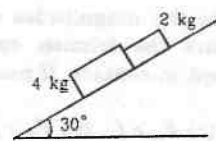


Figure 6-E6

19. Two masses $M_1$ and $M_2$ are connected by a light rod and the system is slipping down a rough incline of angle $\theta$ with the horizontal. The friction coefficient at both the contacts is $\mu$. Find the acceleration of the system and the force by the rod on one of the blocks.

20. A block of mass $M$ is kept on a rough horizontal surface. The coefficient of static friction between the block and the surface is $\mu$. The block is to be pulled by applying a force to it. What minimum force is needed to slide the block ? In which direction should this force act ?

21. The friction coefficient between the board and the floor shown in figure (6-E7) is $\mu$. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor.
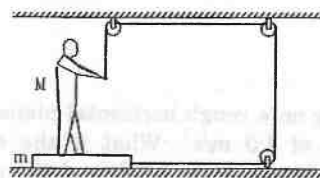


Figure 6-E7

22. A 2 kg block is placed over a 4 kg block and both are placed on a smooth horizontal surface. The coefficient of friction between the blocks is $0.20$. Find the acceleration of the two blocks if a horizontal force of 12 N is applied to (a) the upper block, (b) the lower block. Take $g = 10$ m/s$^2$.

23. Find the accelerations $a_1$, $a_2$, $a_3$ of the three blocks shown in figure (6-E8) if a horizontal force of 10 N is applied on (a) 2 kg block, (b) 3 kg block, (c) 7 kg block. Take $g = 10$ m/s$^2$.
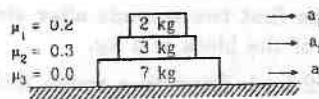


Figure 6-E8

24. The friction coefficient between the two blocks shown in figure (6-E9) is $\mu$ but the floor is smooth. (a) What maximum horizontal force $F$ can be applied without disturbing the equilibrium of the system ? (b) Suppose the horizontal force applied is double of that found in part (a). Find the accelerations of the two masses.
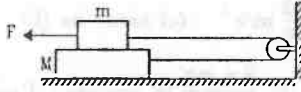
Figure 6-E9

25. Suppose the entire system of the previous question is kept inside an elevator which is coming down with an acceleration $a < g$. Repeat parts (a) and (b).

26. Consider the situation shown in figure (6-E9). Suppose a small electric field $E$ exists in the space in the vertically upward direction and the upper block carries a positive charge $Q$ on its top surface. The friction coefficient between the two blocks is $\mu$ but the floor is smooth. What maximum horizontal force $F$ can be applied without disturbing the equilibrium ?
[Hint : The force on a charge $Q$ by the electric field $E$ is $F = QE$ in the direction of $E$.]

27. A block of mass $m$ slips on a rough horizontal table under the action of a horizontal force applied to it. The coefficient of friction between the block and the table is $\mu$. The table does not move on the floor. Find the total frictional force applied by the floor on the legs of the table. Do you need the friction coefficient between the table and the floor or the mass of the table ?
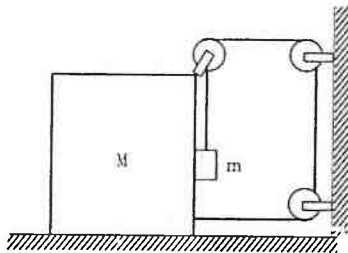


Figure 6-E10

28. Find the acceleration of the block of mass $M$ in the situation of figure (6-E10). The coefficient of friction

between the two blocks is $\mu_1$ and that between the bigger block and the ground is $\mu_2$.

29. A block of mass 2 kg is pushed against a rough vertical wall with a force of 40 N, coefficient of static friction being 0·5. Another horizontal force of 15 N, is applied on the block in a direction parallel to the wall. Will the block move ? If yes, in which direction ? If no, find the frictional force exerted by the wall on the block.

30. A person (40 kg) is managing to be at rest between two vertical walls by pressing one wall $A$ by his hands and feet and the other wall $B$ by his back (figure 6-E11). Assume that the friction coefficient between his body and the walls is 0·8 and that limiting friction acts at all the contacts. (a) Show that the person pushes the two walls with equal force. (b) Find the normal force exerted by either wall on the person. Take $g = 10$ m/s$^2$.



Figure 6-E11

31. Figure (6-E12) shows a small block of mass $m$ kept at the left end of a larger block of mass $M$ and length $l$. The system can slide on a horizontal road. The system is started towards right with an initial velocity $v$. The friction coefficient between the road and the bigger block is $\mu$ and that between the blocks is $\mu/2$. Find the time elapsed before the smaller block separates from the bigger block.
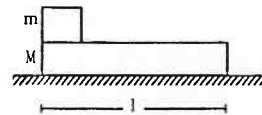


Figure 6-E12

□

ANSWERS

OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (b) | 2. (b) | 3. (c) | 4. (d) | 5. (d) | 6. (a) |
| 7. (c) | 8. (b) | 9. (d) | 10. (a) | | |

OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (b), (d) | 2. (b), (d) | 3. (b), (c), (d) |
| 4. (c), (d) | 5. (a), (b) | |

EXERCISES

1. 0·4
2. 50 m
3. zero
4. 0·11
5. 10 m
6. (a) 13 N (b) zero

7. 17·5 N

8. $2\sqrt{2}$ m/s$^2$

9. 0·21 s

11. (a) 0·4 m/s$^2$    (b) 2·4 N        (c) 4·8 N

12. $\mu_1 = 0.75$, $\mu_2 = 0.06$

13. 96 N in the left string and 68 N in the right

14. 16°

15. (a) $\frac{10}{3}$ s  (b) $\frac{10}{3}$ s

18. 2·7 m/s$^2$, 2·4 m/s$^2$

19. $a = g(\sin\theta - \mu\cos\theta)$, zero

20. $\dfrac{\mu\, mg}{\sqrt{1+\mu^2}}$ at an angle $\tan^{-1}\mu$ with the horizontal

21. $\dfrac{\mu\,(M+m)\,g}{1+\mu}$

22. (a) upper block 4 m/s$^2$, lower block 1 m/s$^2$
    (b) both blocks 2 m/s$^2$

23. (a) $a_1 = 3$ m/s$^2$, $a_2 = a_3 = 0.4$ m/s$^2$
    (b) $a_1 = a_2 = a_3 = \dfrac{5}{6}$ m/s$^2$    (c) same as (b)

24. (a) $2\mu\, mg$      (b) $\dfrac{2\mu\, mg}{M+m}$ in opposite directions

25. (a) $2\mu\, m\,(g-a)$      (b) $\dfrac{2\mu\, m\,(g-a)}{m+M}$

26. $2\mu\,(mg - QE)$

27. $\mu\, mg$

28. $\dfrac{[2m - \mu_2\,(M+m)]g}{M + m\,[5 + 2\,(\mu_1 - \mu_2)]}$

29. it will move at an angle of 53° with the 15 N force

30. (b) 250 N

31. $\sqrt{\dfrac{4\,M\,l}{(M+m)\,\mu\, g}}$
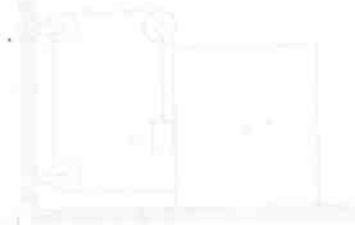
(c) weight $(\Delta m)g$ and

(d) normal force $\mathcal{N}$ by the table.

The tension at $A$ acts along the tangent at $A$ and the tension at $B$ acts along the tangent at $B$. As the small part $ACB$ moves in a circle of radius $R$ at a constant speed $v$, its acceleration is towards the centre (along $CO$) and has a magnitude $(\Delta m)v^2/R$.

Resolving the forces along the radius $CO$,

$$T\cos\left(90° - \frac{\Delta\theta}{2}\right) + T\cos\left(90° - \frac{\Delta\theta}{2}\right) = (\Delta m)\left(\frac{v^2}{R}\right)$$

or,           $$2T\sin\frac{\Delta\theta}{2} = (\Delta m)\left(\frac{v^2}{R}\right). \qquad \ldots \text{(i)}$$

The length of the part $ACB$ is $R\Delta\theta$. As the total mass of the ring is $m$, the mass of the part $ACB$ will be

$$\Delta m = \frac{m}{2\pi R}\,R\Delta\theta = \frac{m\Delta\theta}{2\pi}.$$

Putting $\Delta m$ in (i),

$$2T\sin\frac{\Delta\theta}{2} = \frac{m}{2\pi}\,\Delta\theta\left(\frac{v^2}{R}\right)$$

or,           $$T = \frac{mv^2}{2\pi R}\cdot\frac{\Delta\theta/2}{\sin(\Delta\theta/2)}.$$

As $\Delta\theta$ is very small, $\dfrac{\Delta\theta/2}{\sin(\Delta\theta/2)} \approx 1$ and $T = \dfrac{mv^2}{2\pi R}$.

13. *A table with smooth horizontal surface is turning at an angular speed $\omega$ about its axis. A groove is made on the surface along a radius and a particle is gently placed inside the groove at a distance $a$ from the centre. Find the speed of the particle with respect to the table as its distance from the centre becomes $L$.*

*Solution :* The situation is shown in figure (7-W10).
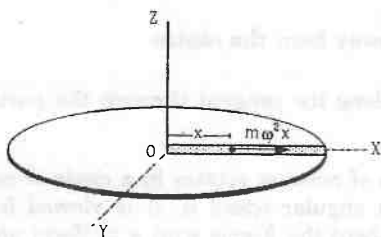


Figure 7-W10

Let us work from the frame of reference of the table. Let us take the origin at the centre of rotation $O$ and

the $X$-axis along the groove (figure 7-W10). The $Y$-axis is along the line perpendicular to $OX$, coplanar with the surface of the table and the $Z$-axis is along the vertical. Suppose at time $t$ the particle in the groove is at a distance $x$ from the origin and is moving along the $X$-axis with a speed $v$. The forces acting on the particle (including the pseudo forces that we must assume because we have taken our frame on the table which is rotating and is noninertial) are

(a) weight $mg$ vertically downward,

(b) normal contact force $\mathcal{N}_1$ vertically upward by the bottom surface of the groove,

(c) normal contact force $\mathcal{N}_2$ parallel to the $Y$-axis by the side walls of the groove,

(d) centrifugal force $m\omega^2 x$ along the $X$-axis, and

(e) coriolis force along $Y$-axis (coriolis force is perpendicular to the velocity of the particle and the axis of rotation.)

As the particle can only move in the groove, its acceleration is along the $X$-axis. The only force along the $X$-axis is the centrifugal force $m\omega^2 x$. All the other forces are perpendicular to the $X$-axis and have no components along the $X$-axis.

Thus the acceleration along the $X$-axis is

$$a = \frac{F}{m} = \frac{m\omega^2 x}{m} = \omega^2 x$$

or,           $$\frac{dv}{dt} = \omega^2 x$$

or,           $$\frac{dv}{dx}\cdot\frac{dx}{dt} = \omega^2 x$$

or,           $$\frac{dv}{dx}\cdot v = \omega^2 x$$

or,           $$v\,dv = \omega^2 x\,dx$$

or,           $$\int_0^v v\,dv = \int_a^L \omega^2 x\,dx$$

or,           $$\left[\frac{1}{2}v^2\right]_0^v = \left[\frac{1}{2}\omega^2 x^2\right]_a^L$$

or,           $$\frac{v^2}{2} = \frac{1}{2}\omega^2(L^2 - a^2)$$

or,           $$v = \omega\sqrt{L^2 - a^2}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. You are driving a motorcycle on a horizontal road. It is moving with a uniform velocity. Is it possible to accelerate the motorcycle without putting higher petrol input rate into the engine ?

2. Some washing machines have cloth driers. It contains a drum in which wet clothes are kept. As the drum rotates, the water particles get separated from the cloth. The general description of this action is that "the centrifugal force throws the water particles away from the drum". Comment on this statement from the viewpoint of an observer rotating with the drum and the observer who is washing the clothes.

3. A small coin is placed on a record rotating at $33\frac{1}{3}$ rev/minute. The coin does not slip on the record. Where does it get the required centripetal force from?

4. A bird while flying takes a left turn, where does it get the centripetal force from?

5. Is it necessary to express all angles in radian while using the equation $\omega = \omega_0 + \alpha t$?

6. After a good meal at a party you wash your hands and find that you have forgotten to bring your handkerchief. You shake your hands vigorously to remove the water as much as you can. Why is water removed in this process?

7. A smooth block loosely fits in a circular tube placed on a horizontal surface. The block moves in a uniform circular motion along the tube (figure 7-Q1). Which wall (inner or outer) will exert a nonzero normal contact force on the block?
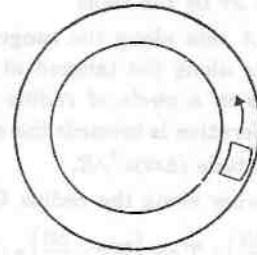


Figure 7-Q1

8. Consider the circular motion of the earth around the sun. Which of the following statements is more appropriate?
(a) Gravitational attraction of the sun on the earth is equal to the centripetal force.
(b) Gravitational attraction of the sun on the earth is the centripetal force.

9. A car driver going at some speed $v$ suddenly finds a wide wall at a distance $r$. Should he apply brakes or turn the car in a circle of radius $r$ to avoid hitting the wall?

10. A heavy mass $m$ is hanging from a string in equilibrium without breaking it. When this same mass is set into oscillation, the string breaks. Explain.

## OBJECTIVE I

1. When a particle moves in a circle with a uniform speed
(a) its velocity and acceleration are both constant
(b) its velocity is constant but the acceleration changes
(c) its acceleration is constant but the velocity changes
(d) its velocity and acceleration both change.

2. Two cars having masses $m_1$ and $m_2$ move in circles of radii $r_1$ and $r_2$ respectively. If they complete the circles in equal time, the ratio of their angular speeds $\omega_1/\omega_2$ is
(a) $m_1/m_2$     (b) $r_1/r_2$     (c) $m_1 r_1/m_2 r_2$     (d) 1.

3. A car moves at a constant speed on a road as shown in figure (7-Q2). The normal force by the road on the car is $N_A$ and $N_B$ when it is at the points $A$ and $B$ respectively.
(a) $N_A = N_B$  (b) $N_A > N_B$  (c) $N_A < N_B$  (d) insufficient information to decide the relation of $N_A$ and $N_B$.



Figure 7-Q2

4. A particle of mass $m$ is observed from an inertial frame of reference and is found to move in a circle of radius $r$ with a uniform speed $v$. The centrifugal force on it is
(a) $\dfrac{mv^2}{r}$ towards the centre
(b) $\dfrac{mv^2}{r}$ away from the centre
(c) $\dfrac{mv^2}{r}$ along the tangent through the particle
(d) zero.

5. A particle of mass $m$ rotates in a circle of radius $a$ with a uniform angular speed $\omega$. It is viewed from a frame rotating about the $Z$-axis with a uniform angular speed $\omega_0$. The centrifugal force on the particle is
(a) $m\omega^2 a$    (b) $m\omega_0^2 a$    (c) $m\left(\dfrac{\omega + \omega_0}{2}\right)^2 a$    (d) $m\omega\,\omega_0\, a$

6. A particle is kept fixed on a turntable rotating uniformly. As seen from the ground, the particle goes in a circle, its speed is 20 cm/s and acceleration is 20 cm/s$^2$. The particle is now shifted to a new position to make the radius half of the original value. The new values of the speed and acceleration will be
(a) 10 cm/s, 10 cm/s$^2$       (b) 10 cm/s, 80 cm/s$^2$
(c) 40 cm/s, 10 cm/s$^2$       (d) 40 cm/s, 40 cm/s$^2$.

7. Water in a bucket is whirled in a vertical circle with a string attached to it. The water does not fall down even when the bucket is inverted at the top of its path. We conclude that in this position

(a) $mg = \dfrac{mv^2}{r}$ \qquad (b) $mg$ is greater than $\dfrac{mv^2}{r}$

(c) $mg$ is not greater than $\dfrac{mv^2}{r}$

(d) $mg$ is not less than $\dfrac{mv^2}{r}$.

8. A stone of mass $m$ tied to a string of length $l$ is rotated in a circle with the other end of the string as the centre. The speed of the stone is $v$. If the string breaks, the stone will move
   (a) towards the centre \qquad (b) away from the centre
   (c) along a tangent \qquad (d) will stop.

9. A coin placed on a rotating turntable just slips if it is placed at a distance of 4 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of
   (a) 1 cm \qquad (b) 2 cm \qquad (c) 4 cm \qquad (d) 8 cm.

10. A motorcyle is going on an overbridge of radius $R$. The driver maintains a constant speed. As the motorcycle is ascending on the overbridge, the normal force on it
    (a) increases \qquad\qquad (b) decreases
    (c) remains the same \qquad (d) fluctuates.

11. Three identical cars $A$, $B$ and $C$ are moving at the same speed on three bridges. The car $A$ goes on a plane bridge, $B$ on a bridge convex upward and $C$ goes on a bridge concave upward. Let $F_A$, $F_B$ and $F_C$ be the normal forces exerted by the cars on the bridges when they are at the middle of bridges.
    (a) $F_A$ is maximum of the three forces.
    (b) $F_B$ is maximum of the three forces.
    (c) $F_C$ is maximum of the three forces.
    (d) $F_A = F_B = F_C$.

12. A train $A$ runs from east to west and another train $B$ of the same mass runs from west to east at the same speed along the equator. $A$ presses the track with a force $F_1$ and $B$ presses the track with a force $F_2$.
    (a) $F_1 > F_2$. \qquad (b) $F_1 < F_2$. \qquad (c) $F_1 = F_2$.
    (d) the information is insufficient to find the relation between $F_1$ and $F_2$.

13. If the earth stops rotating, the apparent value of $g$ on its surface will
    (a) increase everywhere
    (b) decrease everywhere
    (c) remain the same everywhere
    (d) increase at some places and remain the same at some other places.

14. A rod of length $L$ is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane. Let $T_1$ and $T_2$ be the tensions at the points $L/4$ and $3L/4$ away from the pivoted ends.
    (a) $T_1 > T_2$. (b) $T_2 > T_1$. (c) $T_1 = T_2$. (d) The relation between $T_1$ and $T_2$ depends on whether the rod rotates clockwise or anticlockwise.

15. A simple pendulum having a bob of mass $m$ is suspended from the ceiling of a car used in a stunt film shooting. The car moves up along an inclined cliff at a speed $v$ and makes a jump to leave the cliff and lands at some distance. Let $R$ be the maximum height of the car from the top of the cliff. The tension in the string when the car is in air is
    (a) $mg$ \qquad (b) $mg - \dfrac{mv^2}{R}$ \qquad (c) $mg + \dfrac{mv^2}{R}$ \qquad (d) zero.

16. Let $\theta$ denote the angular displacement of a simple pendulum oscillating in a vertical plane. If the mass of the bob is $m$, the tension in the string is $mg\cos\theta$
    (a) always \qquad\qquad\qquad (b) never
    (c) at the extreme positions
    (d) at the mean position.

## OBJECTIVE II

1. An object follows a curved path. The following quantities may remain constant during the motion
   (a) speed \qquad\qquad (b) velocity
   (c) acceleration \qquad (d) magnitude of acceleration.

2. Assume that the earth goes round the sun in a circular orbit with a constant speed of 30 km/s.
   (a) The average velocity of the earth from 1st Jan, 90 to 30th June, 90 is zero.
   (b) The average acceleration during the above period is 60 km/s$^2$.
   (c) The average speed from 1st Jan, 90 to 31st Dec, 90 is zero.
   (d) The instantaneous acceleration of the earth points towards the sun.

3. The position vector of a particle in a circular motion about the origin sweeps out equal area in equal time. Its
   (a) velocity remains constant
   (b) speed remains constant
   (c) acceleration remains constant
   (d) tangential acceleration remains constant.

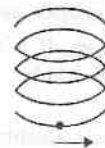4. A particle is going in a spiral path as shown in figure (7-Q3) with constant speed.



Figure 7-Q3

(a) The velocity of the particle is constant.
(b) The acceleration of the particle is constant.

(c) The magnitude of acceleration is constant.

(d) The magnitude of acceleration is decreasing continuously.

5. A car of mass $M$ is moving on a horizontal circular path of radius $r$. At an instant its speed is $v$ and is increasing at a rate $a$.

(a) The acceleration of the car is towards the centre of the path.

(b) The magnitude of the frictional force on the car is greater than $\dfrac{mv^2}{r}$.

(c) The friction coefficient between the ground and the car is not less than $a/g$.

(d) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\dfrac{v^2}{rg}$.

6. A circular road of radius $r$ is banked for a speed $v = 40$ km/hr. A car of mass $m$ attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible.

(a) The car cannot make a turn without skidding.

(b) If the car turns at a speed less than 40 km/hr, it will slip down.

(c) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is equal to $\dfrac{mv^2}{r}$.

(d) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is greater than $mg$ as well as greater than $\dfrac{mv^2}{r}$.

7. A person applies a constant force $\vec{F}$ on a particle of mass $m$ and finds that the particle moves in a circle of radius $r$ with a uniform speed $v$ as seen from an inertial frame of reference.

(a) This is not possible.

(b) There are other forces on the particle.

(c) The resultant of the other forces is $\dfrac{mv^2}{r}$ towards the centre.

(d) The resultant of the other forces varies in magnitude as well as in direction.

# EXERCISES

1. Find the acceleration of the moon with respect to the earth from the following data : Distance between the earth and the moon = $3\cdot85 \times 10^{\,\prime\prime}$ km and the time taken by the moon to complete one revolution around the earth = $27\cdot3$ days.

2. Find the acceleration of a particle placed on the surface of the earth at the equator due to earth's rotation. The diameter of earth = 12800 km and it takes 24 hours for the earth to complete one revolution about its axis.

3. A particle moves in a circle of radius $1\cdot0$ cm at a speed given by $v = 2\cdot0\,t$ where $v$ is in cm/s and $t$ in seconds.
(a) Find the radial acceleration of the particle at $t = 1$ s.
(b) Find the tangential acceleration at $t = 1$ s. (c) Find the magnitude of the acceleration at $t = 1$ s.

4. A scooter weighing 150 kg together with its rider moving at 36 km/hr is to take a turn of radius 30 m. What horizontal force on the scooter is needed to make the turn possible ?

5. If the horizontal force needed for the turn in the previous problem is to be supplied by the normal force by the road, what should be the proper angle of banking ?

6. A park has a radius of 10 m. If a vehicle goes round it at an average speed of 18 km/hr, what should be the proper angle of banking ?

7. If the road of the previous problem is horizontal (no banking), what should be the minimum friction coefficient so that a scooter going at 18 km/hr does not skid ?

8. A circular road of radius 50 m has the angle of banking equal to 30°. At what speed should a vehicle go on this road so that the friction is not used ?

9. In the Bohr model of hydrogen atom, the electron is treated as a particle going in a circle with the centre at the proton. The proton itself is assumed to be fixed in an inertial frame. The centripetal force is provided by the Coulomb attraction. In the ground state, the electron goes round the proton in a circle of radius $5\cdot3 \times 10^{-11}$ m. Find the speed of the electron in the ground state. Mass of the electron = $9\cdot1 \times 10^{-31}$ kg and charge of the electron = $1\cdot6 \times 10^{-19}$ C.

10. A stone is fastened to one end of a string and is whirled in a vertical circle of radius $R$. Find the minimum speed the stone can have at the highest point of the circle.

11. A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1500 at full speed. Consider a particle of mass 1 g sticking at the outer end of a blade. How much force does it experience when the fan runs at full speed ? Who exerts this force on the particle ? How much force does the particle exert on the blade along its surface ?

12. A mosquito is sitting on an L.P. record disc rotating on a turn table at $33\frac{1}{3}$ revolutions per minute. The distance of the mosquito from the centre of the turn table is 10 cm. Show that the friction coefficient between the record and the mosquito is greater than $\pi^2/81$. Take $g = 10$ m/s$^2$.

13. A simple pendulum is suspended from the ceiling of a car taking a turn of radius 10 m at a speed of 36 km/h. Find the angle made by the string of the pendulum with the vertical if this angle does not change during the turn. Take $g = 10$ m/s$^2$.

14. The bob of a simple pendulum of length 1 m has mass 100 g and a speed of 1·4 m/s at the lowest point in its path. Find the tension in the string at this instant.

15. Suppose the bob of the previous problem has a speed of 1·4 m/s when the string makes an angle of 0·20 radian with the vertical. Find the tension at this instant. You can use $\cos\theta \approx 1 - \theta^2/2$ and $\sin\theta \approx \theta$ for small $\theta$.

16. Suppose the amplitude of a simple pendulum having a bob of mass $m$ is $\theta_0$. Find the tension in the string when the bob is at its extreme position.

17. A person stands on a spring balance at the equator. (a) By what fraction is the balance reading less than his true weight ? (b) If the speed of earth's rotation is increased by such an amount that the balance reading is half the true weight, what will be the length of the day in this case ?

18. A turn of radius 20 m is banked for the vehicles going at a speed of 36 km/h. If the coefficient of static friction between the road and the tyre is 0·4, what are the possible speeds of a vehicle so that it neither slips down nor skids up ?

19. A motorcycle has to move with a constant speed on an overbridge which is in the form of a circular arc of radius $R$ and has a total length $L$. Suppose the motorcycle starts from the highest point. (a) What can its maximum velocity be for which the contact with the road is not broken at the highest point ? (b) If the motorcycle goes at speed $1/\sqrt{2}$ times the maximum found in part (a), where will it lose the contact with the road ? (c) What maximum uniform speed can it maintain on the bridge if it does not lose contact anywhere on the bridge ?

20. A car goes on a horizontal circular road of radius $R$, the speed increasing at a constant rate $\dfrac{dv}{dt} = a$. The friction coefficient between the road and the tyre is $\mu$. Find the speed at which the car will skid.

21. A block of mass $m$ is kept on a horizontal ruler. The friction coefficient between the ruler and the block is $\mu$. The ruler is fixed at one end and the block is at a distance $L$ from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end. (a) What can the maximum angular speed be for which the block does not slip ? (b) If the angular speed of the ruler is uniformly increased from zero at an angular acceleration $\alpha$, at what angular speed will the block slip ?

22. A track consists of two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in figure (7-E1). Each part subtends a right angle at its centre. A cycle weighing 100 kg together with the rider travels at a constant speed of 18 km/h on the track. (a) Find the normal contact force by the road on the cycle when it is at B and at D. (b) Find the force of friction exerted by the track on the tyres when the cycle is at B, C and D. (c) Find the normal force between the road and the cycle just before and just after the cycle crosses C. (d) What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed ? Take $g = 10 \text{ m/s}^2$.
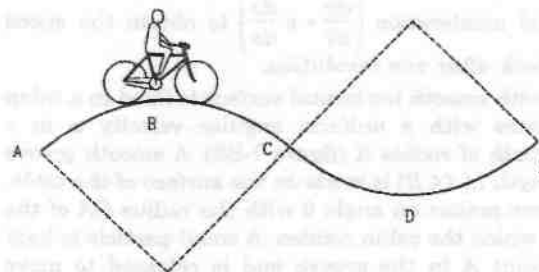


Figure 7-E1

23. In a children's park a heavy rod is pivoted at the centre and is made to rotate about the pivot so that the rod always remains horizontal. Two kids hold the rod near the ends and thus rotate with the rod (figure 7-E2). Let the mass of each kid be 15 kg, the distance between the points of the rod where the two kids hold it be 3·0 m and suppose that the rod rotates at the rate of 20 revolutions per minute. Find the force of friction exerted by the rod on one of the kids.
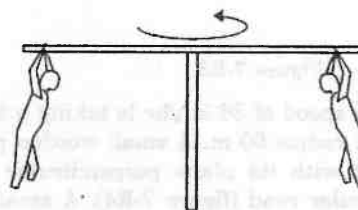


Figure 7-E2

24. A hemispherical bowl of radius $R$ is rotated about its axis of symmetry which is kept vertical. A small block is kept in the bowl at a position where the radius makes an angle $\theta$ with the vertical. The block rotates with the bowl without any slipping. The friction coefficient between the block and the bowl surface is $\mu$. Find the range of the angular speed for which the block will not slip.

25. A particle is projected with a speed $u$ at an angle $\theta$ with the horizontal. Consider a small part of its path near the highest position and take it approximately to be a circular arc. What is the radius of this circle ? This radius is called the radius of curvature of the curve at the point.

26. What is the radius of curvature of the parabola traced out by the projectile in the previous problem at a point where the particle velocity makes an angle $\theta/2$ with the horizontal ?

27. A block of mass $m$ moves on a horizontal circle against the wall of a cylindrical room of radius $R$. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is $\mu$. The block is given an initial speed $v_0$. As a function of the speed $v$ write (a) the normal force by the wall on the block, (b) the frictional force by the wall and (c) the tangential acceleration of the block. (d) Integrate the

tangential acceleration $\left(\dfrac{dv}{dt} = v\,\dfrac{dv}{ds}\right)$ to obtain the speed of the block after one revolution.

28. A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity ω in a circular path of radius $R$ (figure 7-E3). A smooth groove $AB$ of length $L( \ll R)$ is made on the surface of the table. The groove makes an angle θ with the radius $OA$ of the circle in which the cabin rotates. A small particle is kept at the point $A$ in the groove and is released to move along $AB$. Find the time taken by the particle to reach the point $B$.
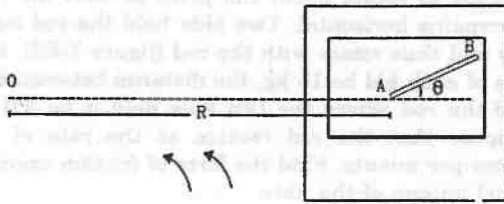


Figure 7-E3

29. A car moving at a speed of 36 km/hr is taking a turn on a circular road of radius 50 m. A small wooden plate is kept on the seat with its plane perpendicular to the radius of the circular road (figure 7-E4). A small block of mass 100 g is kept on the seat which rests against the plate. The friction coefficient between the block and the plate is μ = 0·58. (a) Find the normal contact force exerted by the plate on the block. (b) The plate is slowly turned so that the angle between the normal to the plate and the radius of the road slowly increases. Find the angle at which the block will just start sliding on the plate.

□
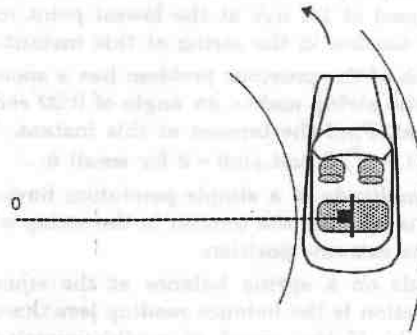


Figure 7-E4

30. A table with smooth horizontal surface is placed in a cabin which moves in a circle of a large radius $R$ (figure 7-E5). A smooth pulley of small radius is fastened to the table. Two masses $m$ and $2m$ placed on the table are connected through a string going over the pulley. Initially the masses are held by a person with the strings along the outward radius and then the system is released from rest (with respect to the cabin). Find the magnitude of the initial acceleration of the masses as seen from the cabin and the tension in the string.
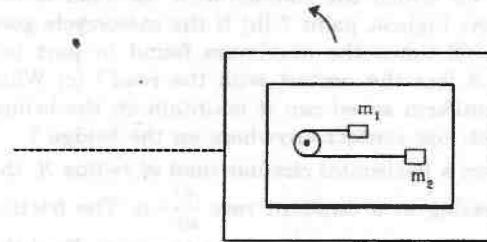


Figure 7-E5

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (d) | 2. (d) | 3. (c) | 4. (d) | 5. (b) | 6. (a) |
| 7. (c) | 8. (c) | 9. (a) | 10. (a) | 11. (c) | 12. (a) |
| 13. (d) | 14. (a) | 15. (d) | 16. (c) | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (d) | 2. (d) | 3. (b), (d) |
| 4. (c) | 5. (b), (c) | 6. (b), (d) |
| 7. (b), (d) | | |

### EXERCISES

1. $2·73 \times 10^{-3} \text{ m/s}^2$

2. $0·0336 \text{ m/s}^2$

3. (a) $4·0 \text{ cm/s}^2$    (b) $2·0 \text{ cm/s}^2$    (c) $\sqrt{20} \text{ cm/s}^2$

4. 500 N

5. $\tan^{-1}(1/3)$

6. $\tan^{-1}(1/4)$

7. 0·25

8. 17 m/s

9. $2·2 \times 10^6 \text{ m/s}$

10. $\sqrt{Rg}$

11. 14·8 N, 14·8 N

13. 45°

14. 1·2 N

15. $1.16$ N

16. $mg \cos\theta_0$

17. (a) $3.5 \times 10^{-3}$                    (b) $2.0$ hour

18. Between $14.7$ km/h and $54$ km/hr

19. (a) $\sqrt{Rg}$,
    (b) a distance $\pi R/3$ along the bridge from the highest point,
    (c) $\sqrt{gR\cos(L/2R)}$

20. $\left[(\mu^2 g^2 - a^2)R^2\right]^{1/4}$

21. (a) $\sqrt{\mu g/L}$                    (b) $\left[\left(\dfrac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$

22. (a) $975$ N, $1025$ N           (b) $0$, $707$ N, $0$
    (c) $682$ N, $732$ N            (d) $1037$

23. $10\,\pi^2$

24. $\left[\dfrac{g(\sin\theta - \mu\cos\theta)}{R\,\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{1/2}$ to $\left[\dfrac{g(\sin\theta + \mu\cos\theta)}{R\,\sin\theta(\cos\theta - \mu\sin\theta)}\right]^{1/2}$

25. $\dfrac{u^2\cos^2\theta}{g}$

26. $\dfrac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$

27. (a) $\dfrac{mv^2}{R}$         (b) $\dfrac{\mu mv^2}{R}$         (c) $-\dfrac{\mu v^2}{R}$         (d) $v_0 e^{-2\pi\mu}$

28. $\sqrt{\dfrac{2L}{\omega^2 R\cos\theta}}$

29. (a) $0.2$ N                    (b) $30°$

30. $\dfrac{\omega^2 R}{3}$, $\dfrac{4}{3}m\omega^2 R$.

or,                 $mg + \mathcal{N} = 2\,mgh/R.$

For $h$ to be minimum, $\mathcal{N}$ should assume the minimum value which can be zero. Thus,

$$2\,mg\,\frac{h_{min}}{R} = mg \quad \text{or,} \quad h_{min} = R/2.$$

**14.** *A heavy particle is suspended by a string of length l. The particle is given a horizontal velocity $v_0$. The string becomes slack at some angle and the particle proceeds on a parabola. Find the value of $v_0$ if the particle passes through the point of suspension.*
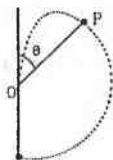


Figure 8-W10

**Solution :** Suppose the string becomes slack when the particle reaches the point $P$ (figure 8-W10). Suppose the string $OP$ makes an angle $\theta$ with the upward vertical. The only force acting on the particle at $P$ is its weight $mg$. The radial component of the force is $mg\cos\theta$. As the particle moves on the circle upto $P$,

$$mg\cos\theta = m\left(\frac{v^2}{l}\right)$$

or,                 $v^2 = g\,l\cos\theta$                 ... (i)

where $v$ is its speed at $P$. Using conservation of energy

$$\frac{1}{2}m\,v_0^2 = \frac{1}{2}m\,v^2 + mgl\,(1 + \cos\theta)$$

or,         $v^2 = v_0^2 - 2\,g\,l\,(1 + \cos\theta)$         ... (ii)

From (i) and (ii), $v_0^2 - 2\,g\,l\,(1 + \cos\theta) = g\,l\cos\theta$

or,                 $v_0^2 = g\,l\,(2 + 3\cos\theta).$                 ... (iii)

Now onwards the particle goes in a parabola under the action of gravity. As it passes through the point of suspension $O$, the equations for horizontal and vertical motions give,

$$l\sin\theta = (v\cos\theta)\,t$$

and         $-l\cos\theta = (v\sin\theta)\,t - \frac{1}{2}g\,t^2$

or,         $-l\cos\theta = (v\sin\theta)\left(\frac{l\sin\theta}{v\cos\theta}\right) - \frac{1}{2}g\left(\frac{l\sin\theta}{v\cos\theta}\right)^2$

or,         $-\cos^2\theta = \sin^2\theta - \frac{1}{2}g\,\frac{l\sin^2\theta}{v^2\cos\theta}$

or,         $-\cos^2\theta = 1 - \cos^2\theta - \frac{1}{2}\frac{g\,l\sin^2\theta}{g\,l\cos^2\theta}$   [From (i)]

or,                 $1 = \frac{1}{2}\tan^2\theta$

or,                 $\tan\theta = \sqrt{2}.$

From (iii), $v_0 = [g\,l\,(2 + \sqrt{3})]^{1/2}.$

## QUESTIONS FOR SHORT ANSWER

1. When you lift a box from the floor and put it on an almirah the potential energy of the box increases, but there is no change in its kinetic energy. Is it a violation of conservation of energy ?

2. A particle is released from the top of an incline of height $h$. Does the kinetic energy of the particle at the bottom of the incline depend on the angle of incline ? Do you need any more information to answer this question in Yes or No ?

3. Can the work by kinetic friction on an object be positive ? Zero ?

4. Can static friction do nonzero work on an object ? If yes, give an example. If no, give reason.

5. Can normal force do a nonzero work on an object. If yes, give an example. If no, give reason.

6. Can kinetic energy of a system be increased without applying any external force on the system ?

7. Is work-energy theorem valid in noninertial frames ?

8. A heavy box is kept on a smooth inclined plane and is pushed up by a force $F$ acting parallel to the plane. Does the work done by the force $F$ as the box goes from $A$ to $B$ depend on how fast the box was moving at $A$ and $B$ ? Does the work by the force of gravity depend on this ?

9. One person says that the potential energy of a particular book kept in an almirah is 20 J and the other says it is 30 J. Is one of them necessarily wrong ?

10. A book is lifted from the floor and is kept in an almirah. One person says that the potential energy of the book is increased by 20 J and the other says it is increased by 30 J. Is one of them necessarily wrong ?

11. In one of the exercises to strengthen the wrist and fingers, a person squeezes and releases a soft rubber ball. Is the work done on the ball positive, negative or zero during compression ? During expansion ?

12. In tug of war, the team that exerts a larger tangential force on the ground wins. Consider the period in which a team is dragging the opposite team by applying a

larger tangential force on the ground. List which of the following works are positive, which are negative and which are zero ?
(a) work by the winning team on the losing team
(b) work by the losing team on the winning team
(c) work by the ground on the winning team
(d) work by the ground on the losing team
(e) total external work on the two teams.

13. When an apple falls from a tree what happens to its gravitational potential energy just as it reaches the ground ? After it strikes the ground ?

14. When you push your bicycle up on an incline the potential energy of the bicyle and yourself increases. Where does this energy come from ?

15. The magnetic force on a charged particle is always perpendicular to its velocity. Can the magnetic force change the velocity of the particle ? Speed of the particle ?

16. A ball is given a speed $v_0$ on a rough horizontal surface. The ball travels through a distance $l$ on the surface and stops. (a) What are the initial and final kinetic energies of the ball ? (b) What is the work done by the kinetic friction ?

17. Consider the situation of the previous question from a frame moving with a speed $v_0$ parallel to the initial velocity of the block. (a) What are the initial and final kinetic energies ? (b) What is the work done by the kinetic friction ?

## OBJECTIVE I

1. A heavy stone is thrown from a cliff of height $h$ with a speed $v$. The stone will hit the ground with maximum speed if it is thrown
(a) vertically downward
(b) vertically upward
(c) horizontally
(d) the speed does not depend on the initial direction.

2. Two springs $A$ and $B(k_A = 2k_B)$ are stretched by applying forces of equal magnitudes at the four ends. If the energy stored in $A$ is $E$, that in $B$ is
(a) $E/2$
(b) $2E$
(c) $E$
(d) $E/4$.

3. Two equal masses are attached to the two ends of a spring of spring constant $k$. The masses are pulled out symmetrically to stretch the spring by a length $x$ over its natural length. The work done by the spring on each mass is
(a) $\frac{1}{2}kx^2$,
(b) $-\frac{1}{2}kx^2$
(c) $\frac{1}{4}kx^2$
(d) $-\frac{1}{4}kx^2$.

4. The negative of the work done by the conservative internal forces on a system equals the change in
(a) total energy
(b) kinetic energy
(c) potential energy
(d) none of these.

5. The work done by the external forces on a system equals the change in
(a) total energy
(b) kinetic energy
(c) potential energy
(d) none of these.

6. The work done by all the forces (external and internal) on a system equals the change in

(a) total energy
(b) kinetic energy
(c) potential energy
(d) none of these.

7. _____ of a two particle system depends only on the separation between the two particles. The most appropriate choice for the blank space in the above sentence is
(a) kinetic energy
(b) total mechanical energy
(c) potential energy
(d) total energy.

8. A small block of mass $m$ is kept on a rough inclined surface of inclination $\theta$ fixed in an elevator. The elevator goes up with a uniform velocity $v$ and the block does not slide on the wedge. The work done by the force of friction on the block in time $t$ will be
(a) zero
(b) $mgvt \cos^2\theta$
(c) $mgvt \sin^2\theta$
(d) $mgvt \sin 2\theta$.

9. A block of mass $m$ slides down a smooth vertical circular track. During the motion, the block is in
(a) vertical equilibrium
(b) horizontal equilibrium
(c) radial equilibrium
(d) none of these.

10. A particle is rotated in a vertical circle by connecting it to a string of length $l$ and keeping the other end of the string fixed. The minimum speed of the particle when the string is horizontal for which the particle will complete the circle is
(a) $\sqrt{gl}$
(b) $\sqrt{2gl}$
(c) $\sqrt{3gl}$
(d) $\sqrt{5gl}$.

## OBJECTIVE II

1. A heavy stone is thrown from a cliff of height $h$ in a given direction. The speed with which it hits the ground
(a) must depend on the speed of projection
(b) must be larger than the speed of projection

(c) must be independent of the speed of projection
(d) may be smaller than the speed of projection.

2. The total work done on a particle is equal to the change in its kinetic energy
(a) always

(b) only if the forces acting on it are conservative

(c) only if gravitational force alone acts on it

(d) only if elastic force alone acts on it.

3. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. If follows that

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

(d) it moves in a circular path.

4. Consider two observers moving with respect to each other at a speed $v$ along a straight line. They observe a block of mass $m$ moving a distance $l$ on a rough surface. The following quantities will be same as observed by the two observers

(a) kinetic energy of the block at time $t$

(b) work done by friction

(c) total work done on the block

(d) acceleration of the block.

5. You lift a suitcase from the floor and keep it on a table. The work done by you on the suitcase does not depend on

(a) the path taken by the suitcase

(b) the time taken by you in doing so

(c) the weight of the suitcase

(d) your weight.

6. No work is done by a force on an object if

(a) the force is always perpendicular to its velocity

(b) the force is always perpendicular to its acceleration

(c) the object is stationary but the point of application of the force moves on the object

(d) the object moves in such a way that the point of application of the force remains fixed.

7. A particle of mass $m$ is attached to a light string of length $l$, the other end of which is fixed. Initially the string is kept horizontal and the particle is given an upward velocity $v$. The particle is just able to complete a circle.

(a) The string becomes slack when the particle reaches its highest point.

(b) The velocity of the particle becomes zero at the highest point.

(c) The kinetic energy of the ball in initial position was $\frac{1}{2} mv^2 = mgl$.

(d) The particle again passes through the intial position.

8. The kinetic energy of a particle continuously increases with time.

(a) The resultant force on the particle must be parallel to the velocity at all instants.

(b) The resultant force on the particle must be at an angle less than 90° all the time.

(c) Its height above the ground level must continuously decrease.

(d) The magnitude of its linear momentum is increasing continuously.

9. One end of a light spring of spring constant $k$ is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2} kx^2$. The possible cases are

(a) the spring was initially compressed by a distance $x$ and was finally in its natural length

(b) it was initially streched by a distance $x$ and finally was in its natural length

(c) it was initially in its natural length and finally in a compressed position

(d) it was initially in its natural length and finally in a stretched position.

10. A block of mass $M$ is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F. The kinetic energy of the block increases by 20 J in 1 s.

(a) The tension in the string is $Mg$.

(b) The tension in the string is $F$.

(c) The work done by the tension on the block is 20 J in the above 1 s.

(d) The work done by the force of gravity is −20 J in the above 1 s.

## EXERCISES

1. The mass of cyclist together with the bike is 90 kg. Calculate the increase in kinetic energy if the speed increases from 6·0 km/h to 12 km/h.

2. A block of mass 2·00 kg moving at a speed of 10·0 m/s accelerates at 3·00 m/s² for 5·00 s. Compute its final kinetic energy.

3. A box is pushed through 4·0 m across a floor offering 100 N resistance. How much work is done by the resisting force ?

4. A block of mass 5·0 kg slides down an incline of inclination 30° and length 10 m. Find the work done by the force of gravity.

5. A constant force of 2·50 N accelerates a stationary particle of mass 15 g through a displacement of 2·50 m. Find the work done and the average power delivered.

6. A particle moves from a point $\vec{r}_1 = (2\text{ m})\,\vec{i} + (3\text{ m})\,\vec{j}$ to another point $\vec{r}_2 = (3\text{ m})\,\vec{i} + (2\text{ m})\,\vec{j}$ during which a certain force $\vec{F} = (5\text{ N})\,\vec{i} + (5\text{ N})\,\vec{j}$ acts on it. Find the work done by the force on the particle during the displacement.

7. A man moves on a straight horizontal road with a block of mass 2 kg in his hand. If he covers a distance of 40 m with an acceleration of 0·5 m/s², find the work done by the man on the block during the motion.

8. A force $F = a + bx$ acts on a particle in the $x$ direction where $a$ and $b$ are constants. Find the work done by this force during a displacement from $x - 0$ to $x = d$.

9. A block of mass $250\,g$ slides down an incline of inclination 37° with a uniform speed. Find the work done against the friction as the block slides through $1·0\,m$.

10. A block of mass $m$ is kept over another block of mass $M$ and the system rests on a horizontal surface (figure 8-E1). A constant horizontal force $F$ acting on the lower block produces an acceleration $\dfrac{F}{2(m+M)}$ in the system, the two blocks always move together. (a) Find the coefficient of kinetic friction between the bigger block and the horizontal surface. (b) Find the frictional force acting on the smaller block. (c) Find the work done by the force of friction on the smaller block by the bigger block during a displacement $d$ of the system.
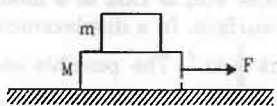


Figure 8-E1

11. A box weighing 2000 N is to be slowly slid through 20 m on a straight track having friction coefficient 0·2 with the box. (a) Find the work done by the person pulling the box with a chain at an angle θ with the horizontal. (b) Find the work when the person has chosen a value of θ which ensures him the minimum magnitude of the force.

12. A block of weight 100 N is slowly slid up on a smooth incline of inclination 37° by a person. Calculate the work done by the person in moving the block through a distance of 2·0 m, if the driving force is (a) parallel to the incline and (b) in the horizontal direction.

13. Find the average frictional force needed to stop a car weighing 500 kg in a distance of 25 m if the initial speed is 72 km/h.

14. Find the average force needed to accelerate a car weighing 500 kg from rest to 72 km/h in a distance of 25 m.

15. A particle of mass $m$ moves on a straight line with its velocity varying with the distance travelled according to the equation $v = a\sqrt{x}$, where $a$ is a constant. Find the total work done by all the forces during a displacement from $x = 0$ to $x = d$.

16. A block of mass $2·0\,kg$ kept at rest on an inclined plane of inclination 37° is pulled up the plane by applying a constant force of 20 N parallel to the incline. The force acts for one second. (a) Show that the work done by the applied force does not exceed 40 J. (b) Find the work done by the force of gravity in that one second if the work done by the applied force is 40 J. (c) Find the kinetic energy of the block at the instant the force ceases to act. Take $g = 10\,m/s^2$.

17. A block of mass $2·0\,kg$ is pushed down an inclined plane of inclination 37° with a force of 20 N acting parallel to the incline. It is found that the block moves on the incline with an acceleration of $10\,m/s^2$. If the block

started from rest, find the work done (a) by the applied force in the first second, (b) by the weight of the block in the first second and (c) by the frictional force acting on the block in the first second. Take $g = 10\,m/s^2$.

18. A 250 g block slides on a rough horizontal table. Find the work done by the frictional force in bringing the block to rest if it is initially moving at a speed of 40 cm/s. If the friction coefficient between the table and the block is 0·1, how far does the block move before coming to rest?

19. Water falling from a 50 m high fall is to be used for generating electric energy. If $1·8 \times 10^5\,kg$ of water falls per hour and half the gravitational potential energy can be converted into electric energy, how many 100 W lamps can be lit?

20. A person is painting his house walls. He stands on a ladder with a bucket containing paint in one hand and a brush in other. Suddenly the bucket slips from his hand and falls down on the floor. If the bucket with the paint had a mass of 6·0 kg and was at a height of 2·0 m at the time it slipped, how much gravitational potential energy is lost together with the paint?

21. A projectile is fired from the top of a 40 m high cliff with an initial speed of 50 m/s at an unknown angle. Find its speed when it hits the ground.

22. The 200 m free style women's swimming gold medal at Seol Olympic 1988 went to Heike Friedrich of East Germany when she set a new Olympic record of 1 minute and 57·56 seconds. Assume that she covered most of the distance with a uniform speed and had to exert 460 W to maintain her speed. Calculate the average force of resistance offered by the water during the swim.

23. The US athlete Florence Griffith-Joyner won the 100 m sprint gold medal at Seol Olympic 1988 setting a new Olympic record of 10·54 s. Assume that she achieved her maximum speed in a very short time and then ran the race with that speed till she crossed the line. Take her mass to be 50 kg. (a) Calculate the kinetic energy of Griffith-Joyner at her full speed. (b) Assuming that the track, the wind etc. offered an average resistance of one tenth of her weight, calculate the work done by the resistance during the run. (c) What power Griffith-Joyner had to exert to maintain uniform speed?

24. A water pump lifts water from a level 10 m below the ground. Water is pumped at a rate of 30 kg/minute with negligible velocity. Calculate the minimum horsepower the engine should have to do this.

25. An unruly demonstrator lifts a stone of mass 200 g from the ground and throws it at his opponent. At the time of projection, the stone is 150 cm above the ground and has a speed of 3·00 m/s. Calculate the work done by the demonstrator during the process. If it takes one second for the demonstrator to lift the stone and throw, what horsepower does he use?

26. In a factory it is desired to lift 2000 kg of metal through a distance of 12 m in 1 minute. Find the minimum horsepower of the engine to be used.

27. A scooter company gives the following specifications about its product.

Weight of the scooter — 95 kg
Maximum speed — 60 km/h
Maximum engine power — 3·5 hp
Pick up time to get the maximum speed — 5 s
Check the validity of these specifications.

28. A block of mass 30·0 kg is being brought down by a chain. If the block acquires a speed of 40·0 cm/s in dropping down 2·00 m, find the work done by the chain during the process.

29. The heavier block in an Atwood machine has a mass twice that of the lighter one. The tension in the string is 16·0 N when the system is set into motion. Find the decrease in the gravitational potential energy during the first second after the system is released from rest.

30. The two blocks in an Atwood machine have masses 2·0 kg and 3·0 kg. Find the work done by gravity during the fourth second after the system is released from rest.

31. Consider the situation shown in figure (8-E2). The system is released from rest and the block of mass 1·0 kg is found to have a speed 0·3 m/s after it has descended through a distance of 1 m. Find the coefficient of kinetic friction between the block and the table.
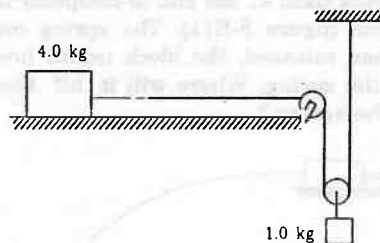


Figure 8-E2

32. A block of mass 100 g is moved with a speed of 5·0 m/s at the highest point in a closed circular tube of radius 10 cm kept in a vertical plane. The cross-section of the tube is such that the block just fits in it. The block makes several oscillations inside the tube and finally stops at the lowest point. Find the work done by the tube on the block during the process.

33. A car weighing 1400 kg is moving at a speed of 54 km/h up a hill when the motor stops. If it is just able to reach the destination which is at a height of 10 m above the point, calculate the work done against friction (negative of the work done by the friction).

34. A small block of mass 200 g is kept at the top of a frictionless incline which is 10 m long and 3·2 m high. How much work was required (a) to lift the block from the ground and put it at the top, (b) to slide the block up the incline ? What will be the speed of the block when it reaches the ground if (c) it falls off the incline and drops vertically on the ground (d) it slides down the incline ? Take $g = 10$ m/s$^2$.

35. In a children's park, there is a slide which has a total length of 10 m and a height of 8·0 m (figure 8-E3). A vertical ladder is provided to reach the top. A boy weighing 200 N climbs up the ladder to the top of the slide and slides down to the ground. The average friction offered by the slide is three tenth of his weight. Find (a) the work done by the ladder on the boy as he goes up, (b) the work done by the slide on the boy as he comes down. (c) Find the work done by forces inside the body of the boy.



Figure 8-E3

36. Figure (8-E4) shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point $A$, how far away from the track will the particle hit the ground ?
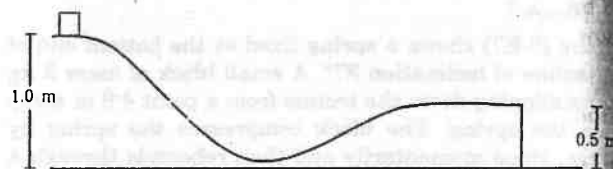


Figure 8-E4

37. A block weighing 10 N travels down a smooth curved track $AB$ joined to a rough horizontal surface (figure 8-E5). The rough surface has a friction coefficient of 0·20 with the block. If the block starts slipping on the track from a point 1·0 m above the horizontal surface, how far will it move on the rough surface ?
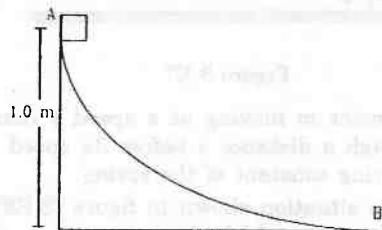


Figure 8-E5

38. A uniform chain of mass $m$ and length $l$ overhangs a table with its two third part on the table. Find the work to be done by a person to put the hanging part back on the table.

39. A uniform chain of length $L$ and mass $M$ overhangs a horizontal table with its two third part on the table. The friction coefficient between the table and the chain is $\mu$. Find the work done by the friction during the period the chain slips off the table.

40. A block of mass 1 kg is placed at the point $A$ of a rough track shown in figure (8-E6). If slightly pushed towards right, it stops at the point $B$ of the track. Calculate the work done by the frictional force on the block during its transit from $A$ to $B$.

Figure 8-E6

41. A block of mass 5·0 kg is suspended from the end of a vertical spring which is stretched by 10 cm under the load of the block. The block is given a sharp impulse from below so that it acquires an upward speed of 2·0 m/s. How high will it rise ? Take $g$ = 10 m/s$^2$.

42. A block of mass 250 g is kept on a vertical spring of spring constant 100 N/m fixed from below. The spring is now compressed to have a length 10 cm shorter than its natural length and the system is released from this position. How high does the block rise ? Take $g$ = 10 m/s$^2$.

43. Figure (8-E7) shows a spring fixed at the bottom end of an incline of inclination 37°. A small block of mass 2 kg starts slipping down the incline from a point 4·8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance of 1 m up the incline. Find (a) the friction coefficient between the plane and the block and (b) the spring constant of the spring. Take $g$ = 10 m/s$^2$.
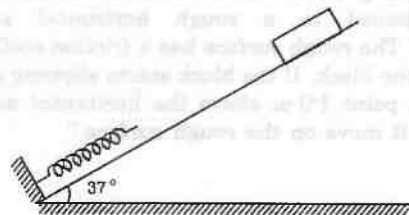


Figure 8-E7

44. A block of mass $m$ moving at a speed $v$ compresses a spring through a distance $x$ before its speed is halved. Find the spring constant of the spring.

45. Consider the situation shown in figure (8-E8). Initially the spring is unstretched when the system is released from rest. Assuming no friction in the pulley, find the maximum elongation of the spring.
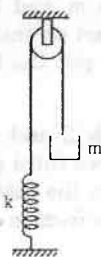


Figure 8-E8

46. A block of mass $m$ is attached to two unstretched springs of spring constants $k_1$ and $k_2$ as shown in figure (8-E9). The block is displaced towards right through a distance

$x$ and is released. Find the speed of the block as it passes through the mean position shown.
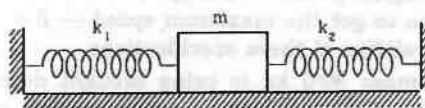


Figure 8-E9

47. A block of mass $m$ sliding on a smooth horizontal surface with a velocity $v$ meets a long horizontal spring fixed at one end and having spring constant $k$ as shown in figure (8-E10). Find the maximum compression of the spring. Will the velocity of the block be the same as $v$ when it comes back to the original position shown ?
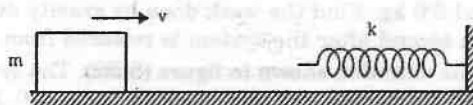


Figure 8-E10

48. A small block of mass 100 g is pressed against a horizontal spring fixed at one end to compress the spring through 5·0 cm (figure 8-E11). The spring constant is 100 N/m. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring ?
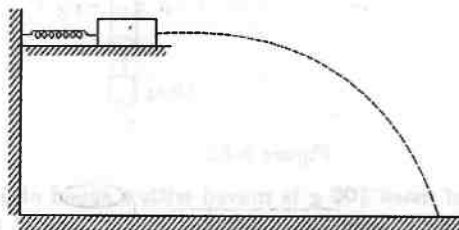


Figure 8-E11

49. A small heavy block is attached to the lower end of a light rod of length $l$ which can be rotated about its clamped upper end. What minimum horizontal velocity should the block be given so that it moves in a complete vertical circle ?
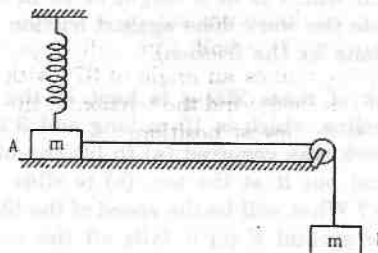
50. Figure (8-E12) shows two blocks $A$ and $B$, each having a mass of 320 g connected by a light string passing over a smooth light pulley. The horizontal surface on which the block $A$ can slide is smooth. The block $A$ is attached



Figure 8-E12

to a spring of spring constant 40 N/m whose other end is fixed to a support 40 cm above the horizontal surface. Initially, the spring is vertical and unstretched when the system is released to move. Find the velocity of the block A at the instant it breaks off the surface below it. Take $g = 10 \text{ m/s}^2$.

51. One end of a spring of natural length h and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height h (figure 8-E13). Initially, the spring makes an angle of 37° with the vertical when the system is released from rest. Find the speed of the ring when the spring becomes vertical.
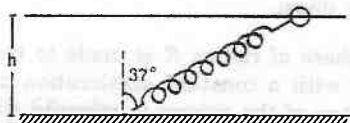


Figure 8-E13

52. Figure (8-E14) shows a light rod of length l rigidly attached to a small heavy block at one end and a hook at the other end. The system is released from rest with the rod in a horizontal position. There is a fixed smooth ring at a depth h below the initial position of the hook and the hook gets into the ring as it reaches there. What should be the minimum value of h so that the block moves in a complete circle about the ring ?
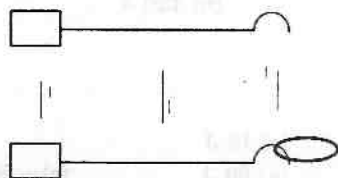


Figure 8-E14

53. The bob of a pendulum at rest is given a sharp hit to impart a horizontal velocity $\sqrt{10\,gl}$ where l is the length of the pendulum. Find the tension in the string when (a) the string is horizontal, (b) the bob is at its highest point and (c) the string makes an angle of 60° with the upward vertical.

54. A simple pendulum consists of a 50 cm long string connected to a 100 g ball. The ball is pulled aside so that the string makes an angle of 37° with the vertical and is then released. Find the tension in the string when the bob is at its lowest position.



Figure 8-E15

55. Figure (8-E15) shows a smooth track, a part of which is a circle of radius R. A block of mass m is pushed against a spring of spring constant k fixed at the left end and

is then released. Find the initial compression of the spring so that the block presses the track with a force mg when it reaches the point P, where the radius of the track is horizontal.

56. The bob of a stationary pendulum is given a sharp hit to impart it a horizontal speed of $\sqrt{3\,gl}$. Find the angle rotated by the string before it becomes slack.

57. A heavy particle is suspended by a 1·5 m long string. It is given a horizontal velocity of $\sqrt{57}$ m/s. (a) Find the angle made by the string with the upward vertical, when it becomes slack. (b) Find the speed of the particle at this instant. (c) Find the maximum height reached by the particle over the point of suspension. Take $g = 10 \text{ m/s}^2$.

58. A simple pendulum of length L having a bob of mass m is deflected from its rest position by an angle θ and released (figure 8-E16). The string hits a peg which is fixed at a distance x below the point of suspension and the bob starts going in a circle centred at the peg. (a) Assuming that initially the bob has a height less than the peg, show that the maximum height reached by the bob equals its initial height. (b) If the pendulum is released with θ = 90° and x = L/2 find the maximum height reached by the bob above its lowest position before the string becomes slack. (c) Find the minimum value of x/L for which the bob goes in a complete circle about the peg when the pendulum is released from θ = 90°.
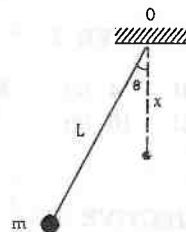


Figure 8-E16

59. A particle slides on the surface of a fixed smooth sphere starting from the topmost point. Find the angle rotated by the radius through the particle, when it leaves contact with the sphere.

60. A particle of mass m is kept on a fixed, smooth sphere of radius R at a position where the radius through the particle makes an angle of 30° with the vertical. The particle is released from this position. (a) What is the force exerted by the sphere on the particle just after the release ? (b) Find the distance travelled by the particle before it leaves contact with the sphere.

61. A particle of mass m is kept on the top of a smooth sphere of radius R. It is given a sharp impulse which imparts it a horizontal speed v. (a) Find the normal force between the sphere and the particle just after the impulse. (b) What should be the minimum value of v for which the particle does not slip on the sphere? (c) Assuming the velocity v to be half the minimum calculated in part (b), find the angle made by the radius

through the particle with the vertical when it leaves the sphere.
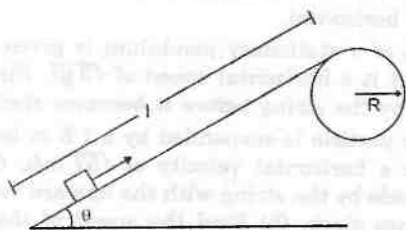


Figure 8-E17

62. Figure (8-E17) shows a smooth track which consists of a straight inclined part of length $l$ joining smoothly with the circular part. A particle of mass $m$ is projected up the incline from its bottom. (a) Find the minimum projection-speed $v_0$ for which the particle reaches the top of the track. (b) Assuming that the projection-speed is $2v_0$ and that the block does not lose contact with the track before reaching its top, find the force acting on it when it reaches the top. (c) Assuming that the projection-speed is only slightly greater than $v_0$, where will the block lose contact with the track?

63. A chain of length $l$ and mass $m$ lies on the surface of a smooth sphere of radius $R > l$ with one end tied to the top of the sphere. (a) Find the gravitational potential energy of the chain with reference level at the centre of the sphere. (b) Suppose the chain is released and slides down the sphere. Find the kinetic energy of the chain, when it has slid through an angle $\theta$. (c) Find the tangential acceleration $\dfrac{dv}{dt}$ of the chain when the chain starts sliding down.

64. A smooth sphere of radius $R$ is made to translate in a straight line with a constant acceleration $a$. A particle kept on the top of the sphere is released from there at zero velocity with respect to the sphere. Find the speed of the particle with respect to the sphere as a function of the angle $\theta$ it slides.

□

# ANSWERS

## OBJECTIVE I

1. (d)  2. (b)  3. (d)  4. (c)  5. (a)  6. (b)
7. (c)  8. (c)  9. (d)  10. (c)

## OBJECTIVE II

1. (a), (b)    2. (a)         3. (c), (d)
4. (d)         5. (a), (b), (d)  6. (a), (c), (d)
7. (a), (d)    8. (b), (d)    9. (a), (b)
10. (b)

## EXERCISES

1. 375 J
2. 625 J
3. 400 J
4. 245 J
5. 6·25 J, 36·1 W
6. zero
7. 40 J
8. $\left\{a + \dfrac{1}{2}bd\right\}d$
9. 1·5 J
10. (a) $\dfrac{F}{2(M+m)g}$  (b) $\dfrac{mF}{2(M+m)}$  (c) $\dfrac{mFd}{2(M+m)}$
11. (a) $\dfrac{40000\ J}{5 + \tan\theta}$  (b) 7690 J

12. (a) 120 J    (b) 120 J
13. 4000 N
14. 4000 N
15. $ma^2 d/2$
16. (b) $-24$ J    (c) 16 J
17. (a) 100 J    (b) 60 J    (c) $-60$ J
18. $-0·02$ J, 8·2 cm
19. 122
20. 118 J
21. 58 m/s
22. 270 N
23. (a) 2250 J    (b) $-4900$ J    (c) 465 W
24. $6·6 \times 10^{-1}$ hp
25. 3·84 J, $5·14 \times 10^{-3}$ hp
26. 5·3 hp
27. Seems to be somewhat overclaimed.
28. $-586$ J
29. 19·6 J
30. 67 J
31. 0·12
32. $-1·45$ J
33. 20300 J
34. (a) 6·4 J    (b) 6·4 J    (c) 8·0 m/s    (d) 8·0 m/s
35. (a) zero    (b) $-600$ J    (c) 1600 J
36. At a horizontal distance of 1 m from the end of the track.
37. 5·0 m

38. $mgl/18$

39. $-2\,\mu\,MgL/9$

40. $-2$ J

41. 20 cm

42. 20 cm

43. (a) 0·5        (b) 1000 N/m

44. $\dfrac{3\,mv^2}{4\,x^2}$

45. $2\,mg/k$

46. $\sqrt{\dfrac{k_1+k_2}{m}}\;x$

47. $v\,\sqrt{m/k}$, No

48. At a horizontal distance of 1 m from the free end of the spring.

49. $2\,\sqrt{gl}$

50. 1·5 m/s

51. $\dfrac{h}{4}\,\sqrt{k/m}$

52. $i$

53. (a) $8\,mg$      (b) $5\,mg$      (c) $6·5\,mg$

54. 1·4 N

55. $\sqrt{\dfrac{3\,mg\,R}{k}}$

56. $\cos^{-1}(-1/3)$

57. (a) $53°$      (b) 3·0 m/s      (c) 1·2 m

58. (b) $5L/6$ above the lowest point     (c) 0·6

59. $\cos^{-1}(2/3)$

60. $\sqrt{3}\,mg/2$      (b) 0·43 $R$

61. (a) $mg-\dfrac{mv^2}{R}$      (b) $\sqrt{Rg}$      (c) $\cos^{-1}(3/4)$

62. (a) $\sqrt{2\,g\,[R(1-\cos\theta)+l\sin\theta]}$   (b) $6\,mg\left(1-\cos\theta+\dfrac{l}{R}\sin\theta\right)$

     (c) The radius through the particle makes an angle $\cos^{-1}(2/3)$ with the vertical.

63. (a) $\dfrac{mR^2g}{l}\sin(l/R)$

     (b) $\dfrac{mR^2g}{l}\left[\sin\left(\dfrac{l}{R}\right)+\sin\theta-\sin\left(\theta+\dfrac{l}{R}\right)\right]$

     (c) $\dfrac{Rg}{l}[1-\cos(l/R)]$

64. $[2\,R(a\sin\theta+g-g\cos\theta)]^{1/2}$

□

Figure 9-W1,

Let $N$ = magnitude of the contact force between the particle and the pan

$T$ = tension in the string
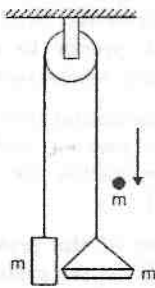
Consider the impulse imparted to the particle. The force is $N$ in upward direction and the impulse is $\int N\,dt$. This should be equal to the change in its momentum.

Thus,                    $\int N\,dt = mv - mV$.                    ...(i)

Similarly considering the impulse imparted to the pan,

$$\int (N - T)dt = mV$$                    ... (ii)

and that to the block,

$$\int T\,dt = mV.$$                    ... (iii)

Adding (ii) and (iii),

$$\int N\,dt = 2\,mV.$$

Comparing with (i),

$$mv - mV = 2mV$$

or,                         $V = v/3.$

□

## QUESTIONS FOR SHORT ANSWER

1. Can the centre of mass of a body be at a point outside the body ?

2. If all the particles of a system lie in $X$-$Y$ plane, is it necessary that the centre of mass be in $X$-$Y$ plane ?

3. If all the particle of a system lie in a cube, is it neccesary that the centre of mass be in the cube ?

4. The centre of mass is defined as $\vec{R} = \dfrac{1}{M} \sum_i m_i \vec{r_i}$. Suppose we define "centre of charge" as $\vec{R_c} = \dfrac{1}{Q} \sum_i q_i \vec{r_i}$ where $q_i$ represents the $i$th charge placed at $\vec{r_i}$ and $Q$ is the total charge of the system.
   (a) can the centre of charge of a two-charge system be outside the line segment joining the charges.
   (b) If all the charges of a system are in $X$-$Y$ plane, is it necessary that the centre of charge be in $X$-$Y$ plane ?
   (c) If all the charges of a system lie in a cube, is it necessary that the centre of charge be in the cube ?

5. The weight $Mg$ of an extended body is generally shown in a diagram to act through the centre of mass. Does it mean that the earth does not attract other particles ?

6. A bob suspended from the ceiling of a car which is accelerating on a horizontal road. The bob stays at rest with respect to the car with the string making an angle $\theta$ with the vertical. The linear momentum of the bob as seen from the road is increasing with time. Is it a violation of conservation of linear momentum ? If not, where is the external force which changes the linear momentum ?

7. You are waiting for a train on a railway platform. Your three year old niece is standing on your iron trunk containing the luggage. Why does the trunck not recoil as she jumps off as on the platform ?

8. In a head-on collision between two particles, is it necessary that the particles will acquire a common velocity at least for one instant ?

9. A collision experiment is done on a horizontal table kept in an elevator. Do you expect a change in the results if the elevator is accelerated up or down because of the noninertial character of the frame ?

10. Two bodies make an elastic head-on collision on a smooth horizontal table kept in a car. Do you expect a change in the result if the car is accelerated on a horizontal road because of the noninertial character of the frame ? Does the equation "Velocity of separation = Velocity of approach" remain valid in an accelerating car ? Does the equation "final momentum = initial momentum" remain valid in the accelerating car ?

11. If the total mechanical energy of a particle is zero, is its linear momentum necessarily zero ? Is it necessarily nonzero ?

12. If the linear momentum of a paricle is known, can you find its kinetic energy ? If the kinetic energy of a particle is known can you find its linear momentum ?

13. What can be said about the centre of mass of a uniform hemisphere without making any calculation ? Will its distance from the centre be more than $r/2$ or less than $r/2$ ?

14. You are holding a cage containing a bird. Do you have to make less effort if the bird flies from its position in the cage and manages to stay in the middle without touching the walls of the cage ? Does it make a difference whether the cage is completely closed or it has rods to let air pass ?

15. A fat person is standing on a light plank floating on a calm lake. The person walks from one end to the other on the plank. His friend sitting on the shore watches him and finds that the person hardly moves any distance because the plank moves backward about the same distance as the person moves on the plank. Explain.

16. A high-jumper successfully clears the bar. Is it possible that his centre of mass crossed the bar from below it. Try it with appropriate figures.

17. Which of the two persons shown in figure (9-Q1) is more likely to fall down ? Which external force is responsible for his falling down ?
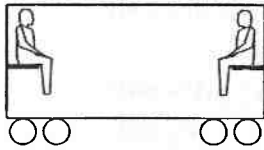


Figure 9-Q1

18. Suppose we define a quantity 'Linear Pomentum' as
    linear pomentum = mass × speed.
The linear pomentum of a system of particles is the sum of linear pomenta of the individual particles. Can we state a principle of conservation of linear pomentum as "linear pomentum of a system remains constant if no external force acts on it" ?

19. Use the definition of linear pomentum from the previous question. Can we state the principle of conservation of linear pomentum for a single particle ?

20. To accelerate a car we ignite petrol in the engine of the car. Since only an external force can accelerate the centre of mass, is it proper to say that "the force generated by the engine accelerates the car" ?

21. A ball is moved on a horizontal table with some velocity. The ball stops after moving some distance. Which external force is responsible for the change in the momentum of the ball ?

22. Consider the situation of the previous problem. Take "the table plus the ball" as the system. Friction between the table and the ball is then an internal force. As the ball slows down, the momentum of the system decreases. Which external force is responsible for this change in the momentum ?

23. When a nucleus at rest emits a beta particle, it is found that the velocities of the recoiling nucleus and the beta particle are not along the same straight line. How can this be possible in view of the principle of conservation of momentum ?

24. A van is standing on a frictionless portion of a horizontal road. To start the engine, the vehicle must be set in motion in the forward direction. How can the persons sitting inside the van do it without coming out and pushing from behind ?

25. In one-dimensional elastic collision of equal masses, the velocities are interchanged. Can velocities in a one-dimensional collision be interchanged if the masses are not equal ?

## OBJECTIVE I

1. Consider the following two equations :

$$(A) \quad \vec{R} = \frac{1}{M} \sum_i m_i \vec{r_i}$$

and $$(B) \quad \vec{a}_{CM} = \frac{\vec{F}}{M}.$$

In a noninertial frame
(a) both are correct
(b) both are wrong
(c) A is correct but B is wrong
(d) B is correct but A is wrong.

2. Consider the following two statements :
(A) Linear momentum of the system remains constant.
(B) Centre of mass of the system remains at rest.
   (a) A implies B and B implies A.
   (b) A does not imply B and B does not imply A.
   (c) A implies B but B does not imply A.
   (d) B implies A but A does not imply B.

3. Consider the following two statements :
(A) Linear momentum of a system of particles is zero.
(B) Kinetic energy of a system of particles is zero.
   (a) A implies B and B implies A.
   (b) A does not imply B and B does not imply A.
   (c) A implies B but B does not imply A.
   (d) B implies A but A does not imply B.

4. Consider the following two statements :
(A) The linear momentum of a particle is independent of the frame of reference.
(B) The kinetic energy of a particle is independent of the frame of reference.
(a) Both A and B are true.   (b) A is true but B is false.
(c) A is false but B is true.   (d) both A and B are false.

5. All the particles of a body are situated at a distance R from the origin. The distance of the centre of mass of the body from the origin is
(a) = R        (b) < R        (c) > R        (d) ≥ R.

6. A circular plate of diameter d is kept in contact with a square plate of edge d as shown in figure (9-Q2). The density of the material and the thickness are same
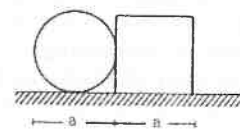


Figure 9-Q2

everywhere. The centre of mass of the composite system will be
(a) inside the circular plate   (b) inside the square plate
(c) at the point of contact     (d) outside the system.

7. Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration $\vec{a}$. The centre of mass has an acceleration
   (a) zero   (b) $\frac{1}{2}\vec{a}$   (c) $\vec{a}$   (d) $2\vec{a}$.

8. Internal forces can change
   (a) the linear momentum but not the kinetic energy
   (b) the kinetic energy but not the linear momentum
   (c) linear momentum as well as kinetic energy
   (d) neither the linear momentum nor the kinetic energy.

9. A bullet hits a block kept at rest on a smooth horizontal surface and gets embedded into it. Which of the following does not change ?
   (a) linear momentum of the block
   (b) kinetic energy of the block
   (c) gravitational potential energy of the block
   (d) temperature of the block.

10. A uniform sphere is placed on a smooth horizontal surface and a horizontal force $F$ is applied on it at a distance $h$ above the surface. The acceleration of the centre
    (a) is maximum when $h = 0$
    (b) is maximum when $h = R$
    (c) is maximum when $h = 2R$
    (d) is independent of $h$.

11. A body falling vertically downwards under gravity breaks in two parts of unequal masses. The centre of mass of the two parts taken together shifts horizontally towards
    (a) heavier piece            (b) lighter piece
    (c) does not shift horizontally
    (d) depends on the vertical velocity at the time of breaking.

12. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The velocity of the centre of mass
    (a) of the box remains constant
    (b) of the box plus the ball system remains constant
    (c) of the ball remains constant
    (d) of the ball relative to the box remains constant.

13. A body at rest breaks into two pieces of equal masses. The parts will move
    (a) in same direction          (b) along different lines
    (c) in opposite directions with equal speeds
    (d) in opposite directions with unequal speeds.

14. A heavy ring of mass $m$ is clamped on the periphery a light circular disc. A small particle having equal mass is clamped at the centre of the disc. The system is rotated in such a way that the centre moves in a circle of radius $r$ with a uniform speed $v$. We conclude that an external force
    (a) $\frac{mv^2}{r}$ must be acting on the central particle
    (b) $\frac{2mv^2}{r}$ must be acting on the central particle
    (c) $\frac{2mv^2}{r}$ must be acting on the system
    (d) $\frac{2mv^2}{r}$ must be acting on the ring.

15. The quantities remaining constant in a collision are
    (a) momentum, kinetic energy and temperature
    (b) momentum and kinetic energy but not temperature
    (c) momentum and temperature but not kinetic energy
    (d) momentum, but neither kinetic energy nor temperature.

16. A nucleus moving with a velocity $\vec{v}$ emits an $\alpha$-particle. Let the velocities of the $\alpha$-particle and the remaining nucleus be $\vec{v}_1$ and $\vec{v}_2$ and their masses be $m_1$ and $m_2$.
    (a) $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ must be parallel to each other.
    (b) None of the two of $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ should be parallel to each other.
    (c) $\vec{v}_1 + \vec{v}_2$ must be parallel to $\vec{v}$.
    (d) $m_1 \vec{v}_1 + m_2 \vec{v}_2$ must be parallel to $\vec{v}$.

17. A shell is fired from a cannon with a velocity $V$ at an angle $\theta$ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces retraces its path to the cannon. The speed of the other piece immediately after the explosion is
    (a) $3V\cos\theta$   (b) $2V\cos\theta$   (c) $\frac{3}{2}V\cos\theta$   (d) $V\cos\theta$.

18. In an elastic collision
    (a) the initial kinetic energy is equal to the final kinetic energy
    (b) the final kinetic energy is less than the initial kinetic energy
    (c) the kinetic energy remains constant
    (d) the kinetic energy first increases then decreases.

19. In an inelastic collsion
    (a) the initial kinetic energy is equal to the final kinetic energy
    (b) the final kinetic energy is less than the initial kinetic energy
    (c) the kinetic energy remains the constant
    (d) the kinetic energy first increases then decreases.

## OBJECTIVE II

1. The centre of mass of a system of particles is at the origin. It follows that
   (a) the number of particles to the right of the origin is equal to the number of particles to the left
   (b) the total mass of the particles to the right of the origin is same as the total mass to the left of the origin

(c) the number of particles on $X$-axis should be equal to the number of particles on $Y$-axis

(d) if there is a particle on the positive $X$-axis, there must be at least one particle on the negative $X$-axis.

2. A body has its centre of mass at the origin. The $x$-coordinates of the particles
   (a) may be all positive     (b) may be all negative
   (c) may be all non-negative
   (d) may be positive for some cass and negative in other cases

3. In which of the following cases the centre of mass of a rod is certainly not at its centre ?
   (a) The density continuously increases from left to right
   (b) The density continuously decreases from left to right
   (c) The density decreases from left to right upto the centre and then increases.
   (d) The density increases from left to right upto the centre and then decreases.

4. If the external forces acting on a system have zero resultant, the centre of mass
   (a) must not move     (b) must not accelerate
   (c) may move     (d) may accelerate.

5. A nonzero external force acts on a system of particles. The velocity and the acceleration of the centre of mass are found to be $v_0$ and $a_0$ at an instant $t$. It is possible that
   (a) $v_0 = 0$, $a_0 = 0$     (b) $v_0 = 0$, $a_0 \neq 0$,
   (c) $v_0 \neq 0$, $a_0 = 0$     (d) $v_0 \neq 0$, $a_0 \neq 0$.

6. Two balls are thrown simultaneously in air. The acceleration of the centre of mass of the two balls while in air
   (a) depends on the direction of the motion of the balls
   (b) depends on the masses of the two balls
   (c) depends on the speeds of the two balls
   (d) is equal to $g$.

7. A block moving in air breaks in two parts and the parts separate

(a) the total momentum must be conserved
(b) the total kinetic energy must be conserved
(c) the total momentum must change
(d) the total kinetic energy must change.

8. In an elastic collision
   (a) the kinetic energy remains constant
   (b) the linear momentum remains constant
   (c) the final kinetic energy is equal to the initial kinetic energy
   (d) the final linear momentum is equal to the initial linear momentum.

9. A ball hits a floor and rebounds after an inelastic collision. In this case
   (a) the momentum of the ball just after the collision is same as that just before the collision
   (b) the mechanical energy of the ball remains the same during the collision
   (c) the total momentum of the ball and the earth is conserved
   (d) the total energy of the ball and the earth remains the same.

10. A body moving towards a finite body at rest collides with it. It is possible that
    (a) both the bodies come to rest
    (b) both the bodies move after collision
    (c) the moving body comes to rest and the stationary body starts moving
    (d) the stationary body remains stationary, the moving body changes its velocity.

11. In head on elastic collision of two bodies of equal masses
    (a) the velocities are interchanged
    (b) the speeds are interchanged
    (c) the momenta are interchanged
    (d) the faster body slows down and the slower body speeds up.

# EXERCISES

1. Three particles of masses $1 \cdot 0$ kg, $2 \cdot 0$ kg and $3 \cdot 0$ kg are placed at the corners $A$, $B$ and $C$ respectively of an equilateral triangle $ABC$ of edge 1 m. Locate the centre of mass of the system.

2. The structure of a water molecule is shown in figure (9-E1). Find the distance of the centre of mass of the molecule from the centre of the oxygen atom.



Figure 9-E1

3. Seven homogeneous bricks, each of length $L$, are arranged as shown in figure (9-E2). Each brick is displaced with respect to the one in contact by $L/10$. Find the $x$-coordinate of the centre of mass relative to the origin shown.
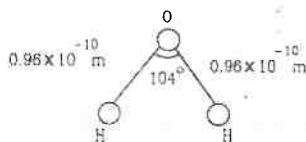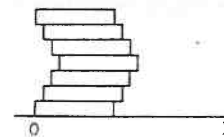


Figure 9-E2

4. A uniform disc of radius $R$ is put over another uniform disc of radius $2R$ of the same thickness and density. The

peripheries of the two discs touch each other. Locate the centre of mass of the system.

5. A disc of radius $R$ is cut out from a larger disc of radius $2R$ in such a way that the edge of the hole touches the edge of the disc. Locate the centre of mass of the residual disk.

6. A square plate of edge $d$ and a circular disc of diameter $d$ are placed touching each other at the midpoint of an edge of the plate as shown in figure (9-Q2). Locate the centre of mass of the combination, assuming same mass per unit area for the two plates.

7. Calculate the velocity of the centre of mass of the system of particles shown in figure (9-E3).
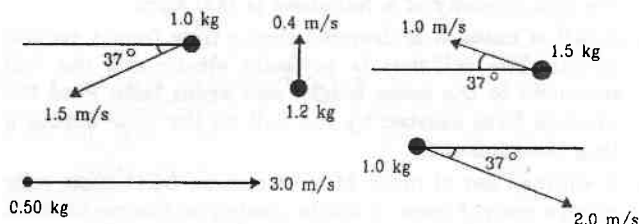


Figure 9-E3

8. Two blocks of masses 10 kg and 20 kg are placed on the $X$-axis. The first mass is moved on the axis by a distance of 2 cm. By what distance should the second mass be moved to keep the position of the centre of mass unchanged ?

9. Two blocks of masses 10 kg and 30 kg are placed along a vertical line. The first block is raised through a height of 7 cm. By what distance should the second mass be moved to raise the centre of mass by 1 cm ?

10. Consider a gravity-free hall in which a tray of mass $M$, carrying a cubical block of ice of mass $m$ and edge $L$, is at rest in the middle (figure 9-E4). If the ice melts, by what distance does the centre of mass of "the tray plus the ice" system descend ?
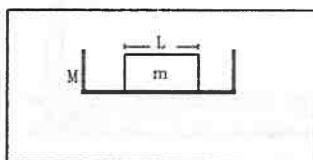


Gravity free hall

Figure 9-E4

11. Find the centre of mass of a uniform plate having semicircular inner and outer boundaries of radii $R_1$ and $R_2$ (figure 9-E5).
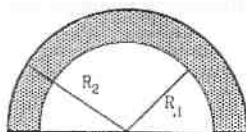


Figure 9-E5

12. Mr. Verma (50 kg) and Mr. Mathur (60 kg) are sitting at the two extremes of a 4 m long boat (40 kg) standing still in water. To discuss a mechanics problem, they come to the middle of the boat. Neglecting friction with water, how far does the boat move on the water during the process ?

13. A cart of mass $M$ is at rest on a frictionless horizontal surface and a pendulum bob of mass $m$ hangs from the roof of the cart (figure 9-E6). The string breaks, the bob falls on the floor, makes several collisions on the floor and finally lands up in a small slot made in the floor. The horizontal distance between the string and the slot is $L$. Find the displacement of the cart during this process.
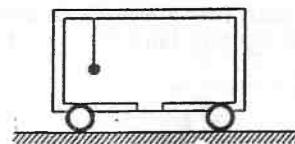


Figure 9-E6

14. The balloon, the light rope and the monkey shown in figure (9-E7) are at rest in the air. If the monkey reaches the top of the rope, by what distance does the balloon descend ? Mass of the balloon = $M$, mass of the monkey = $m$ and the length of the rope ascended by the monkey = $L$.



Figure 9-E7

15. Find the ratio of the linear momenta of two particles of masses 1·0 kg and 4·0 kg if their kinetic energies are equal.

16. A uranium-238 nucleus, initially at rest, emits an alpha particle with a speed of $1·4 \times 10^7$ m/s. Calculate the recoil speed of the residual nucleus thorium-234. Assume that the mass of a nucleus is proportional to the mass number.

17. A man of mass 50 kg starts moving on the earth and acquires a speed of 1·8 m/s. With what speed does the earth recoil ? Mass of earth = $6 \times 10^{24}$ kg.

18. A neutron initially at rest, decays into a proton, an electron and an antineutrino. The ejected electron has a momentum of $1·4 \times 10^{-20}$ kg-m/s and the antineutrino

$6.4 \times 10^{-2}$ kg-m/s. Find the recoil speed of the proton (a) if the electron and the antineutrino are ejected along the same direction and (b) if they are ejected along perpendicular directions. Mass of the proton $= 1.67 \times 10^{-27}$ kg.

19. A man of mass $M$ having a bag of mass $m$ slips from the roof of a tall building of height $H$ and starts falling vertically (figure 9-E8). When at a height $h$ from the ground, he notices that the ground below him is pretty hard, but there is a pond at a horizontal distance $x$ from the line of fall. In order to save himself he throws the bag horizontally (with respect to himself) in the direction opposite to the pond. Calculate the minimum horizontal velocity imparted to the bag so that the man lands in the water. If the man just succeeds to avoid the hard ground, where will the bag land ?
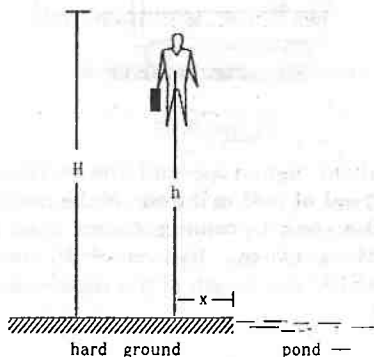


Figure 9-E8

20. A ball of mass 50 g moving at a speed of 2·0 m/s strikes a plane surface at an angle of incidence 45°. The ball is reflected by the plane at equal angle of reflection with the same speed. Calculate (a) the magnitude of the change in momentum of the ball (b) the change in the magnitude of the momentum of the ball.

21. Light in certain cases may be considered as a stream of particles called photons. Each photon has a linear momentum $h/\lambda$ where $h$ is the Planck's constant and $\lambda$ is the wavelength of the light. A beam of light of wavelength $\lambda$ is incident on a plane mirror at an angle of incidence $\theta$. Calculate the change in the linear momentum of a photon as the beam is reflected by the mirror.

22. A block at rest explodes into three equal parts. Two parts start moving along $X$ and $Y$ axes respectively with equal speeds of 10 m/s. Find the initial velocity of the third part.
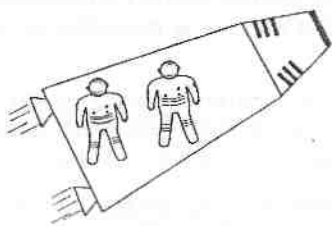


Figure 9-E9

23. Two fat astronauts each of mass 120 kg are travelling in a closed spaceship moving at a speed of 15 km/s in the outer space far removed from all other material objects. The total mass of the spaceship and its contents including the astronauts is 660 kg. If the astronauts do slimming exercise and thereby reduce their masses to 90 kg each, with what velocity will the spaceship move ?

24. During a heavy rain, hailstones of average size 1·0 cm in diameter fall with an average speed of 20 m/s. Suppose 2000 hailstones strike every square meter of a 10 m × 10 m roof perpendicularly in one second and assume that the hailstones do not rebound. Calculate the average force exerted by the falling hailstones on the roof. Density of a hailstone is 900 kg/m$^3$.

25. A ball of mass $m$ is dropped onto a floor from a certain height. The collision is perfectly elastic and the ball rebounds to the same height and again falls. Find the average force exerted by the ball on the floor during a long time interval.

26. A railroad car of mass $M$ is at rest on frictionless rails when a man of mass $m$ starts moving on the car towards the engine. If the car recoils with a speed $v$ backward on the rails, with what velocity is the man approaching the engine ?

27. A gun is mounted on a railroad car. The mass of the car, the gun, the shells and the operator is $50\,m$ where $m$ is the mass of one shell. If the muzzle velocity of the shells is 200 m/s, what is the recoil speed of the car after the second shot ? Neglect friction.

28. Two persons each of mass $m$ are standing at the two extremes of a railroad car of mass $M$ resting on a smooth track (figure 9-E10). The person on left jumps to the left with a horizontal speed $u$ with respect to the state of the car before the jump. Thereafter, the other person jumps to the right, again with the same horizontal speed $u$ with respect to the state of the car before his jump. Find the velocity of the car after both the persons have jumped off.
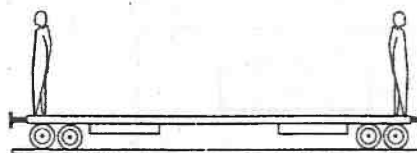


Figure 9-E10

29. Figure (9-E11) shows a small block of mass $m$ which is started with a speed $v$ on the horizontal part of the bigger block of mass $M$ placed on a horizontal floor. The curved part of the surface shown is semicircular. All the surfaces are frictionless. Find the speed of the bigger block when the smaller block reaches the point $A$ of the surface.
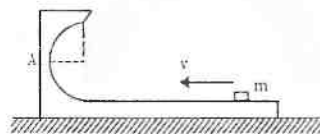


Figure 9-E11

30. In a typical Indian *Bugghi* (a luxury cart drawn by horses), a wooden plate is fixed on the rear on which one person can sit. A bugghi of mass 200 kg is moving at a speed of 10 km/hr. As it overtakes a school boy walking at a speed of 4 km/hr, the boy sits on the wooden plate. If the mass of the boy is 25 kg, what will be the new velocity of the bugghi ?

31. A ball of mass 0·50 kg moving at a speed of 5·0 m/s collides with another ball of mass 1·0 kg. After the collision the balls stick together and remain motionless. What was the velocity of the 1·0 kg block before the collision ?

32. A 60 kg man skating with a speed of 10 m/s collides with a 40 kg skater at rest and they cling to each other. Find the loss of kinetic energy during the collision.

33. Consider a head-on collision between two particles of masses $m_1$ and $m_2$. The initial speeds of the particles are $u_1$ and $u_2$ in the same direction. The collision starts at $t = 0$ and the particles interact for a time interval $\Delta t$. During the collision, the speed of the first particle varies as

$$v(t) = u_1 + \frac{t}{\Delta t}(v_1 - u_1).$$

Find the speed of the second particle as a function of time during the collision.

34. A bullet of mass $m$ moving at a speed $v$ hits a ball of mass $M$ kept at rest. A small part having mass $m'$ breaks from the ball and sticks to the bullet. The remaining ball is found to move at a speed $v_1$ in the direction of the bullet. Find the velocity of the bullet after the collision.

35. A ball of mass $m$ moving at a speed $v$ makes a head-on collision with an identical ball at rest. The kinetic energy of the balls after the collision is three fourths of the original. Find the coefficient of restitution.

36. A block of mass 2·0 kg moving at 2·0 m/s collides head on with another block of equal mass kept at rest. (a) Find the maximum possible loss in kinetic energy due to the collision. (b) If the actual loss in kinetic energy is half of this maximum, find the coefficient of restitution.

37. A particle of mass 100 g moving at an initial speed $u$ collides with another particle of same mass kept initially at rest. If the total kinetic energy becomes 0·2 J after the collision, what could be the minimum and maximum value of $u$.

38. Two friends $A$ and $B$ (each weighing 40 kg) are sitting on a frictionless platform some distance $d$ apart. $A$ rolls a ball of mass 4 kg on the platform towards $B$ which $B$ catches. Then $B$ rolls the ball towards $A$ and $A$ catches it. The ball keeps on moving back and forth between $A$ and $B$. The ball has a fixed speed of 5 m/s on the platform. (a) Find the speed of $A$ after he rolls the ball for the first time. (b) Find the speed of $A$ after he catches the ball for the first time. (c) Find the speeds of $A$ and $B$ after the ball has made 5 round trips and is held by $A$. (d) How many times can $A$ roll the ball ? (e) Where is the centre of mass of the system "$A + B +$ ball" at the end of the $n$th trip ?

39. A ball falls on the ground from a height of 2·0 m and rebounds upto a height of 1·5 m. Find the coefficient of restitution.

40. In a gamma decay process, the internal energy of a nucleus of mass $M$ decreases, a gamma photon of energy $E$ and linear momentum $E/c$ is emitted and the nucleus recoils. Find the decrease in internal energy.

41. A block of mass 2·0 kg is moving on a frictionless horizontal surface with a velocity of 1·0 m/s (figure 9-E12) towards another block of equal mass kept at rest. The spring constant of the spring fixed at one end is 100 N/m. Find the maximum compression of the spring.



1.0 m/s

2.0 kg          2.0 kg
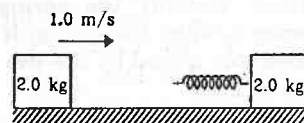
Figure 9-E12

42. A bullet of mass 20 g travelling horizontally with a speed of 500 m/s passes through a wooden block of mass 10·0 kg initially at rest on a level surface. The bullet emerges with a speed of 100 m/s and the block slides 20 cm on the surface before coming to rest. Find the friction coefficient between the block and the surface (figure 9-E13).



500 m/s
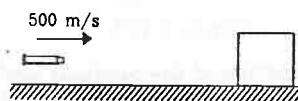
Figure 9-E13

43. A projectile is fired with a speed $u$ at an angle $\theta$ above a horizontal field. The coefficient of restitution of collision between the projectile and the field is $e$. How far from the starting point, does the projectile makes its second collision with the field ?

44. A ball falls on an inclined plane of inclination $\theta$ from a height $h$ above the point of impact and makes a perfectly elastic collision. Where will it hit the plane again ?

45. Solve the previous problem if the coefficient of restitution is $e$. Use $\theta = 45°$, $e = \frac{3}{4}$ and $h = 5$ m.

46. A block of mass 200 g is suspended through a vertical spring. The spring is stretched by 1·0 cm when the block is in equilibrium. A particle of mass 120 g is dropped on the block from a height of 45 cm. The particle sticks to the block after the impact. Find the maximum extension of the spring. Take $g = 10$ m/s$^2$.
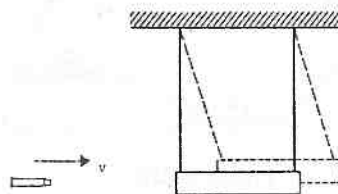


Figure 9-E14

47. A bullet of mass 25 g is fired horizontally into a ballistic pendulum of mass 5·0 kg and gets embedded in it (figure 9-E14). If the centre of the pendulum rises by a distance of 10 cm, find the speed of the bullet.

48. A bullet of mass 20 g moving horizontally at a speed of 300 m/s is fired into a wooden block of mass 500 g suspended by a long string. The bullet crosses the block and emerges on the other side. If the centre of mass of the block rises through a height of 20·0 cm, find the speed of the bullet as it emerges from the block.

49. Two masses $m_1$ and $m_2$ are connected by a spring of spring constant $k$ and are placed on a frictionless horizontal surface. Initially the spring is stretched through a distance $x_0$ when the system is released from rest. Find the distance moved by the two masses before they again come to rest.

50. Two blocks of masses $m_1$ and $m_2$ are connected by a spring of spring constant $k$ (figure 9-E15). The block of mass $m_2$ is given a sharp impulse so that it acquires a velocity $v_0$ towards right. Find (a) the velocity of the centre of mass, (b) the maximum elongation that the spring will suffer.
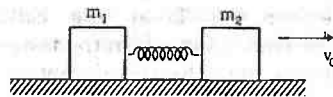


Figure 9-E15

51. Consider the situation of the previous problem. Suppose each of the blocks is pulled by a constant force $F$ instead of any impulse. Find the maximum elongation that the spring will suffer and the distances moved by the two blocks in the process.

52. Consider the situation of the previous problem. Suppose the block of mass $m_1$ is pulled by a constant force $F_1$ and the other block is pulled by a constant force $F_2$. Find the maximum elongation that the spring will suffer.

53. Consider a gravity-free hall in which an experimenter of mass 50 kg is resting on a 5 kg pillow, 8 ft above the floor of the hall. He pushes the pillow down so that it starts falling at a speed of 8 ft/s. The pillow makes a perfectly elastic collision with the floor, rebounds and reaches the experimenter's head. Find the time elapsed in the process.

54. The track shown in figure (9-E16) is frictionless. The block $B$ of mass $2m$ is lying at rest and the block $A$ of mass $m$ is pushed along the track with some speed. The collision between $A$ and $B$ is perfectly elastic. With what velocity should the block $A$ be started to get the sleeping man awakened ?
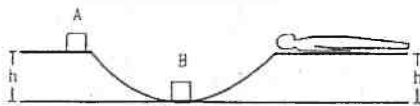


Figure 9-E16

55. A bullet of mass 10 g moving horizontally at a speed of $50\sqrt{7}$ m/s strikes a block of mass 490 g kept on a

frictionless track as shown in figure (9-E17). The bullet remains inside the block and the system proceeds towards the semicircular track of radius 0·2 m. Where will the block strike the horizontal part after leaving the semicircular track ?
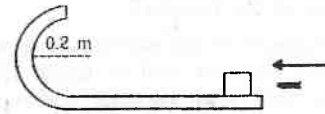


Figure 9-E17

56. Two balls having masses $m$ and $2m$ are fastened to two light strings of same length $l$ (figure 9-E18). The other ends of the strings are fixed at $O$. The strings are kept in the same horizontal line and the system is released from rest. The collision between the balls is elastic. (a) Find the velocities of the balls just after their collision. (b) How high will the balls rise after the collision ?
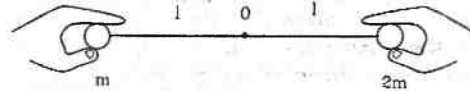


Figure 9-E18

57. A uniform chain of mass $M$ and length $L$ is held vertically in such a way that its lower end just touches the horizontal floor. The chain is released from rest in this position. Any portion that strikes the floor comes to rest. Assuming that the chain does not form a heap on the floor, calculate the force exerted by it on the floor when a length $x$ has reached the floor.

58. The blocks shown in figure (9-E19) have equal masses. The surface of $A$ is smooth but that of $B$ has a friction coefficient of 0·10 with the floor. Block $A$ is moving at a speed of 10 m/s towards $B$ which is kept at rest. Find the distance travelled by $B$ if (a) the collision is perfectly elastic and (b) the collision is perfectly inelastic. Take $g$ = 10 m/s$^2$.
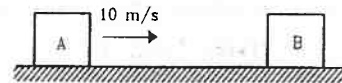


Figure 9-E19

59. The friction coefficient between the horizontal surface and each of the blocks shown in figure (9-E20) is 0·20. The collision between the blocks is perfectly elastic. Find the separation between the two blocks when they come to rest. Take $g$ = 10 m/s$^2$.
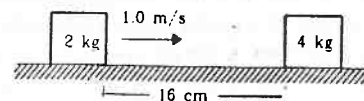


Figure 9-E20

60. A block of mass $m$ is placed on a triangular block of mass $M$, which in turn is placed on a horizontal surface as shown in figure (9-E21). Assuming frictionless

surfaces find the velocity of the triangular block when the smaller block reaches the bottom end.
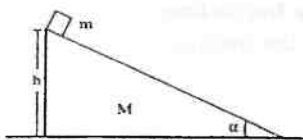


Figure 9-E21

61. Figure (9-E22) shows a small body of mass $m$ placed over a larger mass $M$ whose surface is horizontal near the smaller mass and gradually curves to become vertical. The smaller mass is pushed on the longer one at a speed $v$ and the system is left to itself. Assume that all the surfaces are frictionless. (a) Find the speed of the larger block when the smaller block is sliding on the vertical part. (b) Find the speed of the smaller mass when it breaks off the larger mass at height $h$. (c) Find the maximum height (from the ground) that the smaller mass ascends. (d) Show that the smaller mass will again land on the bigger one. Find the distance traversed by the bigger block during the time when the smaller block was in its flight under gravity.
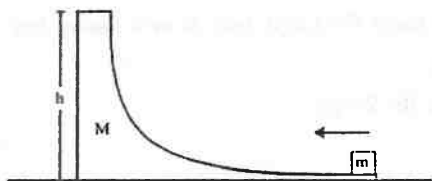


Figure 9-E22

62. A small block of superdense material has a mass of $3 \times 10^{24}$ kg. It is situated at a height $h$ (much smaller than the earth's radius) from where it falls on the earth's surface. Find its speed when its height from the earth's surface has reduced to $h/2$. The mass of the earth is $6 \times 10^{24}$ kg.

63. A body of mass $m$ makes an elastic collision with another identical body at rest. Show that if the collision is not head-on, the bodies go at right angle to each other after the collision.

64. A small particle travelling with a velocity $v$ collides elastically with a spherical body of equal mass and of radius $r$ initially kept at rest. The centre of this spherical body is located a distance $\rho(< r)$ away from the direction of motion of the particle (figure 9-E23). Find the final velocities of the two particles.
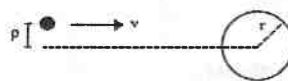


Figure 9-E23

[Hint : The force acts along the normal to the sphere through the contact. Treat the collision as one-dimensional for this direction. In the tangential direction no force acts and the velocities do not change].

□

# ANSWERS

## OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (d) | 3. (d) | 4. (d) | 5. (b) | 6. (b) |
| 7. (b) | 8. (b) | 9. (c) | 10. (d) | 11. (c) | 12. (b) |
| 13. (c) | 14. (c) | 15. (d) | 16. (d) | 17. (a) | 18. (a) |
| 19. (b) | | | | | |

## OBJECTIVE II

| | | |
|---|---|---|
| 1. none | 2. (c), (d) | 3. (a), (b) |
| 4. (b), (c) | 5. (b), (d) | 6. (d) |
| 7. (a), (d) | 8. (b), (c), (d) | 9. (c), (d) |
| 10. (b), (c) | 11. all | |

## EXERCISES

1. Taking $AB$ as the $x$-axis and $A$ as the origin, the centre of mass is at (7/12 m, $\sqrt{3}/4$ m)

2. $6.6 \times 10^{-12}$ m

3. 22 $L/35$

4. At $R/5$ from the centre of the bigger disk towards the centre of the smaller disk

5. At $R/3$ from the centre of the original disk away from the centre of the hole

6. $\dfrac{4d}{4 + \pi}$ right to the centre of the disk

7. 0·20 m/s at 45° below the direction towards right

8. 1 cm

9. 1 cm downward

10. zero

11. $\dfrac{4(R_1^2 + R_1 R_2 + R_2^2)}{3\pi (R_1 + R_2)}$ above the centre

12. 13 cm

13. $mL/(m + M)$

14. $mL/(m + M)$

15. 1 : 2

16. $2\cdot4 \times 10^{\circ}$ m/s

17. $1\cdot5 \times 10^{-23}$ m/s

18. (a) $12\cdot2$ m/s                     (b) $9\cdot2$ m/s

19. $\dfrac{Mx\sqrt{g}}{m\left[\sqrt{2H} - \sqrt{2(H-h)}\right]}$, $Mx/m$ left to the line of fall

20. (a) $0\cdot14$ kg–m/s (b) zero

21. $2h\cos\theta/\lambda$

22. $10\sqrt{2}$ m/s $135°$ below the $X$-axis

23. $15$ km/s

24. $1900$ N

25. $mg$

26. $\left(1 + \dfrac{M}{m}\right)v$

27. $200\left(\dfrac{1}{49} + \dfrac{1}{48}\right)$ m/s

28. $\dfrac{m^{\circ}u}{M(M+m)}$ towards left

29. $\dfrac{mv}{M+m}$

30. $\dfrac{28}{3}$ km/h

31. $2\cdot5$ m/s opposite to the direction of motion of the first ball

32. $1200$ J

33. $u_2 - \dfrac{m_1}{m_2}\dfrac{t}{\Delta t}(v_1 - u_1)$

34. $\dfrac{mv - (M - m')v_t}{m + m'}$ in the initial direction

35. $1/\sqrt{2}$                     36. $2$ J, $1/\sqrt{2}$

37. $2$ m/s, $2\sqrt{2}$ m/s

38. (a) $0\cdot5$ m/s (b) $\dfrac{10}{11}$ m/s (c) $\dfrac{50}{11}$ m/s, $5$ m/s (d) $6$ (e) $\dfrac{10}{21}d$ away from the initial position of $A$ towards $B$

39. $\sqrt{3}/2$

40. $E + \dfrac{E^2}{2Mc^2}$

41. $10$ cm

42. $0\cdot16$

43. $\dfrac{(1 + e)u^{\circ}\sin 2\theta}{g}$

44. $8h\sin\theta$ along the incline

45. $18\cdot5$ m along the incline

46. $6\cdot1$ cm

47. $280$ m/s

48. $250$ m/s

49. $\dfrac{2m_2x_0}{m_1 + m_2}$, $\dfrac{2m_1x_0}{m_1 + m_2}$

50. (a) $\dfrac{m_2v_0}{m_1 + m_2}$  (b) $v_0\left[\dfrac{m_1m_2}{(m_1 + m_2)k}\right]^{1/2}$

51. $2F/k$, $\dfrac{2Fm_2}{k(m_1 + m_2)}$, $\dfrac{2Fm_1}{k(m_1 + m_2)}$

52. $\dfrac{2(m_1F_2 + m_2F_1)}{k(m_1 + m_2)}$

53. $2\cdot22$ s

54. Greater than $\sqrt{2\cdot5\,gh}$

55. At the junction of the straight and the curved parts

56. (a) Light ball $\dfrac{\sqrt{50gl}}{3}$ towards left, heavy ball $\dfrac{\sqrt{2gl}}{3}$ towards right (b) Light ball $2l$ and heavy ball $\dfrac{l}{9}$

57. $3Mgx/L$

58. (a) $50$ m (b) $25$ m

59. $5$ cm

60. $\left[\dfrac{2m^{\circ}gh\cos^{\circ}\alpha}{(M+m)(M+m\sin^{\circ}\alpha)}\right]^{1/2}$

61. (a) $\dfrac{mv}{M+m}$  (b) $\left[\dfrac{(M^2 + Mm + m^2)}{(M+m)^2}v^2 - 2gh\right]^{1/2}$

   (c) $\dfrac{Mv^2}{2g(M+m)}$  (d) $\dfrac{2mv[Mv^2 - 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$

62. $\sqrt{\dfrac{2gh}{3}}$

64. The small particle goes along the tangent with a speed of $v\rho/r$ and the spherical body goes perpendicular to the smaller particle with a speed of $\dfrac{v}{r}\sqrt{r^2 - \rho^2}$

## QUESTIONS FOR SHORT ANSWER

1. Can an object be in pure translation as well as in pure rotation ?

2. A simple pendulum is a point mass suspended by a light thread from a fixed point. The particle is displaced towards one side and then released. It makes small oscillations. Is the motion of such a simple pendulum a pure rotation ? If yes, where is the axis of rotation ?

3. In a rotating body, $a = \alpha r$ and $v = \omega r$. Thus $\dfrac{a}{\alpha} = \dfrac{v}{\omega}$. Can you use the theorems of ratio and proportion studied in algebra so as to write

$$\frac{a + \alpha}{a - \alpha} = \frac{v + \omega}{v - \omega}$$

4. A ball is whirled in a circle by attaching it to a fixed point with a string. Is there an angular rotation of the ball about its centre ? If yes, is this angular velocity equal to the angular velocity of the ball about the fixed point ?

5. The moon rotates about the earth in such a way that only one hemisphere of the moon faces the earth (figure 10-Q1). Can we ever see the "other face" of the moon from the earth ? Can a person on the moon ever see all the faces of the earth ?
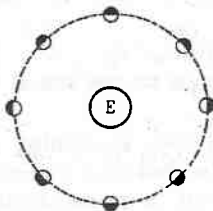


Figure 10-Q1

6. The torque of the weight of any body about any vertical axis is zero. Is it always correct ?

7. The torque of a force $\vec{F}$ about a point is defined as $\vec{\Gamma} = \vec{r} \times \vec{F}$. Suppose $\vec{r}, \vec{F}$ and $\vec{\Gamma}$ are all nonzero. Is $\vec{r} \times \vec{\Gamma} \parallel \vec{F}$ always true ? Is it ever true ?

8. A heavy particle of mass $m$ falls freely near the earth's surface. What is the torque acting on this particle about a point 50 cm east to the line of motion. Does this torque produce any angular acceleration in the particle ?

9. If several forces act on a particle, the total torque on the particle may be obtained by first finding the resultant force and then taking torque of this resultant. Prove this. Is this result valid for the forces acting on different particles of a body in such a way that their lines of action intersect at a common point ?

10. If the sum of all the forces acting on a body is zero, is it necessarily in equilibrium ? If the sum of all the forces on a particle is zero, is it necessarily in equilibrium ?

11. If the angular momentum of a body is found to be zero about a point, is it necessary that it will also be zero about a different point ?

12. If the resultant torque of all the forces acting on a body is zero about a point, is it necessary that it will be zero about any other point ?

13. A body is in translational equilibrium under the action of coplanar forces. If the torque of these forces is zero about a point, is it necessary that it will also be zero about any other point ?

14. A rectangular brick is kept on a table with a part of its length projecting out. It remains at rest if the length projected is slightly less than half the total length but it falls down if the length projected is slightly more than half the total length. Give reason.

15. When a fat person tries to touch his toes, keeping the legs straight, he generally falls. Explain with reference to figure (10-Q2).



Figure 10-Q2

16. A ladder is resting with one end on a vertical wall and the other end on a horizontal floor. Is it more likely to slip when a man stands near the bottom or near the top ?

17. When a body is weighed on an ordinary balance, we demand that the arm should be horizontal if the weights on the two pans are equal. Suppose equal weights are put on the two pans, the arm is kept at an angle with the horizontal and released. Is the torque of the two weights about the middle point (point of support) zero ? Is the total torque zero ? If so, why does the arm rotate and finally become horizontal ?

18. The density of a rod $AB$ continuously increases from $A$ to $B$. Is it easier to set it in rotation by clamping it at $A$ and applying a perpendicular force at $B$ or by clamping it at $B$ and applying the force at $A$ ?

19. When tall buildings are constructed on earth, the duration of day-night slightly increases. Is it true ?

20. If the ice at the poles melts and flows towards the equator, how will it affect the duration of day-night ?

21. A hollow sphere, a solid sphere, a disc and a ring all having same mass and radius are rolled down on an inclined plane. If no slipping takes place, which one will take the smallest time to cover a given length ?

22. A sphere rolls on a horizontal surface. Is there any point of the sphere which has a vertical velocity ?

## OBJECTIVE I

1. Let $\vec{A}$ be a unit vector along the axis of rotation of a purely rotating body and $\vec{B}$ be a unit vector along the velocity of a particle $P$ of the body away from the axis. The value of $\vec{A} \cdot \vec{B}$ is
   (a) 1     (b) −1     (c) 0     (d) None of these.

2. A body is uniformly rotating about an axis fixed in an inertial frame of reference. Let $\vec{A}$ be a unit vector along the axis of rotation and $\vec{B}$ be the unit vector along the resultant force on a particle $P$ of the body away from the axis. The value of $\vec{A} \cdot \vec{B}$ is
   (a) 1     (b) −1     (c) 0     (d) none of these.

3. A particle moves with a constant velocity parallel to the $X$-axis. Its angular momentum with respect to the origin
   (a) is zero     (b) remains constant
   (c) goes on increasing     (d) goes on decreasing.

4. A body is in pure rotation. The linear speed $v$ of a particle, the distance $r$ of the particle from the axis and the angular velocity $\omega$ of the body are related as $\omega = \dfrac{v}{r}$. Thus
   (a) $\omega \propto \dfrac{1}{r}$        (b) $\omega \propto r$
   (c) $\omega = 0$        (d) $\omega$ is independent of $r$.

5. Figure (10-Q2) shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates about the common axis. The strings supporting $A$ and $B$ do not slip on the wheels. If $x$ and $y$ be the distances travelled by $A$ and $B$ in the same time interval, then
   (a) $x = 2y$     (b) $x = y$     (c) $y = 2x$     (d) none of these.



Figure 10-Q2

6. A body is rotating uniformly about a vertical axis fixed in an inertial frame. The resultant force on a particle of the body not on the axis is
   (a) vertical     (b) horizontal and skew with the axis
   (c) horizontal and intersecting the axis
   (d) none of these.

7. A body is rotating nonuniformly about a vertical axis fixed in an inertial frame. The resultant force on a particle of the body not on the axis is
   (a) vertical     (b) horizontal and skew with the axis
   (c) horizontal and intersecting the axis
   (d) none of these.

8. Let $\vec{F}$ be a force acting on a particle having position vector $\vec{r}$. Let $\vec{\Gamma}$ be the torque of this force about the origin, then
   (a) $\vec{r} \cdot \vec{\Gamma} = 0$ and $\vec{F} \cdot \vec{\Gamma} = 0$     (b) $\vec{r} \cdot \vec{\Gamma} = 0$ but $\vec{F} \cdot \vec{\Gamma} \neq 0$
   (c) $\vec{r} \cdot \vec{\Gamma} \neq 0$ but $\vec{F} \cdot \vec{\Gamma} \neq 0$     (d) $\vec{r} \cdot \vec{\Gamma} \neq 0$ and $\vec{F} \cdot \vec{\Gamma} \neq 0$.

9. One end of a uniform rod of mass $m$ and length $l$ is clamped. The rod lies on a smooth horizontal surface and rotates on it about the clamped end at a uniform angular velocity $\omega$. The force exerted by the clamp on the rod has a horizontal component
   (a) $m\omega^2 l$     (b) zero     (c) $mg$     (d) $\dfrac{1}{2} m\omega^2 l$.

10. A uniform rod is kept vertically on a horizontal smooth surface at a point $O$. If it is rotated slightly and released, it falls down on the horizontal surface. The lower end will remain
    (a) at $O$     (b) at a distance less than $l/2$ from $O$
    (c) at a distance $l/2$ from $O$
    (d) at a distance larger than $l/2$ from $O$.

11. A circular disc $A$ of radius $r$ is made from an iron plate of thickness $t$ and another circular disc $B$ of radius $4r$ is made from an iron plate of thickness $t/4$. The relation between the moments of inertia $I_A$ and $I_B$ is
    (a) $I_A > I_B$     (b) $I_A = I_B$     (c) $I_A < I_B$
    (d) depends on the actual values of $t$ and $r$.

12. Equal torques act on the discs $A$ and $B$ of the previous problem, initially both being at rest. At a later instant, the linear speeds of a point on the rim of $A$ and another point on the rim of $B$ are $v_A$ and $v_B$ respectively. We have
    (a) $v_A > v_B$     (b) $v_A = v_B$     (c) $v_A < v_B$
    (d) the relation depends on the actual magnitude of the torques.

13. A closed cylindrical tube containing some water (not filling the entire tube) lies in a horizontal plane. If the tube is rotated about a perpendicular bisector, the moment of inertia of water about the axis
    (a) increases     (b) decreases     (c) remains constant
    (d) increases if the rotation is clockwise and decreases if it is anticlockwise.

14. The moment of inertia of a uniform semicircular wire of mass $M$ and radius $r$ about a line perpendicular to the plane of the wire through the centre is
    (a) $Mr^2$     (b) $\dfrac{1}{2} Mr^2$     (c) $\dfrac{1}{4} Mr^2$     (d) $\dfrac{2}{5} Mr^2$.

15. Let $I_1$ and $I_2$ be the moments of inertia of two bodies of identical geometrical shape, the first made of aluminium and the second of iron.
    (a) $I_1 < I_2$     (b) $I_1 = I_2$     (c) $I_1 > I_2$
    (d) relation between $I_1$ and $I_2$ depends on the actual shapes of the bodies.

16. A body having its centre of mass at the origin has three of its particles at $(a,0,0)$, $(0,a,0)$, $(0,0,a)$. The moments of inertia of the body about the $X$ and $Y$ axes are $0.20$ kg-m$^2$ each. The moment of inertia about the $Z$-axis
    (a) is $0.20$ kg-m$^2$     (b) is $0.40$ kg-m$^2$
    (c) is $0.20\sqrt{2}$ kg-m$^2$
    (d) cannot be deduced with this information.

17. A cubical block of mass $M$ and edge $a$ slides down a rough inclined plane of inclination $\theta$ with a uniform

velocity. The torque of the normal force on the block about its centre has a magnitude

(a) zero    (b) $Mga$    (c) $Mga \sin\theta$    (d) $\frac{1}{2} Mga \sin\theta$.

18. A thin circular ring of mass $M$ and radius $r$ is rotating about its axis with an angular speed $\omega$. Two particles having mass $m$ each are now attached at diametrically opposite points. The angular speed of the ring will become

(a) $\dfrac{\omega M}{M + m}$    (b) $\dfrac{\omega M}{M + 2m}$

(c) $\dfrac{\omega(M - 2m)}{M + 2m}$    (d) $\dfrac{\omega(M + 2m)}{M}$.

19. A person sitting firmly over a rotating stool has his arms stretched. If he folds his arms, his angular momentum about the axis of rotation

(a) increases    (b) decreases
(c) remains unchanged    (d) doubles.

20. The centre of a wheel rolling on a plane surface moves with a speed $v_0$. A particle on the rim of the wheel at the same level as the centre will be moving at speed

(a) zero    (b) $v_0$    (c) $\sqrt{2}v_0$    (d) $2v_0$.

21. A wheel of radius 20 cm is pushed to move it on a rough horizontal surface. It is found to move through a distance of 60 cm on the road during the time it completes one revolution about the centre. Assume that the linear and the angular accelerations are uniform. The frictional force acting on the wheel by the surface is

(a) along the velocity of the wheel
(b) opposite to the velocity of the wheel
(c) perpendicular to the velocity of the wheel
(d) zero.

22. The angular velocity of the engine (and hence of the wheel) of a scooter is proportional to the petrol input per second. The scooter is moving on a frictionless road with uniform velocity. If the petrol input is increased by

10%, the linear velocity of the scooter is increased by
(a) 50%    (b) 10%    (c) 20%    (d) 0%.

23. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of a smooth incline and released. Least time will be taken in reaching the bottom by

(a) the solid sphere    (b) the hollow sphere
(c) the disc    (d) all will take same time.

24. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. Least time will be taken in reaching the bottom by

(a) the solid sphere    (b) the hollow sphere
(c) the disc    (d) all will take same time.

25. In the previous question, the smallest kinetic energy at the bottom of the incline will be achieved by

(a) the solid sphere    (b) the hollow sphere
(c) the disc    (d) all will achieve same kinetic energy.

26. A string of negligible thickness is wrapped several times around a cylinder kept on a rough horizontal surface. A man standing at a distance $l$ from the cylinder holds one end of the string and pulls the cylinder towards him (figure 10-Q3). There is no slipping anywhere. The length of the string passed through the hand of the man while the cylinder reaches his hands is

(a) $l$    (b) $2l$    (c) $3l$    (d) $4l$.
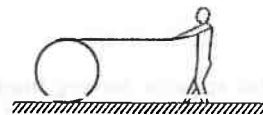


Figure 10-Q3

## OBJECTIVE II

1. The axis of rotation of a purely rotating body
(a) must pass through the centre of mass
(b) may pass through the centre of mass
(c) must pass through a particle of the body
(d) may pass through a particle of the body.

2. Consider the following two equations

(A) $L = I\omega$    (B) $\dfrac{dL}{dt} = \Gamma$

In noninertial frames
(a) both $A$ and $B$ are true    (b) $A$ is true but $B$ is false
(c) $B$ is true but $A$ is false    (d) both $A$ and $B$ are false.

3. A particle moves on a straight line with a uniform velocity. Its angular momentum
(a) is always zero
(b) is zero about a point on the straight line
(c) is not zero about a point away from the straight line
(d) about any given point remains constant.

4. If there is no external force acting on a nonrigid body, which of the following quantities must remain constant?
(a) angular momentum    (b) linear momentum
(c) kinetic energy    (d) moment of inertia.

5. Let $I_A$ and $I_B$ be moments of inertia of a body about two axes $A$ and $B$ respectively. The axis $A$ passes through the centre of mass of the body but $B$ does not.
(a) $I_A < I_B$    (b) If $I_A < I_B$, the axes are parallel.
(c) If the axes are parallel, $I_A < I_B$.
(d) If the axes are not parallel, $I_A > I_B$.

6. A sphere is rotating about a diameter.
(a) The particles on the surface of the sphere do not have any linear acceleration.
(b) The particles on the diameter mentioned above do not have any linear acceleration.
(c) Different particles on the surface have different

angular speeds.

(d) All the particles on the surface have same linear speed.

7. The density of a rod gradually decreases from one end to the other. It is pivoted at an end so that it can move about a vertical axis through the pivot. A horizontal force $F$ is applied on the free end in a direction perpendicular to the rod. The quantities, that do not depend on which end of the rod is pivoted, are
(a) angular acceleration
(b) angular velocity when the rod completes one rotation
(c) angular momentum when the rod completes one rotation
(d) torque of the applied force.

8. Consider a wheel of a bicycle rolling on a level road at a linear speed $v_0$ ( figure 10-Q4).
(a) the speed of the particle $A$ is zero
(b) the speed of $B$, $C$ and $D$ are all equal to $v_0$
(c) the speed of $C$ is $2\,v_0$
(d) the speed of $B$ is greater than the speed of $O$.
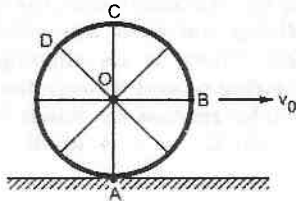


Figure 10-Q4

9. Two uniform solid spheres having unequal masses and unequal radii are released from rest from the same height on a rough incline. If the spheres roll without slipping,
(a) the heavier sphere reaches the bottom first
(b) the bigger sphere reaches the bottom first
(c) the two spheres reach the bottom together
(d) the information given is not sufficient to tell which sphere will reach the bottom first.

10. A hollow sphere and a solid sphere having same mass and same radii are rolled down a rough inclined plane.
(a) The hollow sphere reaches the bottom first.
(b) The solid sphere reaches the bottom with greater speed.
(c) The solid sphere reaches the bottom with greater kinetic energy.
(d) The two spheres will reach the bottom with same linear momentum.

11. A sphere cannot roll on
(a) a smooth horizontal surface
(b) a smooth inclined surface
(c) a rough horizontal surface
(d) a rough inclined surface.

12. In rear-wheel drive cars, the engine rotates the rear wheels and the front wheels rotate only because the car moves. If such a car accelerates on a horizontal road, the friction
(a) on the rear wheels is in the forward direction
(b) on the front wheels is in the backward direction
(c) on the rear wheels has larger magnitude than the friction on the front wheels
(d) on the car is in the backward direction.

13. A sphere can roll on a surface inclined at an angle $\theta$ if the friction coefficient is more than $\frac{2}{7} g \tan\theta$. Suppose the friction coefficient is $\frac{1}{7} g \tan\theta$. If a sphere is released from rest on the incline,
(a) it will stay at rest
(b) it will make pure translational motion
(c) it will translate and rotate about the centre
(d) the angular momentum of the sphere about its centre will remain constant.

14. A sphere is rolled on a rough horizontal surface. It gradually slows down and stops. The force of friction tries to
(a) decrease the linear velocity
(b) increase the angular velocity
(c) increase the linear momentum
(d) decrease the angular velocity.

15. Figure (10-Q5) shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angle of incline $\theta$ is related to the acceleration $a$ of the car as $a - g \tan\theta$. If the sphere is set in pure rolling on the incline,
(a) it will continue pure rolling
(b) it will slip down the plane
(c) its linear velocity will increase
(d) its linear velocity will slowly decrease.
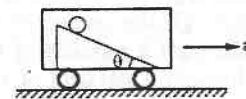


Figure 10-Q5

## EXERCISES

1. A wheel is making revolutions about its axis with uniform angular acceleration. Starting from rest, it reaches 100 rev/sec in 4 seconds. Find the angular acceleration. Find the angle rotated during these four seconds.

2. A wheel rotating with uniform angular acceleration covers 50 revolutions in the first five seconds after the start. Find the angular acceleration and the angular velocity at the end of five seconds.

3. A wheel starting from rest is uniformly accelerated at 4 rad/s$^2$ for 10 seconds. It is allowed to rotate uniformly

for the next 10 seconds and is finally brought to rest in the next 10 seconds. Find the total angle rotated by the wheel.

4. A body rotates about a fixed axis with an angular acceleration of one radian/second/second. Through what angle does it rotate during the time in which its angular velocity increases from 5 rad/s to 15 rad/s.

5. Find the angular velocity of a body rotating with an acceleration of 2 rev/s$^2$ as it completes the 5th revolution after the start.

6. A disc of radius 10 cm is rotating about its axis at an angular speed of 20 rad/s. Find the linear speed of
   (a) a point on the rim,
   (b) the middle point of a radius.

7. A disc rotates about its axis with a constant angular acceleration of 4 rad/s$^2$. Find the radial and tangential accelerations of a particle at a distance of 1 cm from the axis at the end of the first second after the disc starts rotating.

8. A block hangs from a string wrapped on a disc of radius 20 cm free to rotate about its axis which is fixed in a horizontal position. If the angular speed of the disc is 10 rad/s at some instant, with what speed is the block going down at that instant?

9. Three particles, each of mass 200 g, are kept at the corners of an equilateral triangle of side 10 cm. Find the moment of inertia of the system about an axis
   (a) joining two of the particles and
   (b) passing through one of the particles and perpendicular to the plane of the particles.

10. Particles of masses 1 g, 2 g, 3 g, ......., 100 g are kept at the marks 1 cm, 2 cm, 3 cm, ........., 100 cm respectively on a metre scale. Find the moment of inertia of the system of particles about a perpendicular bisector of the metre scale.

11. Find the moment of inertia of a pair of spheres, each having a mass $m$ and radius $r$, kept in contact about the tangent passing through the point of contact.

12. The moment of inertia of a uniform rod of mass 0·50 kg and length 1 m is 0·10 kg-m$^2$ about a line perpendicular to the rod. Find the distance of this line from the middle point of the rod.

13. Find the radius of gyration of a circular ring of radius $r$ about a line perpendicular to the plane of the ring and passing through one of its particles.

14. The radius of gyration of a uniform disc about a line perpendicular to the disc equals its radius. Find the distance of the line from the centre.

15. Find the moment of inertia of a uniform square plate of mass $m$ and edge $a$ about one of its diagonals.

16. The surface density (mass/area) of a circular disc of radius $a$ depends on the distance from the centre as $\rho(r) = A + Br$. Find its moment of inertia about the line perpendicular to the plane of the disc through its centre.

17. A particle of mass $m$ is projected with a speed $u$ at an angle $\theta$ with the horizontal. Find the torque of the weight of the particle about the point of projection when the particle is at the highest point.

18. A simple pendulum of length $l$ is pulled aside to make an angle $\theta$ with the vertical. Find the magnitude of the torque of the weight $w$ of the bob about the point of suspension. When is the torque zero?

19. When a force of 6·0 N is exerted at 30° to a wrench at a distance of 8 cm from the nut, it is just able to loosen the nut. What force $F$ would be sufficient to loosen it if it acts perpendicularly to the wrench at 16 cm from the nut?
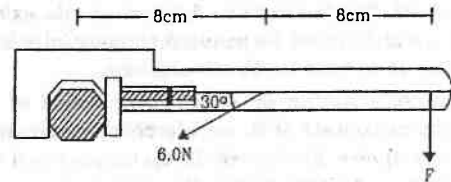


Figure 10-E1

20. Calculate the total torque acting on the body shown in figure (10-E2) about the point $O$.
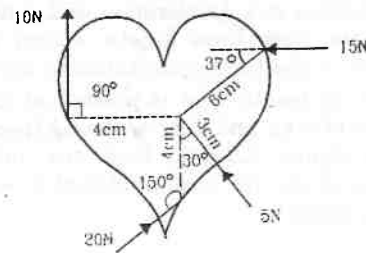


Figure 10-E2

21. A cubical block of mass $m$ and edge $a$ slides down a rough inclinded plane of inclination $\theta$ with a uniform speed. Find the torque of the normal force acting on the block about its centre.

22. A rod of mass $m$ and length $L$, lying horizontally, is free to rotate about a vertical axis through its centre. A horizontal force of constant magnitude $F$ acts on the rod at a distance of $L/4$ from the centre. The force is always perpendicular to the rod. Find the angle rotated by the rod during the time $t$ after the motion starts.

23. A square plate of mass 120 g and edge 5·0 cm rotates about one of the edges. If it has a uniform angular acceleration of 0·2 rad/s$^2$. what torque acts on the plate?

24. Calculate the torque on the square plate of the previous problem if it rotates about a diagonal with the same angular acceleration.

25. A flywheel of moment of inertia 5·0 kg-m$^2$ is rotated at a speed of 60 rad/s. Because of the friction at the axle, it comes to rest in 5·0 minutes. Find (a) the average torque of the friction, (b) the total work done by the friction and (c) the angular momentum of the wheel 1 minute before it stops rotating.

26. Because of the friction between the water in oceans with the earth's surface, the rotational kinetic energy of the earth is continuously decreasing. If the earth's angular speed decreases by 0·0016 rad/day in 100 years, find the

average torque of the friction on the earth. Radius of the earth is 6400 km and its mass is $6 \cdot 0 \times 10^{24}$ kg.

27. A wheel rotating at a speed of 600 rpm (revolutions per minute) about its axis is brought to rest by applying a constant torque for 10 seconds. Find the angular deceleration and the angular velocity 5 seconds after the application of the torque.

28. A wheel of mass 10 kg and radius 20 cm is rotating at an angular speed of 100 rev/min when the motor is turned off. Neglecting the friction at the axle, calculate the force that must be applied tangentially to the wheel to bring it to rest in 10 revolutions.

29. A cylinder rotating at an angular speed of 50 rev/s is brought in contact with an identical stationary cylinder. Because of the kinetic friction, torques act on the two cylinders, accelerating the stationary one and decelerating the moving one. If the common magnitude of the acceleration and deceleration be one revolution per second square, how long will it take before the two cylinders have equal angular speed?

30. A body rotating at 20 rad/s is acted upon by a constant torque providing it a deceleration of 2 rad/s². At what time will the body have kinetic energy same as the initial value if the torque continues to act?

31. A light rod of length 1 m is pivoted at its centre and two masses of 5 kg and 2 kg are hung from the ends as shown in figure (10-E3). Find the initial angular acceleration of the rod assuming that it was horizontal in the beginning.
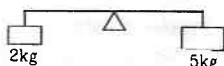


Figure 10-E3

32. Suppose the rod in the previous problem has a mass of 1 kg distributed uniformly over its length.
(a) Find the initial angular acceleration of the rod.
(b) Find the tension in the supports to the blocks of mass 2 kg and 5 kg.

33. Figure (10-E4) shows two blocks of masses $m$ and $M$ connected by a string passing over a pulley. The horizontal table over which the mass $m$ slides is smooth. The pulley has a radius $r$ and moment of inertia $I$ about its axis and it can freely rotate about this axis. Find the acceleration of the mass $M$ assuming that the string does not slip on the pulley.
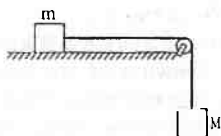


Figure 10-E4

34. A string is wrapped on a wheel of moment of inertia $0 \cdot 20$ kg-m² and radius 10 cm and goes through a light pulley to support a block of mass $2 \cdot 0$ kg as shown in figure (10-E5). Find the acceleration of the block.
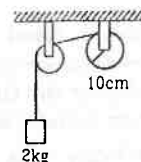


Figure 10-E5

35. Suppose the smaller pulley of the previous problem has its radius $5 \cdot 0$ cm and moment of inertia $0 \cdot 10$ kg-m². Find the tension in the part of the string joining the pulleys.

36. The pulleys in figure (10-E6) are identical, each having a radius $R$ and moment of inertia $I$. Find the acceleration of the block $M$.
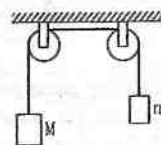


Figure 10-E6

37. The descending pulley shown in figure (10-E7) has a radius 20 cm and moment of inertia $0 \cdot 20$ kg-m². The fixed pulley is light and the horizontal plane frictionless. Find the acceleration of the block if its mass is $1 \cdot 0$ kg.



Figure 10-E7

38. The pulley shown in figure (10-E8) has a radius 10 cm and moment of inertia $0 \cdot 5$ kg-m² about its axis. Assuming the inclined planes to be frictionless, calculate the acceleration of the $4 \cdot 0$ kg block.
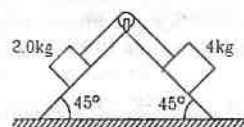


Figure 10-E8

39. Solve the previous problem if the friction coefficient between the $2 \cdot 0$ kg block and the plane below it is $0 \cdot 5$ and the plane below the $4 \cdot 0$ kg block is frictionless.

40. A uniform metre stick of mass 200 g is suspended from the ceiling through two vertical strings of equal lengths fixed at the ends. A small object of mass 20 g is placed on the stick at a distance of 70 cm from the left end. Find the tensions in the two strings.

41. A uniform ladder of length $10 \cdot 0$ m and mass $16 \cdot 0$ kg is resting against a vertical wall making an angle of 37° with it. The vertical wall is frictionless but the ground is rough. An electrician weighing $60 \cdot 0$ kg climbs up the ladder. If he stays on the ladder at a point $8 \cdot 00$ m from

the lower end, what will be the normal force and the force of friction on the ladder by the ground? What should be the minimum coefficient of friction for the elctrician to work safely?

42. Suppose the friction coefficient between the ground and the ladder of the previous probelm is 0·540. Find the maximum weight of a mechanic who could go up and do the work from the same position of the ladder.

43. A 6·5 m long ladder rests against a vertical wall reaching a height of 6·0 m. A 60 kg man stands half way up the ladder. (a) Find the torque of the force exerted by the man on the ladder about the upper end of the ladder. (b) Assuming the weight of the ladder to be negligible as compared to the man and assuming the wall to be smooth, find the force exerted by the ground on the ladder.

44. The door of an almirah is 6 ft high, 1·5 ft wide and weighs 8 kg. The door is supported by two hinges situated at a distance of 1 ft from the ends. If the magnitudes of the forces exerted by the hinges on the door are equal, find this magnitude.

45. A uniform rod of length $L$ rests against a smooth roller as shown in figure (10-E9). Find the friction coefficient between the ground and the lower end if the minimum angle that the rod can make with the horizontal is $\theta$.
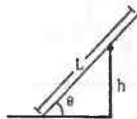


Figure 10-E9

46. A uniform rod of mass 300 g and length 50 cm rotates at a uniform angular speed of 2 rad/s about an axis perpendicular to the rod through an end. Calculate (a) the angular momentum of the rod about the axis of rotation, (b) the speed of the centre of the rod and (c) its kinetic energy.

47. A uniform square plate of mass 2·0 kg and edge 10 cm rotates about one of its diagonals under the action of a constant torque of 0·10 N-m. Calculate the angular momentum and the kinetic energy of the plate at the end of the fifth second after the start.

48. Calculate the ratio of the angular momentum of the earth about its axis due to its spinning motion to that about the sun due to its orbital motion. Radius of the earth = 6400 km and radius of the orbit of the earth about the sun = $1·5 \times 10^8$ km.

49. Two particles of masses $m_1$ and $m_2$ are joined by a light rigid rod of length $r$. The system rotates at an angular speed $\omega$ about an axis through the centre of mass of the system and perpendicular to the rod. Show that the angular momentum of the system is $L = \mu\, r^2\, \omega$ where $\mu$ is the reduced mass of the system defined as
$$\mu = \frac{m_1 m_2}{m_1 + m_2}.$$

50. A dumb-bell consists of two identical small balls of mass 1/2 kg each connected to the two ends of a 50 cm long

light rod. The dumb-bell is rotating about a fixed axis through the centre of the rod and perpendicular to it at an angular speed of 10 rad/s. An implusive force of average magnitude 5·0 N acts on one of the masses in the direction of its velocity for 0·10 s. Find the new angular velocity of the system.

51. A wheel of moment of inertia 0·500 kg-m² and radius 20·0 cm is rotating about its axis at an angular speed of 20·0 rad/s. It picks up a stationary particle of mass 200 g at its edge. Find the new angular speed of the wheel.

52. A diver having a moment of inertia of 6·0 kg-m² about an axis through its centre of mass rotates at an angular speed of 2 rad/s about this axis. If he folds his hands and feet to decrease the moment of inertia to 5·0 kg-m² what will be the new angular speed?

53. A boy is seated in a revolving chair revolving at an angular speed of 120 revolutions per minute. Two heavy balls form part of the revolving system and the boy can pull the balls closer to himself or may push them apart. If by pulling the balls closer, the boy decreases the moment of inertia of the system from 6 kg-m² to 2 kg-m², what will be the new angular speed?

54. A boy is standing on a platform which is free to rotate about its axis. The boy holds an open umbrella in his hand. The axis of the umbrella coincides with that of the platform. The moment of inertia of "the platform plus the boy system" is $3·0 \times 10^{-3}$ kg-m² and that of the umbrella is $2·0 \times 10^{-3}$ kg-m². The boy starts spinning the umbrella about the axis at an angular speed of 2·0 rev/s with respect to himself. Find the angular velocity imparted to the platform.

55. A wheel of moment of inertia 0·10 kg-m² is rotating about a shaft at an angular speed of 160 rev/minute. A second wheel is set into rotation at 300 rev/minute and is coupled to the same shaft so that both the wheels finally rotate with a common angular speed of 200 rev/minute. Find the moment of inertia of the second wheel.

56. A kid of mass $M$ stands at the edge of a platform of radius $R$ which can be freely rotated about its axis. The moment of inertia of the platform is $I$. The system is at rest when a friend throws a ball of mass $m$ and the kid catches it. If the velocity of the ball is $v$ horizontally along the tangent to the edge of the platform when it was caught by the kid, find the angular speed of the platform after the event.

57. Suppose the platform of the previous problem is brought to rest with the ball in the hand of the kid standing on the rim. The kid throws the ball horizontally to his friend in a direction tangential to the rim with a speed $v$ as seen by his friend. Find the angular velocity with which the platform will start rotating.

58. Suppose the platform with the kid in the previous problem is rotating in anticlockwise direction at an angular speed $\omega$. The kid starts walking along the rim with a speed $v$ relative to the platform also in the anticlockwise direction. Find the new angular speed of the platform.

59. A uniform rod of mass $m$ and length $l$ is struck at an end by a force $F$ perpendicular to the rod for a short time interval $t$. Calculate
(a) the speed of the centre of mass, (b) the angular speed of the rod about the centre of mass, (c) the kinetic energy of the rod and (d) the angular momentum of the rod about the centre of mass after the force has stopped to act. Assume that $t$ is so small that the rod does not appreciably change its direction while the force acts.

60. A uniform rod of length $L$ lies on a smooth horizontal table. A particle moving on the table strikes the rod perpendicularly at an end and stops. Find the distance travelled by the centre of the rod by the time it turns through a right angle. Show that if the mass of the rod is four times that of the particle, the collision is elastic.

61. Suppose the particle of the previous problem has a mass $m$ and a speed $v$ before the collision and it sticks to the rod after the collision. The rod has a mass $M$. (a) Find the velocity of the centre of mass $C$ of the system constituting "the rod plus the particle". (b) Find the velocity of the particle with respect to $C$ before the collision. (c) Find the velocity of the rod with respect to $C$ before the collision. (d) Find the angular momentum of the particle and of the rod about the centre of mass $C$ before the collision. (e) Find the moment of inertia of the system about the vertical axis through the centre of mass $C$ after the collision. (f) Find the velocity of the centre of mass $C$ and the angular velocity of the system about the centre of mass after the collision.

62. Two small balls $A$ and $B$, each of mass $m$, are joined rigidly by a light horizontal rod of length $L$. The rod is clamped at the centre in such a way that it can rotate freely about a vertical axis through its centre. The system is rotated with an angular speed $\omega$ about the axis. A particle $P$ of mass $m$ kept at rest sticks to the ball $A$ as the ball collides with it. Find the new angular speed of the rod.

63. Two small balls $A$ and $B$, each of mass $m$, are joined rigidly to the ends of a light rod of length $L$ (figure 10-E10). The system translates on a frictionless horizontal surface with a velocity $v_0$ in a direction perpendicular to the rod. A particle $P$ of mass $m$ kept at rest on the surface sticks to the ball $A$ as the ball collides with it. Find
(a) the linear speeds of the balls $A$ and $B$ after the collision, (b) the velocity of the centre of mass $C$ of the system $A + B + P$ and (c) the angular speed of the system about $C$ after the collision.
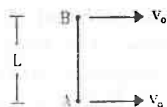


Figure 10-E10

[Hint : The light rod will exert a force on the ball $B$ only along its length.]

64. Suppose the rod with the balls $A$ and $B$ of the previous problem is clamped at the centre in such a way that it can rotate freely about a horizontal axis through the

clamp. The system is kept at rest in the horizontal position. A particle $P$ of the same mass $m$ is dropped from a height $h$ on the ball $B$. The particle collides with $B$ and sticks to it. (a) Find the angular momentum and the angular speed of the system just after the collision. (b) What should be the minimum value of $h$ so that the system makes a full rotation after the collision.

65. Two blocks of masses 400 g and 200 g are connected through a light string going over a pulley which is free to rotate about its axis. The pulley has a moment of inertia $1.6 \times 10^{-4}$ kg-m$^2$ and a radius 2.0 cm. Find (a) the kinetic energy of the system as the 400 g block falls through 50 cm, (b) the speed of the blocks at this instant.

66. The pulley shown in figure (10-E11) has a radius of 20 cm and moment of inertia 0.2 kg-m$^2$. The string going over it is attached at one end to a vertical spring of spring constant 50 N/m fixed from below, and supports a 1 kg mass at the other end. The system is released from rest with the spring at its natural length. Find the speed of the block when it has descended through 10 cm. Take $g = 10$ m/s$^2$.
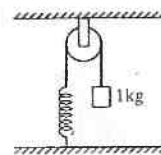


Figure 10-E11

67. A metre stick is held vertically with one end on a rough horizontal floor. It is gently allowed to fall on the floor. Assuming that the end at the floor does not slip, find the angular speed of the rod when it hits the floor.

68. A metre stick weighing 240 g is pivoted at its upper end in such a way that it can freely rotate in a vertical plane through this end (figure 10-E12). A particle of mass 100 g is attached to the upper end of the stick through a light string of length 1 m. Initially, the rod is kept vertical and the string horizontal when the system is released from rest. The particle collides with the lower end of the stick and sticks there. Find the maximum angle through which the stick will rise.
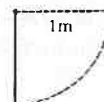


Figure 10-E12

69. A uniform rod pivoted at its upper end hangs vertically. It is displaced through an angle of 60° and then released. Find the magnitude of the force acting on a particle of mass $dm$ at the tip of the rod when the rod makes an angle of 37° with the vertical.

70. A cylinder rolls on a horizontal plane surface. If the speed of the centre is 25 m/s, what is the speed of the highest point ?

71. A sphere of mass $m$ rolls on a plane surface. Find its kinetic energy at an instant when its centre moves with speed $v$.

72. A string is wrapped over the edge of a uniform disc and the free end is fixed with the ceiling. The disc moves down, unwinding the string. Find the downward acceleration of the disc.

73. A small spherical ball is released from a point at a height $h$ on a rough track shown in figure (10-E13). Assuming that it does not slip anywhere, find its linear speed when it rolls on the horizontal part of the track.



Figure 10-E13

74. A small disc is set rolling with a speed $v$ on the horizontal part of the track of the previous problem from right to left. To what height will it climb up the curved part?

75. A sphere starts rolling down an incline of inclination $\theta$. Find the speed of its centre when it has covered a distance $l$.

76. A hollow sphere is released from the top of an inclined plane of inclination $\theta$. (a) What should be the minimum coefficient of friction between the sphere and the plane to prevent sliding? (b) Find the kinetic energy of the ball as it moves down a length $l$ on the incline if the friction coefficient is half the value calculated in part (a).

77. A solid sphere of mass $m$ is released from rest from the rim of a hemispherical cup so that it rolls along the surface. If the rim of the hemisphere is kept horizontal, find the normal force exerted by the cup on the ball when the ball reaches the bottom of the cup.

78. Figure (10-E14) shows a rough track, a portion of which is in the form of a cylinder of radius $R$. With what minimum linear speed should a sphere of radius $r$ be set rolling on the horizontal part so that it completely goes round the circle on the cylindrical part.



Figure 10-E14

79. Figure (10-E15) shows a small spherical ball of mass $m$ rolling down the loop track. The ball is released on the linear portion at a vertical height $H$ from the lowest point. The circular part shown has a radius $R$.
(a) Find the kinetic energy of the ball when it is at a point $A$ where the radius makes an angle $\theta$ with the horizontal.
(b) Find the radial and the tangential accelerations of the centre when the ball is at $A$.

(c) Find the normal force and the frictional force acting on the ball if $H = 60$ cm, $R = 10$ cm, $\theta = 0$ and $m = 70$ g.
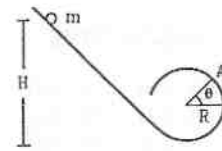


Figure 10-E15

80. A thin spherical shell of radius $R$ lying on a rough horizontal surface is hit sharply and horizontally by a cue. Where should it be hit so that the shell does not slip on the surface?

81. A uniform wheel of radius $R$ is set into rotation about its axis at an angular speed $\omega$. This rotating wheel is now placed on a rough horizontal surface with its axis horizontal. Because of friction at the contact, the wheel accelerates forward and its rotation decelerates till the wheel starts pure rolling on the surface. Find the linear speed of the wheel after it starts pure rolling.

82. A thin spherical shell lying on a rough horizontal surface is hit by a cue in such a way that the line of action passes through the centre of the shell. As a result the shell starts moving with a linear speed $v$ without any initial angular velocity. Find the linear speed of the shell after it starts pure rolling on the surface.

83. A hollow sphere of radius $R$ lies on a smooth horizontal surface. It is pulled by a horizontal force acting tangentially from the highest point. Find the distance travelled by the sphere during the time it makes one full rotation.

84. A solid sphere of mass 0·50 kg is kept on a horizontal surface. The coefficient of static friction between the surfaces in contact is 2/7. What maximum force can be applied at the highest point in the horizontal direction so that the sphere does not slip on the surface?

85. A solid sphere is set into motion on a rough horizontal surface with a linear speed $v$ in the forward direction and an angular speed $v/R$ in the anticlockwise direction as shown in figure (10-E16). Find the the linear speed of the sphere (a) when it stops rotating and (b) when slipping finally ceases and pure rolling starts.
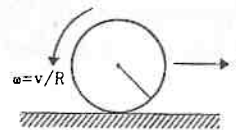


Figure 10-E16

86. A solid sphere rolling on a rough horizontal surface with a linear speed $v$ collides elastically with a fixed, smooth, vertical wall. Find the speed of the sphere after it has started pure rolling in the backward direction.

# ANSWERS

## OBJECTIVE I

1. (c)    2. (c)    3. (b)    4. (d)    5. (c)    6. (c)
7. (b)    8. (a)    9. (d)    10. (c)    11. (c)    12. (a)
13. (a)    14. (a)    15. (a)    16. (d)    17. (d)    18. (b)
19. (c)    20. (c)    21. (a)    22. (d)    23. (d)    24. (d)
25. (b)    26. (b)

## OBJECTIVE II

1. (b), (d)          2. (b)              3. (b), (c), (d)
4. (a), (b)          5. (c)              6. (b)
7. (d)               8. (a), (c), (d)    9. (c)
10. (b)              11. (b)             12. (a), (b), (c)
13. (c)              14. (a), (b)        15. (a)

## EXERCISES

1. $25$ rev/s$^2$, $400\,\pi$ rad
2. $4$ rev/s$^2$, $20$ rev/s
3. $800$ rad
4. $100$ rad
5. $2\sqrt{5}$ rev/s
6. $2$ m/s, $1$ m/s
7. $16$ cm/s$^2$, $4$ cm/s$^2$
8. $2$ m/s
9. $1.5 \times 10^{-3}$ kg-m$^2$, $4.0 \times 10^{-3}$ kg-m$^2$
10. $0.43$ kg-m$^2$
11. $\dfrac{14\,mr^2}{5}$
12. $0.34$ m
13. $\sqrt{2}\,r$
14. $r/\sqrt{2}$
15. $ma^2/12$
16. $2\pi\left(\dfrac{Aa^2}{4} + \dfrac{Ba^2}{5}\right)$
17. $mu^2\sin\theta\cos\theta$ perpendicular to the plane of motion
18. $wl\sin\theta$, when the bob is at the lowest point
19. $1.5$ N
20. $0.54$ N-m
21. $\dfrac{1}{2}\,mg\,a\sin\theta$
22. $\dfrac{3\,Ft^2}{2\,ml}$
23. $2.0 \times 10^{-5}$ N-m
24. $0.5 \times 10^{-5}$ N-m
25. (a) $1.0$ N-m       (b) $9.0$ kJ       (c) $60$ kg-m$^2$/s
26. $5.8 \times 10^{20}$ N-m
27. $1$ rev/s$^2$,   $5$ rev/s

28. $0.87$ N
29. $25$ s
30. $20$ s
31. $8.4$ rad/s$^2$
32. $8.0$ rad/s$^2$,  $27.6$ N,  $29$ N
33. $\dfrac{Mg}{M + m + I/r^2}$
34. $0.89$ m/s$^2$
35. $6.3$ N
36. $\dfrac{(M - m)g}{M + m + 2\,I/r^2}$
37. $10$ m/s$^2$
38. $0.25$ m/s$^2$
39. $0.125$ m/s$^2$
40. $1.04$ N in the left string and $1.12$ N in the right
41. $745$ N, $412$ N,  $0.553$
42. $44.0$ kg
43. (a) $740$ N-m
    (b) $590$ N vertical and $120$ N horizontal
44. $43$ N
45. $\dfrac{L\cos\theta\sin^2\theta}{2\,h - L\cos^2\theta\sin\theta}$
46. (a) $0.05$ kg-m$^2$/s       (b) $50$ cm/s       (c) $0.05$ J
47. $0.5$ kg-m$^2$/s, $75$ J
48. $2.66 \times 10^{-7}$
50. $12$ rad/s
51. $19.7$ rad/s
52. $2.4$ rad/s
53. $360$ rev/minute
54. $0.8$ rev/s
55. $0.04$ kg-m$^2$
56. $\dfrac{mvR}{I + (M + m)R^2}$
57. $\dfrac{mvR}{I + MR^2}$
58. $\omega - \dfrac{Mvr}{I + MR^2}$
59. (a) $\dfrac{Ft}{m}$   (b) $\dfrac{6\,Ft}{ml}$   (c) $\dfrac{2\,F^2t^2}{m}$   (d) $\dfrac{Flt}{2}$
60. $\pi L/12$
61. (a) $\dfrac{mv}{M + m}$   (b) $\dfrac{Mv}{M + m}$   (c) $-\dfrac{mv}{M + m}$
    (d) $\dfrac{M^2mvl}{2(M + m)^2}$, $\dfrac{Mm^2vl}{2(M + m)^2}$   (e) $\dfrac{M(M + 4m)L^2}{12(M + m)}$
    (f) $\dfrac{mv}{M + m}$, $\dfrac{6\,mv}{(M + 4m)L}$
62. $2\,\omega/3$

63. (a) $\dfrac{v_0}{2}$, $v_0$    (b) $\dfrac{2}{3} v_0$  along the initial motion of the rod

    (c) $\dfrac{v_0}{2L}$

64. (a) $\dfrac{mL\sqrt{gh}}{\sqrt{2}}$, $\dfrac{\sqrt{8gh}}{3L}$      (b) $\dfrac{3}{2} L$

65. (a) 0·98 J    (b) 1·4 m/s

66. 0·5 m/s

67. 5·4 rad/s

68. 41°

69. 0·9 $\sqrt{2}\, dm\, g$

70. 50 m/s

71. $\dfrac{7}{10} mv^2$

72. $\dfrac{2}{3} g$

73. $\sqrt{10\,gh/7}$

74. $\dfrac{3v^2}{4g}$

75. $\sqrt{\dfrac{10}{7} g\, l \sin\theta}$

76. (a) $\dfrac{2}{5} \tan\theta$    (b) $\dfrac{7}{8} mgl \sin\theta$

77. 17 $mg/7$

78. $\sqrt{\dfrac{27}{7} g(R - r)}$

79. (a) $mg(H - R - R\sin\theta)$,

    (b) $\dfrac{10}{7} g\left(\dfrac{H}{R} - 1 - \sin\theta\right)$, $-\dfrac{5}{7} g\cos\theta$

    (c) 4·9 N, 0·196 N upward

80. 2 $R/3$ above the centre

81. $\omega R/3$

82. 3 $v/5$

83. 4 $\pi R/3$

84. 3·3 N

85. (a) 3 $v/5$                    (b) 3 $v/7$

86. 3 $v/7$

□

or, $\qquad v^2 = \dfrac{GM}{r} = \dfrac{g R^2}{r} = \dfrac{(9 \cdot 8 \text{ m/s}^2)(6400 \text{ km})^2}{(8000 \text{ km})}$

giving $\qquad v = 7 \cdot 08$ km/s.

The time period is $\dfrac{2 \pi r}{v} = \dfrac{2 \pi (8000 \text{ km})}{(7 \cdot 08 \text{ km/s})} \approx 118$ minutes.

13. *Two satellites $S_1$ and $S_2$ revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8 h respectively. The radius of the orbit of $S_1$ is $10^4$ km. When $S_2$ is closest to $S_1$, find (a) the speed of $S_2$ relative to $S_1$ and (b) the angular speed of $S_2$ as observed by an astronaut in $S_1$.*

*Solution* : Let the mass of the planet be $M$, that of $S_1$ be $m_1$ and of $S_2$ be $m_2$. Let the radius of the orbit of $S_1$ be $R_1$ ($= 10^4$ km) and of $S_2$ be $R_2$.

Let $v_1$ and $v_2$ be the linear speeds of $S_1$ and $S_2$ with respect to the planet. Figure (11-W5) shows the situation.
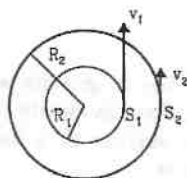


Figure 11-W5

As the square of the time period is proportional to the cube of the radius,

$$\left(\dfrac{R_2}{R_1}\right)^3 = \left(\dfrac{T_2}{T_1}\right)^2 = \left(\dfrac{8 \text{ h}}{1 \text{ h}}\right)^2 = 64$$

or, $\qquad \dfrac{R_2}{R_1} = 4$

or, $\qquad R_2 = 4R_1 = 4 \times 10^4$ km.

Now the time period of $S_1$ is 1 h. So,

$$\dfrac{2 \pi R_1}{v_1} = 1 \text{ h}$$

or, $\qquad v_1 = \dfrac{2 \pi R_1}{1 \text{ h}} = 2 \pi \times 10^4 \text{ km/h} .$

similarly, $\qquad v_2 = \dfrac{2 \pi R_2}{8 h} = \pi \times 10^4 \text{ km/h} .$

(a) At the closest separation, they are moving in the same direction. Hence the speed of $S_2$ with respect to $S_1$ is $|v_2 - v_1| = \pi \times 10^4$ km/h.

(b) As seen from $S_1$, the satellite $S_2$ is at a distance $R_2 - R_1 = 3 \times 10^4$ km at the closest separation. Also it is moving at $\pi \times 10^4$ km/h in a direction perpendicular to the line joining them. Thus, the angular speed of $S_2$ as observed by $S_1$ is

$$\omega = \dfrac{\pi \times 10^4 \text{ km/h}}{3 \times 10^4 \text{ km}} = \dfrac{\pi}{3} \text{ rad/h}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. Can two particles be in equilibrium under the action of their mutual gravitational force ? Can three particles be ? Can one of the three particles be ?

2. Is there any meaning of "Weight of the earth" ?

3. If heavier bodies are attracted more strongly by the earth, why don't they fall faster than the lighter bodies ?

4. Can you think of two particles which do not exert gravitational force on each other ?

5. The earth revolves round the sun because the sun attracts the earth. The sun also attracts the moon and this force is about twice as large as the attraction of the earth on the moon. Why does the moon not revolve round the sun ? Or does it ?

6. At noon, the sun and the earth pull the objects on the earth's surface in opposite directions. At midnight the sun and the earth pull these objects in same direction. Is the weight of an object, as measured by a spring balance on the earth's surface, more at midnight as compared to its weight at noon ?

7. An apple falls from a tree. An insect in the apple finds that the earth is falling towards it with an

acceleration $g$. Who exerts the force needed to accelerate the earth with this acceleration $g$ ?

8. Suppose the gravitational potential due to a small system is $k/r^2$ at a distance $r$ from it. What will be the gravitational field ? Can you think of any such system ? What happens if there were negative masses ?

9. The gravitational potential energy of a two-particle system is derived in this chapter as $U = - \dfrac{Gm_1m_2}{r}$. Does it follow from this equation that the potential energy for $r = \infty$ must be zero ? Can we choose the potential energy for $r = \infty$ to be 20 J and still use this formula ? If no, what formula should be used to calculate the gravitational potential energy at separation $r$ ?

10. The weight of an object is more at the poles than at the equator. Is it benificial to purchase goods at equator and sell them at the pole ? Does it matter whether a spring balance is used or an equal-beam balance is used ?

11. The weight of a body at the poles is greater than the weight at the equator. Is it the actual weight or the apparent weight we are talking about ? Does your

answer depend on whether only the earth's rotation is taken into account or the flattening of the earth at the poles is also taken into account ?

12. If the radius of the earth decreases by 1% without changing its mass, will the acceleration due to gravity at the surface of the earth increase or decrease ? If so, by what per cent ?

13. A nut becomes loose and gets detached from a satellite revolving around the earth. Will it land on the earth ? If yes, where will it land ? If no, how can an astronaut make it land on the earth ?

14. Is it necessary for the plane of the orbit of a satellite to pass through the centre of the earth ?

15. Consider earth satellites in circular orbits. A geostationary satellite must be at a height of about 36000 km from the earth's surface. Will any satellite moving at this height be a geostationary satellite ? Will

any satellite moving at this height have a time period of 24 hours ?

16. No part of India is situated on the equator. Is it possible to have a geostationary satellite which always remains over New Delhi ?

17. As the earth rotates about its axis, a person living in his house at the equator goes in a circular orbit of radius equal to the radius of the earth. Why does he/she not feel weightless as a satellite passenger does ?

18. Two satellites going in equatorial plane have almost same radii. As seen from the earth one move from east to west and the other from west to east. Will they have the same time period as seen from the earth ?. If not, which one will have less time period ?

19. A spacecraft consumes more fuel in going from the earth to the moon than it takes for a return trip. Comment on this statement.

## OBJECTIVE I

1. The acceleration of moon with respect to earth is $0.0027 \text{ m/s}^2$ and the acceleration of an apple falling on earth's surface is about $10 \text{ m/s}^2$. Assume that the radius of the moon is one fourth of the earth's radius. If the moon is stopped for an instant and then released, it will fall towards the earth. The initial acceleration of the moon towards the earth will be

    (a) $10 \text{ m/s}^2$  (b) $0.0027 \text{ m/s}^2$  (c) $6.4 \text{ m/s}^2$  (d) $5.0 \text{ m/s}^2$.

2. The acceleration of the moon just before it strikes the earth in the previous question is

    (a) $10 \text{ m/s}^2$  (b) $0.0027 \text{ m/s}^2$  (c) $6.4 \text{ m/s}^2$  (d) $5.0 \text{ m/s}^2$

3. Suppose, the acceleration due to gravity at the earth's surface is $10 \text{ m/s}^2$ and at the surface of Mars it is $4.0 \text{ m/s}^2$. A 60 kg passenger goes from the earth to the Mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of figure (11-Q1) best represents the weight (net gravitational force) of the passenger as a function of time.

    (a) $A$.          (b) $B$.          (c) $C$          (d) $D$.
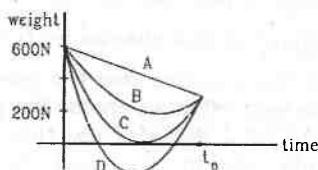


Figure 11-Q1

4. Consider a planet in some solar system which has a mass double the mass of the earth and density equal to the average density of the earth. An object weighing $W$ on the earth will weigh

    (a) $W$  (b) $2W$  (c) $W/2$  (d) $2^{1/3} W$ at the planet.

5. If the acceleration due to gravity at the surface of the earth is $g$, the work done in slowly lifting a body of mass $m$ from the earth's surface to a height $R$ equal to the radius of the earth is

    (a) $\frac{1}{2} mgR$     (b) $2mgR$     (c) $mgR$     (d) $\frac{1}{4} mgR$.

6. A person brings a mass of 1 kg from infinity to a point $A$. Initially the mass was at rest but it moves at a speed of 2 m/s as it reaches $A$. The work done by the person on the mass is $-3 J$. The potential at $A$ is
    (a) $-3$ J/kg  (b) $-2$ J/kg  (c) $-5$ J/kg  (d) none of these.

7. Let $V$ and $E$ be the gravitational potential and gravitational field at a distance $r$ from the centre of a uniform spherical shell. Consider the following two statements :
    (A) The plot of $V$ against $r$ is discontinuous.
    (B) The plot of $E$ against $r$ is discontinuous.
    (a) Both $A$ and $B$ are correct.
    (b) $A$ is correct but $B$ is wrong.
    (c) $B$ is correct but $A$ is wrong.
    (d) Both $A$ and $B$ are wrong.

8. Let $V$ and $E$ represent the gravitational potential and field at a distance $r$ from the centre of a uniform solid sphere. Consider the two statements:
    (A) the plot of $V$ against $r$ is discontinuous.
    (B) The plot of $E$ against $r$ is discontinuous.
    (a) Both $A$ and $B$ are correct.
    (b) $A$ is correct but $B$ is wrong.
    (c) $B$ is correct but $A$ is wrong.
    (d) Both $A$ and $B$ are wrong.

9. Take the effect of bulging of earth and its rotation in account. Consider the following statements :
    (A) There are points outside the earth where the value of $g$ is equal to its value at the equator.
    (B) There are points outside the earth where the value of $g$ is equal to its value at the poles.

(a) Both $A$ and $B$ are correct.
(b) $A$ is correct but $B$ is wrong.
(c) $B$ is correct but $A$ is wrong.
(d) Both $A$ and $B$ are wrong.

10. The time period of an earth-satellite in circular orbit is independent of
(a) the mass of the satellite  (b) radius of the orbit
(c) none of them  (d) both of them.

11. The magnitude of gravitational potential energy of the moon-earth system is $U$ with zero potential energy at infinite separation. The kinetic energy of the moon with respect to the earth is $K$.
(a) $U < K$.  (b) $U > K$.  (c) $U = K$.

12. Figure (11-Q2) shows the elliptical path of a planet about the sun. The two shaded parts have equal area. If $t_1$ and $t_2$ be the time taken by the planet to go from $a$ to $b$ and from $c$ to $d$ respectively,
(a) $t_1 < t_2$  (b) $t_1 = t_2$  (c) $t_1 > t_2$
(d) insufficient information to deduce the relation between $t_1$ and $t_2$.
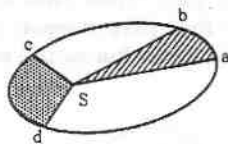


Figure 11-Q2

13. A person sitting in a chair in a satellite feels weightless because
(a) the earth does not attract the objects in a satellite
(b) the normal force by the chair on the person balances the earth's attraction
(c) the normal force is zero
(d) the person in satellite is not accelerated.

14. A body is suspended from a spring balance kept in a satellite. The reading of the balance is $W_1$ when the satellite goes in an orbit of radius $R$ and is $W_2$ when it goes in an orbit of radius $2R$.
(a) $W_1 = W_2$.  (b) $W_1 < W_2$.  (c) $W_1 > W_2$.  (d) $W_1 \neq W_2$

15. The kinetic energy needed to project a body of mass $m$ from the earth's surface to infinity is
(a) $\frac{1}{4} mgR$  (b) $\frac{1}{2} mgR$  (c) $mgR$  (d) $2 mgR$.

16. A particle is kept at rest at a distance $R$ (earth's radius) above the earth's surface. The minimum speed with which it should be projected so that it does not return is
(a) $\sqrt{\dfrac{GM}{4R}}$  (b) $\sqrt{\dfrac{GM}{2R}}$  (c) $\sqrt{\dfrac{GM}{R}}$  (d) $\sqrt{\dfrac{2GM}{R}}$.

17. A satellite is orbiting the earth close to its surface. A particle is to be projected from the satellite to just escape from the earth. The escape speed from the earth is $v_e$. Its speed with respect to the satellite
(a) will be less than $v_e$
(b) will be more than $v_e$
(c) will be equal to $v_e$
(d) will depend on direction of projection.

## OBJECTIVE II

1. Let $V$ and $E$ denote the gravitational potential and gravitational field at a point. It is possible to have
(a) $V = 0$ and $E = 0$  (b) $V = 0$ and $E \neq 0$
(c) $V \neq 0$ and $E = 0$  (d) $V \neq 0$ and $E \neq 0$.

2. Inside a uniform spherical shell
(a) the gravitational potential is zero
(b) the gravitational field is zero
(c) the gravitational potential is same everywhere
(d) the gravitational field is same everywhere.

3. A uniform spherical shell gradually shrinks maintaining its shape. The gravitational poential at the centre
(a) increases  (b) decreases
(c) remains constant  (d) oscillates.

4. Consider a planet moving in an elliptical orbit round the sun. The work done on the planet by the gravitational force of the sun
(a) is zero in any small part of the orbit

(b) is zero in some parts of the orbit
(c) is zero in one complete revolution
(d) is zero in no part of the motion.

5. Two satellites $A$ and $B$ move round the earth in the same orbit. The mass of $B$ is twice the mass of $A$.
(a) Speeds of $A$ and $B$ are equal.
(b) The potential energy of earth + $A$ is same as that of earth + $B$.
(c) The kinetic energy of $A$ and $B$ are equal.
(d) The total energy of earth + $A$ is same as that of earth + $B$.

6. Which of the following quantities remain constant in a planetory motion (consider elliptical orbits) as seen from the sun ?
(a) Speed.  (b) Angular speed.
(c) Kinetic energy.  (d) Angular momentum.

## EXERCISES

1. Two spherical balls of mass 10 kg each are placed 10 cm apart. Find the gravitational force of attraction between them.

2. Four particles having masses $m$, $2m$, $3m$ and $4m$ are placed at the four corners of a square of edge $a$. Find

the gravitational force acting on a particle of mass $m$ placed at the centre.

3. Three equal masses $m$ are placed at the three corners of an equilateral triangle of side $a$. Find the force exerted by this system on another particle of mass $m$ placed at (a) the mid-point of a side, (b) at the centre of the triangle.

4. Three uniform spheres each having a mass $M$ and radius $a$ are kept in such a way that each touches the other two. Find the magnitude of the gravitational force on any of the spheres due to the other two.

5. Four particles of equal masses $M$ move along a circle of radius $R$ under the action of their mutual gravitational attraction. Find the speed of each particle.

6. Find the acceleration due to gravity of the moon at a point 1000 km above the moon's surface. The mass of the moon is $7·4 \times 10^{22}$ kg and its radius is 1740 km.

7. Two small bodies of masses 10 kg and 20 kg are kept a distance 1·0 m apart and released. Assuming that only mutual gravitational forces are acting, find the speeds of the particles when the separation decreases to 0·5 m.

8. A semicircular wire has a length $L$ and mass $M$. A particle of mass $m$ is placed at the centre of the circle. Find the gravitational attraction on the particle due to the wire.

9. Derive an expression for the gravitational field due to a uniform rod of length $L$ and mass $M$ at a point on its perpendicular bisector at a distance $d$ from the centre.

10. Two concentric spherical shells have masses $M_1$, $M_2$ and radii $R_1$, $R_2$ $(R_1 < R_2)$. What is the force exerted by this system on a particle of mass $m_1$ if it is placed at a distance $(R_1 + R_2)/2$ from the centre ?

11. A tunnel is dug along a diameter of the earth. Find the force on a particle of mass $m$ placed in the tunnel at a distance $x$ from the centre.

12. A tunnel is dug along a chord of the earth at a perpendicular distance $R/2$ from the earth's centre. The wall of the tunnel may be assumed to be frictionless. Find the force exerted by the wall on a particle of mass $m$ when it is at a distance $x$ from the centre of the tunnel.

13. A solid sphere of mass $m$ and radius $r$ is placed inside a hollow thin spherical shell of mass $M$ and radius $R$ as shown in figure (11-E1). A particle of mass $m'$ is placed on the line joining the two centres at a distance $x$ from the point of contact of the sphere and the shell. Find the magnitude of the resultant gravitational force on this particle due to the sphere and the shell if (a) $r < x < 2r$, (b) $2r < x < 2R$ and (c) $x > 2R$.
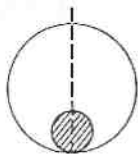


Figure 11-E1

14. A uniform metal sphere of radius $a$ and mass $M$ is surrounded by a thin uniform spherical shell of equal mass and radius $4a$ (figure 11-E2). The centre of the shell falls on the surface of the inner sphere. Find the gravitational field at the points $P_1$ and $P_2$ shown in the figure.
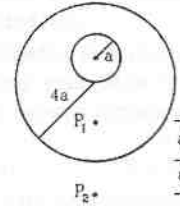


Figure 11-E2

15. A thin spherical shell having uniform density is cut in two parts by a plane and kept separated as shown in figure (11-E3). The point $A$ is the centre of the plane section of the first part and $B$ is the centre of the plane section of the second part. Show that the gravitational field at $A$ due to the first part is equal in magnitude to the gravitational field at $B$ due to the second part.
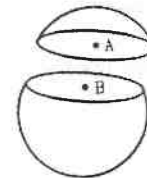


Figure 11-E3

16. Two small bodies of masses 2·00 kg and 4·00 kg are kept at rest at a separation of 2·0 m. Where should a particle of mass 0·10 kg be placed to experience no net gravitational force from these bodies ? The particle is placed at this point. What is the gravitational potential energy of the system of three particles with usual reference level ?

17. Three particles of mass $m$ each are placed at the three corners of an equilateral triangle of side $a$. Find the work which should be done on this system to increase the sides of the triangle to $2a$.

18. A particle of mass 100 g is kept on the surface of a uniform sphere of mass 10 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle away from the sphere.

19. The gravitational field in a region is given by $\vec{E} = (5 \text{ N/kg}) \vec{i} + (12 \text{ N/kg}) \vec{j}$. (a) Find the magnitude of the gravitational force acting on a particle of mass 2 kg placed at the origin. (b) Find the potential at the points (12 m, 0) and (0, 5 m) if the potential at the origin is taken to be zero. (c) Find the change in gravitational potential energy if a particle of mass 2 kg is taken from the origin to the point (12 m, 5 m). (d) Find the change in potential energy if the particle is taken from (12 m, 0) to (0, 5 m).

20. The gravitational pontential in a region is given by $V = (20 \text{ N/kg})(x + y)$. (a) Show that the equation is dimensionally correct. (b) Find the gravitational field at the point $(x, y)$. Leave your answer in terms of the unit vectors $\vec{i}, \vec{j}, \vec{k}$. (c) Calculate the magnitude of the gravitational force on a particle of mass 500 g placed at the origin.

21. The gravitational field in a region is given by $E = (2\vec{i} + 3\vec{j})$ N/kg. Show that no work is done by the gravitational field when a particle is moved on the line $3y + 2x = 5$.

   [Hint : If a line $y = mx + c$ makes angle $\theta$ with the X-axis, $m = \tan\theta$.]

22. Find the height over the earth's surface at which the weight of a body becomes half of its value at the surface.

23. What is the acceleration due to gravity on the top of Mount Everest ? Mount Everest is the highest mountain peak of the world at the height of 8848 m. The value at sea level is $9.80 \text{ m/s}^2$.

24. Find the acceleration due to gravity in a mine of depth 640 m if the value at the surface is $9.800 \text{ m/s}^2$. The radius of the earth is 6400 km.

25. A body is weighed by a spring balance to be 1·000 kg at the north pole. How much will it weigh at the equator ? Account for the earth's rotation only.

26. A body stretches a spring by a particular length at the earth's surface at equator. At what height above the south pole will it stretch the same spring by the same length ? Assume the earth to be spherical.

27. At what rate should the earth rotate so that the apparent $g$ at the equator becomes zero ? What will be the length of the day in this situation ?

28. A pendulum having a bob of mass $m$ is hanging in a ship sailing along the equator from east to west. When the ship is stationary with respect to water the tension in the string is $T_0$. (a) Find the speed of the ship due to rotation of the earth about its axis. (b) Find the difference between $T_0$ and the earth's attraction on the bob. (c) If the ship sails at speed $v$, what is the tension in the string ? Angular speed of earth's rotation is $\omega$ and radius of the earth is $R$.

29. The time taken by Mars to revolve round the sun is 1·88 years. Find the ratio of average distance between Mars and the sun to that between the earth and the sun.

30. The moon takes about 27·3 days to revolve round the earth in a nearly circular orbit of radius $3.84 \times 10^5$ km. Calculate the mass of the earth from these data.

31. A Mars satellite moving in an orbit of radius $9.4 \times 10^3$ km takes 27540 s to complete one revolution. Calculate the mass of Mars.

32. A satellite of mass 1000 kg is supposed to orbit the earth at a height of 2000 km above the earth's surface. Find (a) its speed in the orbit, (b) its kinetic energy, (c) the potential energy of the earth-satellite system and (d) its time period. Mass of the earth = $6 \times 10^{24}$ kg.

33. (a) Find the radius of the circular orbit of a satellite moving with an angular speed equal to the angular speed of earth's rotation. (b) If the satellite is directly above the north pole at some instant, find the time it takes to come over the equatorial plane. Mass of the earth = $6 \times 10^{24}$ kg.

34. What is the true weight of an object in a geostationary satellite that weighed exactly 10·0 N at the north pole ?

35. The radius of a planet is $R_1$ and a satellite revolves round it in a circle of radius $R_2$. The time period of revolution is $T$. Find the acceleration due to the gravitation of the planet at its surface.

36. Find the minimum colatitude which can directly receive a signal from a geostationary satellite.

37. A particle is fired vertically upward from earth's surface and it goes upto a maximum height of 6400 km. Find the initial speed of the particle.

38. A particle is fired vertically upward with a speed of 15 km/s. With what speed will it move in intersteller space. Assume only earth's gravitational field.

39. A mass of $6 \times 10^{24}$ kg (equal to the mass of the earth) is to be compressed in a sphere in such a way that the escape velocity from its surface is $3 \times 10^8$ m/s. What should be the radius of the sphere ?

□

## ANSWERS

### OBJECTIVE I

1. (b)  2. (c)  3. (c)  4. (d)  5. (a)  6. (c)
7. (c)  8. (d)  9. (b)  10. (a)  11. (b)  12. (b)
13. (c)  14. (a)  15. (c)  16. (c)  17. (d)

### OBJECTIVE II

1. all  2. (b), (c), (d)  3. (b)  4. (b), (c)  5. (a)  6. (d)

### EXERCISES

1. $6.67 \times 10^{-7}$ N

2. $\dfrac{4\sqrt{2}\, Gm^2}{a^2}$

3. (a) $\dfrac{4Gm^2}{3a^2}$, (b) zero

4. $\dfrac{\sqrt{3}\, GM^2}{4a^2}$

5. $\sqrt{\dfrac{GM}{R}\left[\dfrac{2\sqrt{2}+1}{4}\right]}$

6. $0.65 \text{ m/s}^2$

7. $4.2 \times 10^{-5} \text{ m/s}$ and $2.1 \times 10^{-5} \text{ m/s}$

8. $\dfrac{2\pi\, GMm}{L^2}$

9. $\dfrac{2\, Gm}{d\sqrt{L^2 + 4\, d^2}}$

10. $\dfrac{4GM_1 m}{(R_1 + R_2)^2}$

11. $\dfrac{GM_e m}{R^3}\, x$

12. $\dfrac{GM_e m}{2R^2}$

13. (a) $\dfrac{Gmm'(x-r)}{r^3}$  (b) $\dfrac{Gmm'}{(x-r)^2}$  (c) $\dfrac{GMm'}{(x-R)^2} + \dfrac{Gmm'}{(x-r)^2}$

14. $\dfrac{GM}{16a^2}$, $\dfrac{61\, GM}{900a^2}$

16. $0.83$ m from the $2.00$ kg body towards the other body, $-3.06 \times 10^{-10}$ J

17. $\dfrac{3Gm^2}{2a}$

18. $6.67 \times 10^{-10}$ J

19. (a) 26 N  (b) $-60$ J/kg, $-60$ J/kg  (c) $-240$ J  (d) zero

20. (b) $-20(\vec{i} + \vec{j})$ N/kg     (c) $10\sqrt{2}$ N

22. $(\sqrt{2} - 1)$ times the radius of the earth

23. $9.77 \text{ m/s}^2$

24. $9.799 \text{ m/s}^2$

25. $0.997$ kg

26. 10 km approx.

27. $1.237 \times 10^{-3}$ rad/sec, $1.41$ h

28. (a) $\omega R$ (b) $m\omega^2 R$ (c) $T_0 + 2\, m\omega v$ approx.

29. $1.52$

30. $6.02 \times 10^{24}$ kg

31. $6.5 \times 10^{23}$ kg

32. (a) $6.90$ km/s (b) $2.38 \times 10^{10}$ J (c) $-4.76 \times 10^{10}$ J with usual reference (d) $2.01$ hours

33. (a) 42300 km   (b) 6 hours

34. $0.23$ N

35. $\dfrac{4\pi^2 R_2^3}{T^2 R_1^2}$

36. $\sin^{-1}(0.15)$

37. $7.9$ km/s

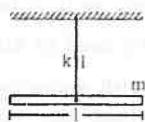38. $10.0$ km/s

39. $\approx 9$ mm

□

Figure 12-W15

**Solution :**

(a) The oscillations take place about the horizontal line through the point of suspension and perpendicular to the plane of the figure. The moment of inertia of the rod about this line is

$$\frac{ml^2}{12} + ml^2 = \frac{13}{12} ml^2.$$

The time period $= 2\pi \sqrt{\dfrac{I}{mgl}} = 2\pi \sqrt{\dfrac{13\, ml^2}{12\, mgl}}$

$$= 2\pi \sqrt{\frac{13\, l}{12\, g}}.$$

(b) The angular oscillations take place about the suspension wire. The moment of inertia about this line is $ml^2/12$. The time period is

$$2\pi \sqrt{\frac{I}{k}} = 2\pi \sqrt{\frac{ml^2}{12\, k}}.$$

21. *A particle is subjected to two simple harmonic motions*

$$x_1 = A_1 \sin\omega t$$

*and* $\qquad x_2 = A_2 \sin(\omega t + \pi/3).$

*Find (a) the displacement at t = 0, (b) the maximum speed of the particle and (c) the maximum acceleration of the particle.*

**Solution :**

(a) At $t = 0$, $\qquad x_1 = A_1 \sin\omega t = 0$

and $x_2 = A_2 \sin(\omega t + \pi/3)$

$$= A_2 \sin(\pi/3) = \frac{A_2 \sqrt{3}}{2}.$$

Thus, the resultant displacement at $t = 0$ is

$$x = x_1 + x_2 = \frac{A_2 \sqrt{3}}{2}.$$

(b) The resultant of the two motions is a simple harmonic motion of the same angular frequency $\omega$. The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2 A_1 A_2 \cos(\pi/3)}$$

$$= \sqrt{A_1^2 + A_2^2 + A_1 A_2}.$$

The maximum speed is

$$v_{max} = A\, \omega = \omega \sqrt{A_1^2 + A_2^2 + A_1 A_2}.$$

(c) The maximum acceleration is

$$a_{max} = A\, \omega^2 = \omega^2 \sqrt{A_1^2 + A_2^2 + A_1 A_2}.$$

22. *A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.*

**Solution :** Let the amplitudes of the individual motions be $A$ each. The resultant amplitude is also $A$. If the phase difference between the two motions is $\delta$,

$$A = \sqrt{A^2 + A^2 + 2A \cdot A \cdot \cos\delta}$$

or, $\qquad = A \sqrt{2(1 + \cos\delta)} = 2A \cos\dfrac{\delta}{2}$

or, $\qquad \cos\dfrac{\delta}{2} = \dfrac{1}{2}$

or, $\qquad \delta = 2\,\pi/3.$

□

# QUESTIONS FOR SHORT ANSWER

1. A person goes to bed at sharp 10·00 pm every day. Is it an example of periodic motion ? If yes, what is the time period ? If no, why ?

2. A particle executing simple harmonic motion comes to rest at the extreme positions. Is the resultant force on the particle zero at these positions according to Newton's first law ?

3. Can simple harmonic motion take place in a noninertial frame? If yes, should the ratio of the force applied with the displacement be constant ?

4. A particle executes simple harmonic motion. If you are told that its velocity at this instant is zero, can you say what is its displacement ? If you are told that its velocity

at this instant is maximum, can you say what is its displacement ?

5. A small creature moves with constant speed in a vertical circle on a bright day. Does its shadow formed by the sun on a horizontal plane move in a simple harmonic motion ?

6. A particle executes simple harmonic motion. Let $P$ be a point near the mean position and $Q$ be a point near an extreme. The speed of the particle at $P$ is larger than the speed at $Q$. Still the particle crosses $P$ and $Q$ equal number of times in a given time interval. Does it make you unhappy ?

7. In measuring time period of a pendulum, it is advised to measure the time between consecutive passage through the mean position in the same direction. This is said to result in better accuracy than measuring time between consecutive passage through an extreme position. Explain.

8. It is proposed to move a particle in simple harmonic motion on a rough horizontal surface by applying an external force along the line of motion. Sketch the graph of the applied force against the position of the particle. Note that the applied force has two values for a given position depending on whether the particle is moving in positive or negative direction.

9. Can the potential energy in a simple harmonic motion be negative ? Will it be so if we choose zero potential energy at some point other than the mean position ?

10. The energy of a system in simple harmonic motion is given by $E = \frac{1}{2} m \omega^2 A^2$. Which of the following two statements is more appropriate ?
(A) The energy is increased because the amplitude is increased.
(B) The amplitude is increased because the energy is increased.

11. A pendulum clock gives correct time at the equator. Will it gain time or loose time as it is taken to the poles ?

12. Can a pendulum clock be used in an earth satellite ?

13. A hollow sphere filled with water is used as the bob of a pendulum. Assume that the equation for simple pendulum is valid with the distance between the point of suspension and centre of mass of the bob acting as the effective length of the pendulum. If water slowly leaks out of the bob, how will the time period vary ?

14. A block of known mass is suspended from a fixed support through a light spring. Can you find the time period of vertical oscillation only by measuring the extension of the spring when the block is in equilibrium ?

15. A platoon of soldiers marches on a road in steps according to the sound of a marching band. The band is stopped and the soldiers are ordered to break the steps while crossing a bridge. Why ?

16. The force acting on a particle moving along $X$-axis is $F = -k(x - v_0 t)$ where $k$ is a positive constant. An observer moving at a constant velocity $v_0$ along the $X$-axis looks at the particle. What kind of motion does he find for the particle ?

## OBJECTIVE I

1. A student says that he had applied a force $F = -k\sqrt{x}$ on a particle and the particle moved in simple harmonic motion. He refuses to tell whether $k$ is a constant or not. Assume that he has worked only with positive $x$ and no other force acted on the particle.
(a) As $x$ increases $k$ increases.
(b) As $x$ increases $k$ decreases.
(c) As $x$ increases $k$ remains constant.
(d) The motion cannot be simple harmonic.

2. The time period of a particle in simple harmonic motion is equal to the time between consecutive appearances of the particle at a particular point in its motion. This point is
(a) the mean position
(b) an extreme position
(c) between the mean position and the positive extreme
(d) between the mean position and the negative extreme.

3. The time period of a particle in simple harmonic motion is equal to the smallest time between the particle acquiring a particular velocity $\vec{v}$. The value of $v$ is
(a) $v_{max}$
(b) 0
(c) between 0 and $v_{max}$
(d) between 0 and $-v_{max}$.

4. The displacement of a particle in simple harmonic motion in one time period is
(a) $A$      (b) $2A$      (c) $4A$      (d) zero.

5. The distance moved by a particle in simple harmonic motion in one time period is
(a) $A$      (b) $2A$      (c) $4A$      (d) zero.

6. The average acceleration in one time period in a simple harmonic motion is
(a) $A \omega^2$      (b) $A \omega^2/2$      (c) $A \omega^2/\sqrt{2}$      (d) zero.

7. The motion of a particle is given by $x = A \sin\omega t + B \cos\omega t$. The motion of the particle is
(a) not simple harmonic
(b) simple harmonic with amplitude $A + B$
(c) simple harmonic with amplitude $(A + B)/2$
(d) simple harmonic with amplitude $\sqrt{A^2 + B^2}$.

8. The displacement of a particle is given by $\vec{r} = A(\vec{i}\cos\omega t + \vec{j}\sin\omega t)$. The motion of the particle is
(a) simple harmonic      (b) on a straight line
(c) on a circle      (d) with constant acceleration.

9. A particle moves on the $X$-axis according to the equation $x = A + B \sin\omega t$. The motion is simple harmonic with amplitude
(a) $A$      (b) $B$      (c) $A + B$      (d) $\sqrt{A^2 + B^2}$.

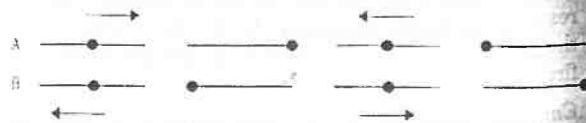10. Figure (12-Q1) represents two simple harmonic motions.



Figure 12-Q1

The parameter which has different values in the two motions is

(a) amplitude       (b) frequency

(c) phase       (d) maximum velocity.

11. The total mechanical energy of a spring-mass system in simple harmonic motion is $E = \frac{1}{2} m \omega^2 A^2$. Suppose the oscillating particle is replaced by another particle of double the mass while the amplitude $A$ remains the same. The new mechanical energy will

(a) become $2E$       (b) become $E/2$

(c) become $\sqrt{2}E$       (d) remain $E$.

12. The average energy in one time period in simple harmonic motion is

(a) $\frac{1}{2} m \omega^2 A^2$       (b) $\frac{1}{4} m \omega^2 A^2$

(c) $m \omega^2 A^2$       (d) zero.

13. A particle executes simple harmonic motion with a frequency $v$. The frequency with which the kinetic energy oscillates is

(a) $v/2$    (b) $v$    (c) $2 v$    (d) zero.

14. A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is $T$. If the spring is divided in two equal parts and one part is used to continue the simple harmonic motion, the time period will

(a) remain $T$       (b) become $2T$

(c) become $T/2$       (d) become $T/\sqrt{2}$.

15. Two bodies $A$ and $B$ of equal mass are suspended from two separate massless springs of spring constant $k_1$ and $k_2$ respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of $A$ to that of $B$ is

(a) $k_1/k_2$       (b) $\sqrt{k_1/k_2}$

(c) $k_2/k_1$       (d) $\sqrt{k_2/k_1}$.

16. A spring-mass system oscillates with a frequency $v$. If it is taken in an elevator slowly accelerating upward, the frequency will

(a) increase       (b) decrease

(c) remain same       (d) become zero.

17. A spring-mass system oscillates in a car. If the car accelerates on a horizontal road, the frequency of oscillation will

(a) increase       (b) decrease

(c) remain same       (d) become zero.

18. A pendulum clock that keeps correct time on the earth is taken to the moon. It will run

(a) at correct rate       (b) 6 times faster

(c) √6 times faster       (d) √6 times slower.

19. A wall clock uses a vertical spring-mass system to measure the time. Each time the mass reaches an extreme position, the clock advances by a second. The clock gives correct time at the equator. If the clock is taken to the poles it will

(a) run slow       (b) run fast

(c) stop working       (d) give correct time.

20. A pendulum clock keeping correct time is taken to high altitudes,

(a) it will keep correct time

(b) its length should be increased to keep correct time

(c) its length should be decreased to keep correct time

(d) it cannot keep correct time even if the length is changed.

21. The free end of a simple pendulum is attached to the ceiling of a box. The box is taken to a height and the pendulum is oscillated. When the bob is at its lowest point, the box is released to fall freely. As seen from the box during this period, the bob will

(a) continue its oscillation as before

(b) stop

(c) will go in a circular path

(d) move on a straight line.

## OBJECTIVE II

1. Select the correct statements.

(a) A simple harmonic motion is necessarily periodic.

(b) A simple harmonic motion is necessarily oscillatory.

(c) An oscillatory motion is necessarily periodic.

(d) A periodic motion is necessarily oscillatory.

2. A particle moves in a circular path with a uniform speed. Its motion is

(a) periodic       (b) oscillatory

(c) simple harmonic       (d) angular simple harmonic.

3. A particle is fastened at the end of a string and is whirled in a vertical circle with the other end of the string being fixed. The motion of the particle is

(a) periodic       (b) oscillatory

(c) simple harmonic       (d) angular simple harmonic.

4. A particle moves in a circular path with a continuously increasing speed. Its motion is

(a) periodic       (b) oscillatory

(c) simple harmonic       (d) none of them.

5. The motion of a torsional pendulum is

(a) periodic       (b) oscillatory

(c) simple harmonic       (d) angular simple harmonic.

6. Which of the following quantities are always negative in a simple harmonic motion ?

(a) $\vec{F} . \vec{a}$.    (b) $\vec{v} . \vec{r}$.    (c) $\vec{a} . \vec{r}$.    (d) $\vec{F} . \vec{r}$.

7. Which of the following quantities are always positive in a simple harmonic motion ?

(a) $\vec{F} . \vec{a}$.    (b) $\vec{v} . \vec{r}$.    (c) $\vec{a} . \vec{r}$.    (d) $\vec{F} . \vec{r}$.

8. Which of the following quantities are always zero in a simple harmonic motion ?

(a) $\vec{F} \times \vec{a}$.    (b) $\vec{v} \times \vec{r}$.    (c) $\vec{a} \times \vec{r}$.    (d) $\vec{F} \times \vec{r}$.

9. Suppose a tunnel is dug along a diameter of the earth. A particle is dropped from a point, a distance $h$ directly

above the tunnel. The motion of the particle as seen from the earth is

(a) simple harmonic          (b) parabolic
(c) on a straight line        (d) periodic.

10. For a particle executing simple harmonic motion, the acceleration is proportional to
(a) displacement from the mean position
(b) distance from the mean position
(c) distance travelled since $t = 0$
(d) speed.

11. A particle moves in the $X$-$Y$ plane according to the equation

$$\vec{r} = (\vec{i} + 2\vec{j}) A \cos\omega t.$$

The motion of the particle is
(a) on a straight line        (b) on an ellipse
(c) periodic                  (d) simple harmonic.

12. A particle moves on the $X$-axis according to the equation $x = x_0 \sin^2 \omega t$. The motion is simple harmonic
(a) with amplitude $x_0$      (b) with amplitude $2x_0$

(c) with time period $\dfrac{2\pi}{\omega}$     (d) with time period $\dfrac{\pi}{\omega}$.

13. In a simple harmonic motion
(a) the potential energy is always equal to the kinetic energy
(b) the potential energy is never equal to the kinetic energy

(c) the average potential energy in any time interval is equal to the average kinetic energy in that time interval
(d) the average potential energy in one time period is equal to the average kinetic energy in this period.

14. In a simple harmonic motion
(a) the maximum potential energy equals the maximum kinetic energy
(b) the minimum potential energy equals the minimum kinetic energy
(c) the minimum potential energy equals the maximum kinetic energy
(d) the maximum potential energy equals the minimum kinetic energy.

15. An object is released from rest. The time it takes to fall through a distance $h$ and the speed of the object as it falls through this distance are measured with a pendulum clock. The entire apparatus is taken on the moon and the experiment is repeated
(a) the measured times are same
(b) the measured speeds are same
(c) the actual times in the fall are equal
(d) the actual speeds are equal.

16. Which of the following will change the time period as they are taken to moon ?
(a) A simple pendulum.        (b) A physical pendulum.
(c) A torsional pendulum.     (d) A spring-mass system.

## EXERCISES

1. A particle executes simple harmonic motion with an amplitude of 10 cm and time period 6 s. At $t = 0$ it is at position $x = 5$ cm going towards positive $x$-direction. Write the equation for the displacement $x$ at time $t$. Find the magnitude of the acceleration of the particle at $t = 4$ s.

2. The position, velocity and acceleration of a particle executing simple harmonic motion are found to have magnitudes 2 cm, 1 m/s and 10 m/s$^2$ at a certain instant. Find the amplitude and the time period of the motion.

3. A particle executes simple harmonic motion with an amplitude of 10 cm. At what distance from the mean position are the kinetic and potential energies equal ?

4. The maximum speed and acceleration of a particle executing simple harmonic motion are 10 cm/s and 50 cm/s$^2$. Find the position(s) of the particle when the speed is 8 cm/s.

5. A particle having mass 10 g oscillates according to the equation $x = (2.0 \text{ cm}) \sin[(100 \text{ s}^{-1})t + \pi/6]$. Find (a) the amplitude, the time period and the spring constant (b) the position, the velocity and the acceleration at $t = 0$.

6. The equation of motion of a particle started at $t = 0$ is given by $x = 5 \sin (20 t + \pi/3)$ where $x$ is in centimetre and $t$ in second. When does the particle
(a) first come to rest

(b) first have zero acceleration
(c) first have maximum speed ?

7. Consider a particle moving in simple harmonic motion according to the equation

$$x = 2.0 \cos(50 \pi t + \tan^{-1} 0.75)$$

where $x$ is in centimetre and $t$ in second. The motion is started at $t = 0$. (a) When does the particle come to rest for the first time ? (b) When does the acceleration have its maximum magnitude for the first time ? (c) When does the particle come to rest for the second time ?

8. Consider a simple harmonic motion of time period $T$. Calculate the time taken for the displacement to change value from half the amplitude to the amplitude.

9. The pendulum of a clock is replaced by a spring-mass system with the spring having spring constant 0.1 N/m. What mass should be attached to the spring ?

10. A block suspended from a vertical spring is in equilibrium. Show that the extension of the spring equals the length of an equivalent simple pendulum i.e. a pendulum having frequency same as that of the block.

11. A block of mass 0.5 kg hanging from a vertical spring executes simple harmonic motion of amplitude 0.1 m and time period 0.314 s. Find the maximum force exerted by the spring on the block.

12. A body of mass 2 kg suspended through a vertical spring executes simple harmonic motion of period 4 s. If the oscillations are stopped and the body hangs in

equilibrium, find the potential energy stored in the spring.

3. A spring stores 5 J of energy when stretched by 25 cm. It is kept vertical with the lower end fixed. A block fastened to its other end is made to undergo small oscillations. If the block makes 5 oscillations each second, what is the mass of the block ?

4. A small block of mass $m$ is kept on a bigger block of mass $M$ which is attached to a vertical spring of spring constant $k$ as shown in the figure. The system oscillates vertically. (a) Find the resultant force on the smaller block when it is displaced through a distance $x$ above its equilibrium position. (b) Find the normal force on the smaller block at this position. When is this force smallest in magnitude ? (c) What can be the maximum amplitude with which the two blocks may oscillate together ?
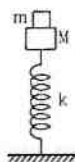


Figure 12-E1

15. The block of mass $m_1$ shown in figure (12-E2) is fastened to the spring and the block of mass $m_2$ is placed against it. (a) Find the compression of the spring in the equilibrium position. (b) The blocks are pushed a further distance $(2/k) (m_1 + m_2)g \sin\theta$ against the spring and released. Find the position where the two blocks separate. (c) What is the common speed of blocks at the time of separation ?
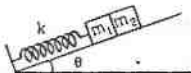


Figure 12-E2

16. In figure (12-E3) $k = 100$ N/m, $M = 1$ kg and $F = 10$ N. (a) Find the compression of the spring in the equilibrium position. (b) A sharp blow by some external agent imparts a speed of 2 m/s to the block towards left. Find the sum of the potential energy of the spring and the kinetic energy of the block at this instant. (c) Find the time period of the resulting simple harmonic motion. (d) Find the amplitude. (e) Write the potential energy of the spring when the block is at the left extreme. (f) Write the potential energy of the spring when the block is at the right extreme.
The answers of (b), (e) and (f) are different. Explain why this does not violate the principle of conservation of energy.
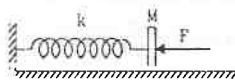


Figure 12-E3

17. Find the time period of the oscillation of mass $m$ in figures (12-E4 a, b, c) What is the equivalent spring constant of the pair of springs in each case ?
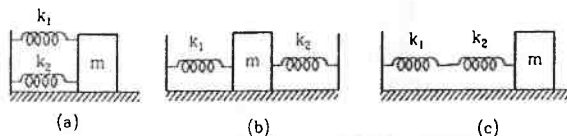


Figure 12-E4

18. The spring shown in figure (12-E5) is unstretched when a man starts pulling on the cord. The mass of the block is $M$. If the man exerts a constant force $F$, find (a) the amplitude and the time period of the motion of the block, (b) the energy stored in the spring when the block passes through the equilibrium position and (c) the kinetic energy of the block at this position.
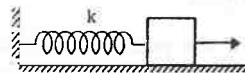


Figure 12-E5

19. A particle of mass $m$ is attatched to three springs $A$, $B$ and $C$ of equal force constants $k$ as shown in figure (12-E6). If the particle is pushed slightly against the spring $C$ and released, find the time period of oscillation.
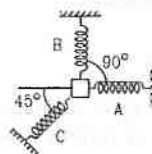


Figure 12-E6

20. Repeat the previous exercise if the angle between each pair of springs is 120° initially.

21. The springs shown in the figure (12-E7) are all unstretched in the beginning when a man starts pulling the block. The man exerts a constant force $F$ on the block. Find the amplitude and the frequency of the motion of the block.
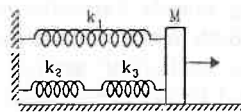


Figure 12-E7

22. Find the elastic potential energy stored in each spring shown in figure (12-E8), when the block is in equilibrium. Also find the time period of vertical oscillation of the block.
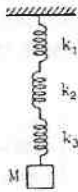
Figure 12-E8

23. The string, the spring and the pulley shown in figure (12-E9) are light. Find the time period of the mass $m$.
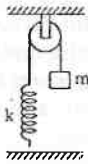


Figure 12-E9

24. Solve the previous problem if the pulley has a moment of inertia $I$ about its axis and the string does not slip over it.

25. Consider the situation shown in figure (12-E10). Show that if the blocks are displaced slightly in opposite directions and released, they will execute simple harmonic motion. Calculate the time period.
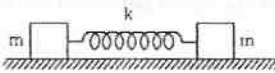


Figure 12-E10

26. A rectangular plate of sides $a$ and $b$ is suspended from a ceiling by two parallel strings of length $L$ each (figure 12-E11). The separation between the strings is $d$. The plate is displaced slightly in its plane keeping the strings tight. Show that it will execute simple harmonic motion. Find the time period.
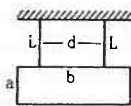


Figure 12-E11

27. A 1 kg block is executing simple harmonic motion of amplitude 0·1 m on a smooth horizontal surface under the restoring force of a spring of spring constant 100 N/m. A block of mass 3 kg is gently placed on it at the instant it passes through the mean position. Assuming that the two blocks move together, find the frequency and the amplitude of the motion.
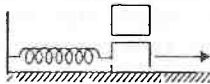


Figure 12-E12

28. The left block in figure (12-E13) moves at a speed $v$ towards the right block placed in equilibrium. All collisions to take place are elastic and the surfaces are frictionless. Show that the motions of the two blocks are periodic. Find the time period of these periodic motions. Neglect the widths of the blocks.
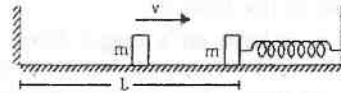


Figure 12-E13

29. Find the time period of the motion of the particle shown in figure (12-E14). Neglect the small effect of the bend near the bottom.
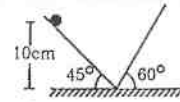


Figure 12-E14

30. All the surfaces shown in figure (12-E15) are frictionless. The mass of the car is $M$, that of the block is $m$ and the spring has spring constant $k$. Initially, the car and the block are at rest and the spring is stretched through a length $x_0$ when the system is released. (a) Find the amplitudes of the simple harmonic motion of the block and of the car as seen from the road. (b) Find the time period(s) of the two simple harmonic motions.
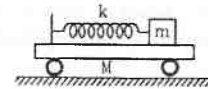


Figure 12-E15

31. A uniform plate of mass $M$ stays horizontally and symmetrically on two wheels rotating in opposite directions (figure 12-E16). The separation between the wheels is $L$. The friction coefficient between each wheel and the plate is $\mu$. Find the time period of oscillation of the plate if it is slightly displaced along its length and released.



Figure 12-E16

32. A pendulum having time period equal to two seconds is called a seconds pendulum. Those used in pendulum clocks are of this type. Find the length of a seconds pendulum at a place where $g = \pi^2$ m/s$^2$.

33. The angle made by the string of a simple pendulum with the vertical depends on time as $\theta = \frac{\pi}{90} \sin[(\pi \text{ s}^{-1}) t]$. Find the length of the pendulum if $g = \pi^2$ m/s$^2$.

34. The pendulum of a certain clock has time period 2·04 s. How fast or slow does the clock run during 24 hours ?

35. A pendulum clock giving correct time at a place where $g = 9·800$ m/s$^2$ is taken to another place where it loses 24 seconds during 24 hours. Find the value of $g$ at this new place.

36. A simple pendulum is constructed by hanging a heavy ball by a 5·0 m long string. It undergoes small oscillations. (a) How many oscillations does it make per second ? (b) What will be the frequency if the system is taken on the moon where acceleration due to gravitation of the moon is 1·67 m/s$^2$.

37. The maximum tension in the string of an oscillating pendulum is double of the minimum tension. Find the angular amplitude.

38. A small block oscillates back and forth on a smooth concave surface of radius $R$ (figure 12-E17). Find the time period of small oscillation.



Figure 12-E17

39. A spherical ball of mass $m$ and radius $r$ rolls without slipping on a rough concave surface of large radius $R$. It makes small oscillations about the lowest point. Find the time period.

40. A simple pendulum of length 40 cm is taken inside a deep mine. Assume for the time being that the mine is 1600 km deep. Calculate the time period of the pendulum there. Radius of the earth = 6400 km.

41. Assume that a tunnel is dug across the earth (radius = $R$) passing through its centre. Find the time a particle takes to cover the length of the tunnel if (a) it is projected into the tunnel with a speed of $\sqrt{gR}$ (b) it is released from a height $R$ above the tunnel (c) it is thrown vertically upward along the length of tunnel with a speed of $\sqrt{gR}$.

42. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance $R/2$ from the earth's centre where $R$ is the radius of the earth. The wall of the tunnel is frictionless. (a) Find the gravitational force exerted by the earth on a particle of mass $m$ placed in the tunnel at a distance $x$ from the centre of the tunnel. (b) Find the component of this force along the tunnel and perpendicular to the tunnel. (c) Find the normal force exerted by the wall on the particle. (d) Find the resultant force on the particle. (e) Show that the motion of the particle in the tunnel is simple harmonic and find the time period.

43. A simple pendulum of length $l$ is suspended through the ceiling of an elevator. Find the time period of small oscillations if the elevator (a) is going up with an acceleration $a_0$ (b) is going down with an acceleration $a_0$ and (c) is moving with a uniform velocity.

44. A simple pendulum of length 1 feet suspended from the ceiling of an elevator takes $\pi/3$ seconds to complete one oscillation. Find the acceleration of the elevator.

45. A simple pendulum fixed in a car has a time period of 4 seconds when the car is moving uniformly on a horizontal road. When the accelerator is pressed, the time period changes to 3·99 seconds. Making an approximate analysis, find the acceleration of the car.

46. A simple pendulum of length $l$ is suspended from the ceiling of a car moving with a speed $v$ on a circular horizontal road of radius $r$. (a) Find the tension in the string when it is at rest with respect to the car. (b) Find the time period of small oscillation.

47. The ear-ring of a lady shown in figure (12-E18) has a 3 cm long light suspension wire. (a) Find the time period of small oscillations if the lady is standing on the ground. (b) The lady now sits in a merry-go-round moving at 4 m/s in a circle of radius 2 m. Find the time period of small oscillations of the ear-ring.



Figure 12-E18

48. Find the time period of small oscillations of the following systems. (a) A metre stick suspended through the 20 cm mark. (b) A ring of mass $m$ and radius $r$ suspended through a point on its periphery. (c) A uniform square plate of edge $a$ suspended through a corner. (d) A unifrom disc of mass $m$ and radius $r$ suspended through a point $r/2$ away from the centre.

49. A uniform rod of length $l$ is suspended by an end and is made to undergo small oscillations. Find the length of the simple pendulum having the time period equal to that of the rod.

50. A uniform disc of radius $r$ is to be suspended through a small hole made in the disc. Find the minimum possible time period of the disc for small oscillations. What should be the distance of the hole from the centre for it to have minimum time period ?

51. A hollow sphere of radius 2 cm is attached to an 18 cm long thread to make a pendulum. Find the time period of oscillation of this pendulum. How does it differ from the time period calculated using the formula for a simple pendulum ?

52. A closed circular wire hung on a nail in a wall undergoes small oscillations of amplitude 2$^0$ and time period 2 s. Find (a) the radius of the circular wire, (b) the speed of the particle farthest away from the point of suspension as it goes through its mean position, (c) the acceleration of this particle as it goes through its mean position and (d) the acceleration of this particle when it is at an extreme position. Take $g = \pi^2$ m/s$^2$.

53. A uniform disc of mass $m$ and radius $r$ is suspended through a wire attached to its centre. If the time period of the torsional oscillations be $T$, what is the torsional constant of the wire.

**54.** Two small balls, each of mass $m$ are connected by a light rigid rod of length $L$. The system is suspended from its centre by a thin wire of torsional constant $k$. The rod is rotated about the wire through an angle $\theta_0$ and released. Find the tension in the rod as the system passes through the mean position.
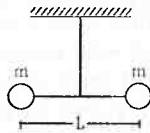


Figure 12-E19

**55.** A particle is subjected to two simple harmonic motions of same time period in the same direction. The amplitude of the first motion is $3.0$ cm and that of the second is $4.0$ cm. Find the resultant amplitude if the phase difference between the motions is (a) $0°$, (b) $60°$, (c) $90°$.

**56.** Three simple harmonic motions of equal amplitudes $A$ and equal time periods in the same direction combine. The phase of the second motion is $60°$ ahead of the first and the phase of the third motion is $60°$ ahead of the second. Find the amplitude of the resultant motion.

**57.** A particle is subjected to two simple harmonic motions given by

$$x_1 = 2.0 \sin(100 \pi t) \text{ and } x_2 = 2.0 \sin(120 \pi t + \pi/3)$$

where $x$ is in centimeter and $t$ in second. Find the displacement of the particle at (a) $t = 0.0125$ (b) $t = 0.025$.

**58.** A particle is subjected to two simple harmonic motions, one along the $X$-axis and the other on a line making an angle of $45°$ with the $X$-axis. The two motions are given by

$$x = x_0 \sin \omega t \text{ and } s = s_0 \sin \omega t$$

Find the amplitude of the resultant motion.

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (a) | 2. (b) | 3. (a) | 4. (d) | 5. (c) | 6. (d) |
| 7. (d) | 8. (c) | 9. (b) | 10. (c) | 11. (d) | 12. (a) |
| 13. (c) | 14. (d) | 15. (d) | 16. (c) | 17. (c) | 18. (d) |
| 19. (d) | 20. (c) | 21. (c). | | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (b) | 2. (a) | 3. (a) |
| 4. (d) | 5. (a), (b), (d) | 6. (c), (d) |
| 7. (a) | 8. all | 9. (c), (d) |
| 10. (a) | 11. (a), (c), (d) | 12. (d) |
| 13. (d) | 14. (a), (b) | 15. (a), (b) |
| 16. (a), (b). | | |

### EXERCISES

1. $x = (10 \text{ cm}) \sin\left(\dfrac{2\pi}{6\text{ s}} t + \dfrac{\pi}{6}\right)$, $\approx 11$ cm/s$^2$

2. $4.9$ cm, $0.28$ s

3. $5\sqrt{2}$ cm

4. $\pm 1.2$ cm from the mean position

5. (a) $2.0$ cm, $0.063$ s, $100$ N/m

   (b) $1.0$ cm, $1.73$ m/s, $100$ m/s$^2$

6. (a) $\dfrac{\pi}{120}$ s (b) $\dfrac{\pi}{30}$ s (c) $\dfrac{\pi}{30}$ s

7. (a) $1.6 \times 10^{-2}$ s (b) $1.6 \times 10^{-2}$ s (c) $3.6 \times 10^{-2}$ s

8. $T/6$

9. $\approx 10$ g

11. $25$ N

12. $40$ J

13. $0.16$ kg

14. (a) $\dfrac{mhx}{M+m}$ (b) $mg - \dfrac{mkx}{M+m}$, at the highest point

   (c) $g\dfrac{(M+m)}{k}$

15. (a) $\dfrac{(m_1 + m_2)g \sin\theta}{k}$

   (b) When the spring acquires its natural length

   (c) $\sqrt{\dfrac{3}{k}(m_1 + m_2)}\, g \sin\theta$

16. (a) $10$ cm (b) $2.5$ J (c) $\pi/5$ s

   (d) $20$ cm (e) $4.5$ J (f) $0.5$ J

17. (a) $2\pi \sqrt{\dfrac{m}{k_1 + k_2}}$ (b) $2\pi \sqrt{\dfrac{m}{k_1 + k_2}}$ (c) $2\pi \sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$

18. (a) $\dfrac{F}{k}$, $2\pi \sqrt{\dfrac{M}{k}}$, (b) $\dfrac{F^2}{2k}$ (c) $\dfrac{F^2}{2k}$

19. $2\pi \sqrt{\dfrac{m}{2k}}$

20. $2\pi \sqrt{\dfrac{2m}{3k}}$

21. $\dfrac{F(k_2 + k_3)}{k_1 k_2 + k_2 k_3 + k_3 k_1}$, $\dfrac{1}{2\pi} \sqrt{\dfrac{k_1 k_2 + k_2 k_3 + k_3 k_1}{M(k_2 + k_3)}}$

22. $\dfrac{M^2g^2}{2k_1}$, $\dfrac{M^2g^2}{2k_2}$ and $\dfrac{M^2g^2}{2k_3}$ from above, time period

$$= 2\pi\sqrt{M\left(\dfrac{1}{k_1} + \dfrac{1}{k_2} + \dfrac{1}{k_3}\right)}$$

23. $2\pi\sqrt{\dfrac{m}{k}}$

24. $2\pi\sqrt{\dfrac{(m + I/r^2)}{k}}$

25. $2\pi\sqrt{\dfrac{m}{2k}}$

26. $2\pi\sqrt{\dfrac{L}{g}}$

27. $\dfrac{5}{2\pi}$ Hz, 5 cm

28. $\left(\pi\sqrt{\dfrac{m}{k}} + \dfrac{2L}{v}\right)$

29. $\approx 0.73$ s

30. (a) $\dfrac{Mx_0}{M + m}$, $\dfrac{mx_0}{M + m}$ (b) $2\pi\sqrt{\dfrac{mM}{k(M + m)}}$

31. $2\pi\sqrt{\dfrac{l}{2\mu g}}$

32. 1 m

33. 1 m

34. 28.8 minutes slow

35. 9.795 m/s$^2$

36. (a) $0.70/\pi$ (b) $1/(2\pi\sqrt{3})$ Hz

37. $\cos^{-1}(3/4)$

38. $2\pi\sqrt{R/g}$

39. $2\pi\sqrt{\dfrac{7(R - r)}{5g}}$

40. 1.47 s

41. $\dfrac{\pi}{2}\sqrt{\dfrac{R}{g}}$ in each case

42. (a) $\dfrac{GMm}{R^3}\sqrt{x^2 + R^2/4}$ (b) $\dfrac{GMm}{R^3}x$, $\dfrac{GMm}{2R^2}$

  (c) $\dfrac{GMm}{2R^2}$, (d) $\dfrac{GMm}{R^3}x$ (e) $2\pi\sqrt{R^3/(GM)}$

43. (a) $2\pi\sqrt{\dfrac{l}{g + a_0}}$  (b) $2\pi\sqrt{\dfrac{l}{g - a_0}}$  (c) $2\pi\sqrt{\dfrac{l}{g}}$

44. 4 feet/s$^2$ upwards

45. $g/10$

46. (a) $ma$ (b) $2\pi\sqrt{l/a}$ where $a = \left[g^2 + \dfrac{v^4}{r^2}\right]^{1/2}$

47. (a) 0.34 s (b) 0.30 s

48. (a) 1.51 s (b) $2\pi\sqrt{\dfrac{2r}{g}}$ (c) $2\pi\sqrt{\dfrac{\sqrt{8}a}{3g}}$ (d) $2\pi\sqrt{\dfrac{3r}{2g}}$

49. $2l/3$

50. $2\pi\sqrt{\dfrac{r\sqrt{2}}{g}}$, $r/\sqrt{2}$

51. 0.89 s, it is about 0.3% larger than the calculated value

52. (a) 50 cm (b) 11 cm/s

  (c) 1.2 cm/s$^2$ towards the point of suspension

  (d) 34 cm/s$^2$ towards the mean position

53. $\dfrac{2\pi^2mr^2}{T^2}$

54. $\left[\dfrac{k^2\theta_0^4}{L} + m^2g^2\right]^{1/2}$

55. (a) 7.0 cm (b) 6.1 cm (c) 5.0 cm

56. $2A$

57. (a) $-2.41$ cm (b) 0.27 cm

58. $\left[x_0^2 + s_0^2 + \sqrt{2}\,x_0s_0\right]^{1/2}$

□

between A and B where the areas of cross-section are 30 cm $^2$ and 15 cm $^2$ respectively. Find the rate of flow of water through the tube.

**Solution :** Let the velocity at $A = v_A$ and that at $B = v_B$.

By the equation of continuity, $\dfrac{v_B}{v_A} = \dfrac{30 \text{ cm}^2}{15 \text{ cm}^2} = 2$.

By Bernoulli's equation,

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

or, $\qquad P_A - P_B = \dfrac{1}{2}\rho(2v_A)^2 - \dfrac{1}{2}\rho v_A^2 = \dfrac{3}{2}\rho v_A^2$

or, $\qquad 600\,\dfrac{\text{N}}{\text{m}^2} = \dfrac{3}{2}\left(1000\,\dfrac{\text{kg}}{\text{m}^3}\right)v_A^2$

or, $\qquad v_A = \sqrt{0.4 \text{ m}^2/\text{s}^2} = 0.63 \text{ m/s}$.

The rate of flow = $(30 \text{ cm}^2)(0.63 \text{ m/s}) = 1890 \text{ cm}^3/\text{s}$.

11. *The area of cross-section of a large tank is $0.5 \text{ m}^2$. It has an opening near the bottom having area of cross-section $1 \text{ cm}^2$. A load of 20 kg is applied on the water at the top. Find the velocity of the water coming out of the opening at the time when the height of water level is 50 cm above the bottom. Take $g = 10 \text{ m/s}^2$.*

**Solution :**
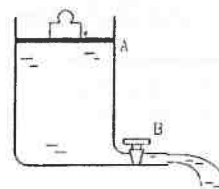


Figure 13-W7

As the area of cross-section of the tank is large compared to that of the opening, the speed of water in the tank will be very small as compared to the speed at the opening. The pressure at the surface of water in the tank is that due to the atmosphere plus due to the load.

$$P_A = P_0 + \frac{(20 \text{ kg})(10 \text{ m/s}^2)}{0.5 \text{ m}^2} = P_0 + 400 \text{ N/m}^2.$$

At the opening, the pressure is that due to the atmosphere.

Using Bernoulli's equation,

$$P_A + \rho g h + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

or, $P_0 + 400 \text{ N/m}^2 + (1000 \text{ kg/m}^3)(10 \text{ m/s}^2)(0.5 \text{ m}) + 0$

$$= P_0 + \frac{1}{2}(1000 \text{ kg/m}^3)v_B^2$$

or, $\qquad 5400 \text{ N/m}^2 = (500 \text{ kg/m}^3)v_B^2$

or, $\qquad v_B = 3.3 \text{ m/s}$.

---

# QUESTIONS FOR SHORT ANSWER

1. Is it always true that the molecules of a dense liquid are heavier than the molecules of a lighter liquid ?

2. If someone presses a pointed needle against your skin, you are hurt. But if someone presses a rod against your skin with the same force, you easily tolerate. Explain.

3. In the derivation of $P_1 - P_2 = \rho g z$, it was assumed that the liquid is incompressible. Why will this equation not be strictly valid for a compressible liquid ?

4. Suppose the density of air at Madras is $\rho_0$ and atomspheric pressure is $P_0$. If we go up, the density and the pressure both decrease. Suppose we wish to calculate the pressure at a height 10 km above Madras. If we use the equation $P_0 - P = \rho_0 g z$, will we get a pressure more than the actual or less than the actual ? Neglect the variation in $g$. Does your answer change if you also consider the variation in $g$ ?

5. The free surface of a liquid resting in an inertial frame is horizontal. Does the normal to the free surface pass through the centre of the earth ? Think separately if the liquid is (a) at the equator (b) at a pole (c) somewhere else.

6. A barometer tube reads 76 cm of mercury. If the tube is gradually inclined keeping the open end immersed in the mercury reservoir, will the length of mercury column be 76 cm, more than 76 cm or less than 76 cm ?

7. A one meter long glass tube is open at both ends. One end of the tube is dipped into a mercury cup, the tube is kept vertical and the air is pumped out of the tube by connecting the upper end to a suction pump. Can mercury be pulled up into the pump by this process ?

8. A satellite revolves round the earth. Air pressure inside the satellite is maintained at 76 cm of mercury. What will be the height of mercury column in a barometer tube 1 m long placed in the satellite ?

9. Consider the barometer shown in figure (13-Q1). If a small hole is made at a point $P$ in the barometer tube, will the mercury come out from this hole ?
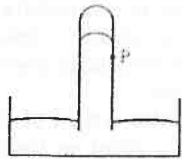
Figure 13-Q1

10. Is Archimedes' principle valid in an elevator accelerating up ? In a car accelerating on a level road ?

11. Why is it easier to swim in sea water than in fresh water ?

12. A glass of water has an ice cube floating in water. The water level just touches the rim of the glass. Will the water overflow when the ice melts ?

13. A ferry boat loaded with rocks has to pass under a bridge. The maximum height of the rocks is slightly more than the height of the bridge so that the boat just fails to pass under the bridge. Should some of the rocks be removed or some more rocks be added ?

14. Water is slowly coming out from a vertical pipe. As the water descends after coming out, its area of cross-section reduces. Explain this on the basis of the equation of continuity.

15. While watering a distant plant, a gardener partially closes the exit hole of the pipe by putting his finger on it. Explain why this results in the water stream going to a larger distance.

16. A Gipsy car has a canvass top. When the car runs at high speed, the top bulges out. Explain.

## OBJECTIVE I

1. A liquid can easily change its shape but a solid can not because
   (a) the density of a liquid is smaller than that of a solid
   (b) the forces between the molecules is stronger in solid than in liquids
   (c) the atoms combine to form bigger molecules in a solid
   (d) the average separation between the molecules is larger in solids.

2. Consider the equations
$$P = \lim_{\Delta s \to 0} \frac{F}{\Delta S} \text{ and } P_1 - P_2 = \rho g z.$$
   In an elevator accelerating upward
   (a) both the equations are valid
   (b) the first is valid but not the second
   (c) the second is valid but not the first
   (d) both are invalid.

3. The three vessels shown in figure (13-Q2) have same base area. Equal volumes of a liquid are poured in the three vessels. The force on the base will be
   (a) maximum in vessel $A$ (b) maximum in vessel $B$
   (c) maximum in vessel $C$ (d) equal in all the vessels.
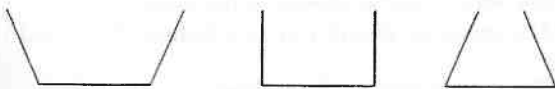


Figure 13-Q2

4. Equal mass of three liquids are kept in three identical cylindrical vessels $A$, $B$ and $C$. The densities are $\rho_A$, $\rho_B$, $\rho_C$ with $\rho_A < \rho_B < \rho_C$. The force on the base will be
   (a) maximum in vessel $A$ (b) maximum in vessel $B$
   (c) maximum in vessel $C$ (d) equal in all the vessels.

5. Figure (13-Q3) shows a siphon. The liquid shown is water. The pressure difference $P_B - P_A$ between the points $A$ and $B$ is

(a) 400 N/m$^2$      (b) 3000 N/m$^1$
(c) 1000 N/m$^1$     (d) zero.
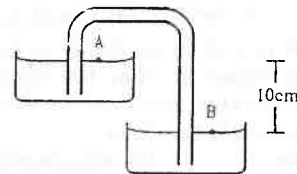


Figure 13-Q3

6. A beaker containing a liquid is kept inside a big closed jar. If the air inside the jar is continuously pumped out, the pressure in the liquid near the bottom of the liquid will
   (a) increase      (b) decrease      (c) remain constant
   (d) first decrease and then increase.

7. The pressure in a liquid at two points in the same horizontal plane are equal. Consider an elevator accelerating upward and a car accelerating on a horizontal road. The above statement is correct in
   (a) the car only
   (b) the elevator only
   (c) both of them
   (d) neither of them.

8. Suppose the pressure at the surface of mercury in a barometer tube is $P_1$ and the pressure at the surface of mercury in the cup is $P_2$.
   (a) $P_1 = 0$, $P_2 =$ atmospheric pressure
   (b) $P_1 =$ atmospheric pressure, $P_2 = 0$
   (c) $P_1 = P_2 =$ atmospheric pressure
   (d) $P_1 = P_2 = 0$.

9. A barometer kept in an elevator reads 76 cm when it is at rest. If the elevator goes up with increasing speed,

the reading will be
(a) zero     (b) 76 cm     (c) < 76 cm     (d) > 76 cm.

10. A barometer kept in an elevator accelerating upward reads 76 cm. The air pressure in the elevator is
(a) 76 cm     (b) < 76 cm     (c) > 76 cm     (d) zero.

11. To construct a barometer, a tube of length 1 m is filled completely with mercury and is inverted in a mercury cup. The barometer reading on a particular day is 76 cm. Suppose a 1 m tube is filled with mercury up to 76 cm and then closed by a cork. It is inverted in a mercury cup and the cork is removed. The height of mercury column in the tube over the surface in the cup will be
(a) zero     (b) 76 cm     (c) > 76 cm     (d) < 76 cm.

12. A 20 N metal block is suspended by a spring balance. A beaker containing some water is placed on a weighing machine which reads 40 N. The spring balance is now lowered so that the block gets immersed in the water. The spring balance now reads 16 N. The reading of the weighing machine will be
(a) 36 N     (b) 60 N     (c) 44 N     (d) 56 N.

13. A piece of wood is floating in water kept in a bottle. The bottle is connected to an air pump. Neglect the compressibility of water. When more air is pushed into the bottle from the pump, the piece of wood will float with
(a) larger part in the water (b) lesser part in the water
(c) same part in the water   (d) it will sink.

14. A metal cube is placed in an empty vessel. When water is filled in the vessel so that the cube is completely immersed in the water, the force on the bottom of the vessel in contact with the cube
(a) will increase     (b) will decrease
(c) will remain the same     (d) will become zero.

15. A wooden object floats in water kept in a beaker. The object is near a side of the beaker (figure 13-Q4). Let $P_1, P_2, P_3$ be the pressures at the three points $A$, $B$ and $C$ of the bottom as shown in the figure.
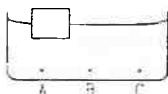


Figure 13-Q4

(a) $P_1 = P_2 = P_3$.
(c) $P_1 > P_2 > P_3$.
(b) $P_1 < P_2 < P_3$.
(d) $P_1 = P_3 \neq P_2$.

16. A closed cubical box is completely filled with water and is accelerated horizontally towards right with an acceleration $a$. The resultant normal force by the water on the top of the box
(a) passes through the centre of the top
(b) passes through a point to the right of the centre
(c) passes through a point to the left of the centre
(d) becomes zero.

17. Consider the situation of the previous problem. Let the water push the left wall by a force $F_1$ and the right wall by a force $F_2$.
(a) $F_1 = F_2$.     (b) $F_1 > F_2$.     (c) $F_1 < F_2$.
(d) The information is insufficient to know the relation between $F_1$ and $F_2$.

18. Water enters through end $A$ with a speed $v_1$ and leaves through end $B$ with a speed $v_2$ of a cylindrical tube $AB$. The tube is always completely filled with water. In case I the tube is horizontal, in case II it is vertical with the end $A$ upward and in case III it is vertical with the end $B$ upward. We have $v_1 = v_2$ for
(a) case I     (b) case II     (c) case III     (d) each case.

19. Bernoulli's theorem is based on conservation of
(a) momentum                (b) mass
(c) energy                  (d) angular momentum.

20. Water is flowing through a long horizontal tube. Let $P_A$ and $P_B$ be the pressures at two points $A$ and $B$ of the tube.
(a) $P_A$ must be equal to $P_B$.
(b) $P_A$ must be greater than $P_B$.
(c) $P_A$ must be smaller than $P_B$.
(d) $P_A = P_B$ only if the cross-sectional area at $A$ and $B$ are equal.

21. Water and mercury are filled in two cylindrical vessels up to same height. Both vessels have a hole in the wall near the bottom. The velocity of water and mercury coming out of the holes are $v_1$ and $v_2$ respectively.
(a) $v_1 = v_2$.                 (b) $v_1 = 13.6 \, v_2$.
(c) $v_1 = v_2/13.6$.            (d) $v_1 = \sqrt{13.6} \, v_2$.

22. A large cylindrical tank has a hole of area $A$ at its bottom. Water is poured in the tank by a tube of equal cross-sectional area $A$ ejecting water at the speed $v$.
(a) The water level in the tank will keep on rising.
(b) No water can be stored in the tank.
(c) The water level will rise to a height $v^2/2g$ and then stop.
(d) The water level will oscillate.

OBJECTIVE II

1. A solid floats in a liquid in a partially dipped position.
(a) The solid exerts a force equal to its weight on the liquid.
(b) The liquid exerts a force of buoyancy on the solid which is equal to the weight of the solid.
(c) The weight of the displaced liquid equals the weight of the solid.
(d) The weight of the dipped part of the solid is equal to the weight of the displaced liquid.

2. The weight of an empty balloon on a spring balance is $W_1$. The weight becomes $W_2$ when the balloon is filled with air. Let the weight of the air itself be $w$. Neglect the thickness of the balloon when it is filled with air. Also neglect the difference in the densities of air inside and outside the balloon.
   (a) $W_2 = W_1$.      (b) $W_2 = W_1 + w$.
   (c) $W_2 < W_1 + w$.      (d) $W_2 > W_1$.

3. A solid is completely immersed in a liquid. The force exerted by the liquid on the solid will
   (a) increase if it is pushed deeper inside the liquid
   (b) change if its orientation is changed
   (c) decrease if it is taken partially out of the liquid
   (d) be in the vertically upward direction.

4. A closed vessel is half filled with water. There is a hole near the top of the vessel and air is pumped out from this hole.
   (a) The water level will rise up in the vessel.
   (b) The pressure at the surface of the water will decrease.
   (c) The force by the water on the bottom of the vessel will decrease.
   (d) The density of the liquid will decrease.

5. In a streamline flow,
   (a) the speed of a particle always remains same
   (b) the velocity of a particle always remains same
   (c) the kinetic energies of all the particles arriving at a given point are the same
   (d) the momenta of all the particles arriving at a given point are the same.

6. Water flows through two identical tubes $A$ and $B$. A volume $V_0$ of water passes through the tube $A$ and $2\,V_0$ through $B$ in a given time. Which of the following may be correct ?
   (a) Flow in both the tubes are steady.
   (b) Flow in both the tubes are turbulent.
   (c) Flow is steady in $A$ but turbulent in $B$.
   (d) Flow is steady in $B$ but turbulent in $A$.

7. Water is flowing in streamline motion through a tube with its axis horizontal. Consider two points $A$ and $B$ in the tube at the same horizontal level.
   (a) The pressures at $A$ and $B$ are equal for any shape of the tube.
   (b) The pressures are never equal.
   (c) The pressures are equal if the tube has a uniform cross-section.
   (d) The pressures may be equal even if the tube has a nonuniform cross-section.

8. There is a small hole near the bottom of an open tank filled with a liquid. The speed of the water ejected does not depend on
   (a) area of the hole      (b) density of the liquid
   (c) height of the liquid from the hole
   (d) acceleration due to gravity.

## EXERCISES

1. The surface of water in a water tank on the top of a house is 4 m above the tap level. Find the pressure of water at the tap when the tap is closed. Is it necessary to specify that the tap is closed ? Take $g = 10 \text{ m/s}^2$.

2. The heights of mercury surfaces in the two arms of the manometer shown in figure (13-E1) are 2 cm and 8 cm. Atmospheric pressure $= 1.01 \times 10^5 \text{ N/m}^2$. Find (a) the pressure of the gas in the cylinder and (b) the pressure of mercury at the bottom of the U tube.
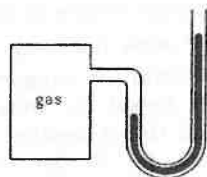


Figure 13-E1

3. The area of cross-section of the wider tube shown in figure (13-E2) is 900 cm$^2$. If the boy standing on the
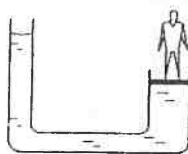


Figure 13-E2

piston weighs 45 kg, find the difference in the levels of water in the two tubes.

4. A glass full of water has a bottom of area 20 cm$^2$, top of area 20 cm$^2$, height 20 cm and volume half a litre.
   (a) Find the force exerted by the water on the bottom.
   (b) Considering the equilibrium of the water, find the resultant force exerted by the sides of the glass on the water. Atmospheric pressure $= 1.0 \times 10^5 \text{ N/m}^2$. Density of water $= 1000 \text{ kg/m}^3$ and $g = 10 \text{ m/s}^2$. Take all numbers to be exact.
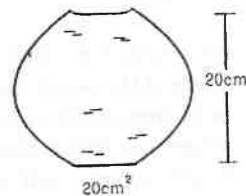


Figure 13-E3

5. Suppose the glass of the previous problem is covered by a jar and the air inside the jar is completely pumped out. (a) What will be the answers to the problem ? (b) Show that the answers do not change if a glass of different shape is used provided the height, the bottom area and the volume are unchanged.

6. If water be used to construct a barometer, what would be the height of water column at standard atmospheric pressure (76 cm of mercury) ?

7. Find the force exerted by the water on a 2 m² plane surface of a large stone placed at the bottom of a sea 500 m deep. Does the force depend on the orientation of the surface ?

8. Water is filled in a rectangular tank of size 3 m × 2 m × 1 m. (a) Find the total force exerted by the water on the bottom surface of the tank. (b) Consider a vertical side of area 2 m × 1 m. Take a horizontal strip of width $\delta x$ metre in this side, situated at a depth of $x$ metre from the surface of water. Find the force by the water on this strip. (c) Find the torque of the force calculated in part (b) about the bottom edge of this side. (d) Find the total force by the water on this side. (e) Find the total torque by the water on the side about the bottom edge. Neglect the atmospheric pressure and take $g = 10$ m/s².

9. An ornament weighing 36 g in air, weighs only 34 g in water. Assuming that some copper is mixed with gold to prepare the ornament, find the amount of copper in it. Specific gravity of gold is 19·3 and that of copper is 8·9.

10. Refer to the previous problem. Suppose, the goldsmith argues that he has not mixed copper or any other material with gold, rather some cavities might have been left inside the ornament. Calculate the volume of the cavities left that will allow the weights given in that problem.

11. A metal piece of mass 160 g lies in equilibrium inside a glass of water (figure 13-E4). The piece touches the bottom of the glass at a small number of points. If the density of the metal is 8000 kg/m³, find the normal force exerted by the bottom of the glass on the metal piece.
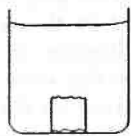


Figure 13-E4

12. A ferry boat has internal volume 1 m³ and weight 50 kg. (a) Neglecting the thickness of the wood, find the fraction of the volume of the boat immersed in water. (b) If a leak develops in the bottom and water starts coming in, what fraction of the boat's volume will be filled with water before water starts coming in from the sides ?

13. A cubical block of ice floating in water has to support a metal piece weighing 0·5 kg. What can be the minimum edge of the block so that it does not sink in water ? Specific gravity of ice = 0·9.

14. A cube of ice floats partly in water and partly in K.oil (figure 13-E5). Find the ratio of the volume of ice immersed in water to that in K.oil. Specific gravity of K.oil is 0·8 and that of ice is 0·9.
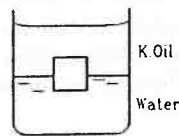


Figure 13-E5

15. A cubical box is to be constructed with iron sheets 1 mm in thickness. What can be the minimum value of the external edge so that the cube does not sink in water? Density of iron = 8000 kg/m³ and density of water = 1000 kg/m³.

16. A cubical block of wood weighing 200 g has a lead piece fastened underneath. Find the mass of the lead piece which will just allow the block to float in water. Specific gravity of wood is 0·8 and that of lead is 11·3.

17. Solve the previous problem if the lead piece is fastened on the top surface of the block and the block is to float with its upper surface just dipping into water.

18. A cubical metal block of edge 12 cm floats in mercury with one fifth of the height inside the mercury. Water is poured till the surface of the block is just immersed in it. Find the height of the water column to be poured. Specific gravity of mercury = 13·6.

19. A hollow spherical body of inner and outer radii 6 cm and 8 cm respectively floats half submerged in water. Find the density of the material of the sphere.

20. A solid sphere of radius 5 cm floats in water. If a maximum load of 0·1 kg can be put on it without wetting the load, find the specific gravity of the material of the sphere.

21. Find the ratio of the weights, as measured by a spring balance, of a 1 kg block of iron and a 1 kg block of wood. Density of iron = 7800 kg/m³, density of wood = 800 kg/m³ and density of air = 1·293 kg/m³.

22. A cylindrical object of outer diameter 20 cm and mass 2 kg floats in water with its axis vertical. If it is slightly depressed and then released, find the time period of the resulting simple harmonic motion of the object.

23. A cylindrical object of outer diameter 10 cm, height 20 cm and density 8000 kg/m³ is supported by a vertical spring and is half dipped in water as shown in figure(13-E6). (a) Find the elongation of the spring in equilibrium condition. (b) If the object is slightly depressed and released, find the time period of resulting oscillations of the object. The spring constant = 500 N/m.
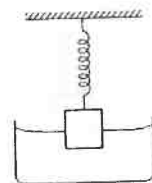


Figure 13-E6

24. A wooden block of mass 0·5 kg and density 800 kg/m³ is fastened to the free end of a vertical spring of spring constant 50 N/m fixed at the bottom. If the entire system is completely immersed in water, find (a) the elongation (or compression) of the spring in equilibrium and (b) the time-period of vertical oscillations of the block when it is slightly depressed and released.

25. A cube of ice of edge 4 cm is placed in an empty cylindrical glass of inner diameter 6 cm. Assume that the ice melts uniformly from each side so that it always retains its cubical shape. Remembering that ice is lighter than water, find the length of the edge of the ice cube at the instant it just leaves contact with the bottom of the glass.

26. A U-tube containing a liquid is accelerated horizontally with a constant acceleration $a_0$. If the separation between the vertical limbs is $l$, find the difference in the heights of the liquid in the two arms.

27. At Deoprayag (Garhwal, UP) river Alaknanda mixes with the river Bhagirathi and becomes river Ganga. Suppose Alaknanda has a width of 12 m, Bhagirathi has a width of 8 m and Ganga has a width of 16 m. Assume that the depth of water is same in the three rivers. Let the average speed of water in Alaknanda be 20 km/h and in Bhagirathi be 16 km/h. Find the average speed of water in the river Ganga.

28. Water flows through a horizontal tube of variable cross-section (figure 13-E7). The area of cross-section at $A$ and $B$ are 4 mm² and 2 mm² respectively. If 1 cc of water enters per second through $A$, find (a) the speed of water at $A$, (b) the speed of water at $B$ and (c) the pressure difference $P_A - P_B$.



Figure 13-E7

29. Suppose the tube in the previous problem is kept vertical with $A$ upward but the other conditions remain the same. The separation between the cross-sections at $A$ and $B$ is 15/16 cm. Repeat parts (a), (b) and (c) of the previous problem. Take $g = 10$ m/s².

30. Suppose the tube in the previous problem is kept vertical with $B$ upward. Water enters through $B$ at the rate of 1 cm³/s. Repeat parts (a), (b) and (c). Note that the speed decreases as the water falls down.

31. Water flows through a tube shown in figure (13-E8). The areas of cross-section at $A$ and $B$ are 1 cm² and 0·5 cm² respectively. The height difference between $A$ and $B$ is 5 cm. If the speed of water at $A$ is 10 cm/s find (a) the speed at $B$ and (b) the difference in pressures at $A$ and $B$.



Figure 13-E8

32. Water flows through a horizontal tube as shown in figure (13-E9). If the difference of heights of water column in the vertical tubes is 2 cm, and the areas of cross-section at $A$ and $B$ are 4 cm² and 2 cm² respectively, find the rate of flow of water across any section.
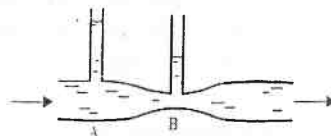


Figure 13-E9

33. Water flows through the tube shown in figure (13-E10). The areas of cross-section of the wide and the narrow portions of the tube are 5 cm² and 2 cm² respectively. The rate of flow of water through the tube is 500 cm³/s. Find the difference of mercury levels in the U-tube.
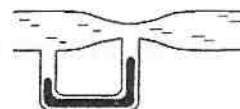


Figure 13-E10

34. Water leaks out from an open tank through a hole of area 2 mm² in the bottom. Suppose water is filled up to a height of 80 cm and the area of cross-section of the tank is 0·4 m². The pressure at the open surface and at the hole are equal to the atmospheric pressure. Neglect the small velocity of the water near the open surface in the tank. (a) Find the initial speed of water coming out of the hole. (b) Find the speed of water coming out when half of water has leaked out. (c) Find the volume of water leaked out during a time interval $dt$ after the height remained is $h$. Thus find the decrease in height $dh$ in terms of $h$ and $dt$. (d) From the result of part (c) find the time required for half of the water to leak out.

35. Water level is maintained in a cylindrical vessel upto a fixed height $H$. The vessel is kept on a horizontal plane. At what height above the bottom should a hole be made in the vessel so that the water stream coming out of the hole strikes the horizontal plane at the greatest distance from the vessel (figure 13-E11).
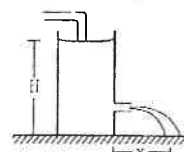


Figure 13.11

**ANSWERS**

### OBJECTIVE I

1. (b)    2. (b)    3. (c)    4. (d)    5. (d)    6. (b)
7. (b)    8. (a)    9. (c)    10. (c)    11. (d)    12. (c)
13. (c)    14. (c)    15. (a)    16. (c)    17. (b)    18. (d)
19. (c)    20. (d)    21. (a)    22. (c)

### OBJECTIVE II

1. (a), (b), (c)        2. (a), (c)        3. (c), (d)
4. (b), (c)        5. (c), (d)        6. (a), (b), (c)
7. (c), (d)        8. (a), (b)

### EXERCISES

1. 40000 N/m$^2$, Yes

2. (a) $1.09 \times 10^5$ N/m$^2$        (b) $1.12 \times 10^5$ N/m$^2$

3. 50 cm

4. (a) 204 N        (b) 1 N upward

5. 4 N, 1 N upward

6. 1033.6 cm

7. $10^7$ N, No

8. (a) 60000 N,  (b) $20000 \times \delta x$ N
   (c) $20000 \times (1-x)\delta x$ N–m  (d) 10000 N,
   (e) 10000/3 N–m

9. 2.2.g

10. 0.112 cm$^3$

11. 1.4 N

12. (a) $\dfrac{1}{20}$        (b) $\dfrac{19}{20}$

13. 17 cm

14. 1 : 1

15. 4.8 cm

16. 54.8 g

17. 50 g

18. 10.4 cm

19. 865 kg/m$^3$

20. 0.8

21. 1.0015

22. 0.5 s

23. (a) 23.5 cm        (b) 0.93 s

24. (a) 2.5 cm        (b) $\pi/5$ s

25. 2.26 cm

26. $a_0 l/g$

27. 23 km/h

28. (a) 25 cm/s,    (b) 50 cm/s    (c) 94 N/m$^2$

29. (a) 25 cm/s,    (b) 50 cm/s,    (d) zero

30. (a) 25 cm/s,    (b) 50 cm/s,    (c) 188 N/m$^2$

31. (a) 20 cm/s,    (b) 485 N/m$^2$

32. 146 cc/s,

33. 1.97 cm

34. (a) 4 m/s,        (b) $\sqrt{8}$ m/s
    (c) $(2 \text{ mm}^2)\sqrt{2gh}\ dt$,  $\sqrt{2gh} \times 5 \times 10^{-6}\ dt$
    (d) 6.5 hours

35. $H/2$.

**Solution :** The velocity gradient in vertical direction is

$$\frac{dv}{dx} = \frac{18 \text{ km/hr}}{5 \text{ m}} = 1 \cdot 0 \text{ s}^{-1}.$$

The magnitude of the force of viscosity is

$$F = \eta A \frac{dv}{dx}.$$

The shearing stress is

$$F/A = \eta \frac{dv}{dx} = (10^{-2} \text{ poise}) (1 \cdot 0 \text{ s}^{-1}) = 10^{-3} \text{ N/m}^2.$$

17. *Find the terminal velocity of a rain drop of radius 0·01 mm. The coefficient of viscosity of air is $1 \cdot 8 \times 10^{-5}$ N-s/m$^2$ and its density is $1 \cdot 2$ kg/m$^3$. Density of water = 1000 kg/m$^3$. Take g = 10 m/s$^2$.*

**Solution :** The forces on the rain drop are

(a) the weight $\frac{4}{3} \pi r^3 \rho g$ downward,

(b) the force of buoyancy $\frac{4}{3} \pi r^3 \sigma g$ upward,

(c) the force of viscosity $6\pi\eta rv$ upward.

Here $\rho$ is the density of water and $\sigma$ is the density of air. At terminal velocity the net force is zero. As the density of air is much smaller than the density of water, the force of buoyancy may be neglected.

Thus, at terminal velocity

$$6\pi\eta rv = \frac{4}{3} \pi r^3 \rho g$$

or,

$$v = \frac{2 \, r^2 \rho g}{9\eta}.$$

$$= \frac{2 \times (0 \cdot 01 \text{ mm})^2 \times (1000 \text{ kg/m}^3)(10 \text{ m/s}^2)}{9 \times (1 \cdot 8 \times 10^{-5} \text{ N-s/m}^2)}$$

$$= 1 \cdot 2 \text{ cm/s}.$$

□

# QUESTIONS FOR SHORT ANSWER

1. The ratio stress/strain remains constant for small deformation of a metal wire. When the deformation is made larger, will this ratio increase or decrease ?

2. When a block of mass $M$ is suspended by a long wire of length $L$, the elastic potential energy stored in the wire is $\frac{1}{2} \times$ stress $\times$ strain $\times$ volume. Show that it is equal to $\frac{1}{2} Mgl$, where $l$ is the extension. The loss in gravitational potential energy of the Mass-earth system is $Mgl$. Where does the remaining $\frac{1}{2} Mgl$ energy go ?

3. When the skeleton of an elephant and the skeleton of a mouse are prepared in the same size, the bones of the elephant are shown thicker than those of the mouse. Explain why the bones of an elephant are thicker than proportionate. The bones are expected to withstand the stress due to the weight of the animal.

4. The yield point of a typical solid is about 1%. Suppose you are lying horizontally and two persons are pulling your hands and two person are pulling your legs along your own length. How much will be the increase in your length if the strain is 1% ? Do you think your yield point is 1% or, much less than that ?

5. When rubber sheets are used in a shock absorber, what happens to the energy of vibration ?

6. If a compressed spring is dissolved in acid, what happens to the elastic potential energy of the spring ?

7. A steel blade placed gently on the surface of water floats on it. If the same blade is kept well inside the water, it sinks. Explain.

8. When some wax is rubbed on a cloth, it becomes waterproof. Explain.

9. The contact angle between pure water and pure silver is 90°. If a capillary tube made of silver is dipped at one end in pure water, will the water rise in the capillary ?

10. It is said that a liquid rises or is depressed in a capillary due to the surface tension. If a liquid neither rises nor depresses in a capillary, can we conclude that the surface tension of the liquid is zero ?

11. The contact angle between water and glass is 0°. When water is poured in a glass to the maximum of its capacity, the water surface is convex upward. The angle of contact in such a situation is more than 90°. Explain.

12. A uniform vertical tube of circular cross-section contains a liquid. The contact angle is 90°. Consider a diameter of the tube lying in the surface of the liquid. The surface to the right of this diameter pulls the surface on the left of it. What keeps the surface on the left in equilibrium ?

13. When a glass capillary tube is dipped at one end in water, water rises in the tube. The gravitational potential energy is thus increased. Is it a violation of conservation of energy ?

14. If a mosquito is dipped into water and released, it is not able to fly till it is dry again. Explain.

15. The force of surface tension acts tangentially to the surface whereas the force due to air pressure acts perpendicularly on the surface. How is then the force due to excess pressure inside a bubble balanced by the force due to the surface tension ?

16. When the size of a soap bubble is increased by pushing more air in it, the surface area increases. Does it mean

that the average separation between the surface molecules is increased ?

17. Frictional force between solids operates even when they do not move with respect to each other. Do we have viscous force acting between two layers even if there is no relative motion ?

18. Water near the bed of a deep river is quiet while that near the surface flows. Give reasons.

19. If water in one flask and castor oil in other are violently shaken and kept on a table, which will come to rest earlier ?

# OBJECTIVE I

1. A rope 1 cm in diameter breaks if the tension in it exceeds 500 N. The maximum tension that may be given to a similar rope of diameter 2 cm is
   (a) 500 N  (b) 250 N  (c) 1000 N  (d) 2000 N.

2. The breaking stress of a wire depends on
   (a) material of the wire  (b) length of the wire
   (c) radius of the wire  (d) shape of the cross-section.

3. A wire can sustain the weight of 20 kg before breaking. If the wire is cut into two equal parts, each part can sustain a weight of
   (a) 10 kg  (b) 20 kg  (c) 40 kg  (d) 80 kg.

4. Two wires $A$ and $B$ are made of same material. The wire $A$ has a length $l$ and diameter $r$ while the wire $B$ has a length $2l$ and diameter $r/2$. If the two wires are stretched by the same force, the elongation in $A$ divided by the elongation in $B$ is
   (a) 1/8  (b) 1/4  (c) 4  (d) 8.

5. A wire elongates by $1.0$ mm when a load $W$ is hanged from it. If this wire goes over a pulley and two weights $W$ each are hung at the two ends, the elongation of the wire will be
   (a) 0.5 m  (b) 1.0 mm  (c) 2.0 mm  (d) 4.0 mm.

6. A heavy uniform rod is hanging vertically from a fixed support. It is stretched by its own weight. The diameter of the rod is
   (a) smallest at the top and gradually increases down the rod
   (b) largest at the top and gradually decreases down the rod
   (c) uniform everywhere
   (d) maximum in the middle.

7. When a metal wire is stretched by a load, the fractional change in its volume $\Delta V/V$ is proportional to
   (a) $\dfrac{\Delta l}{l}$  (b) $\left(\dfrac{\Delta l}{l}\right)^2$  (c) $\sqrt{\Delta l/l}$  (d) none of these.

8. The length of a metal wire is $l_1$ when the tension in it is $T_1$ and is $l_2$ when the tension is $T_2$. The natural length of the wire is
   (a) $\dfrac{l_1 + l_2}{2}$  (b) $\sqrt{l_1 l_2}$  (c) $\dfrac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$  (d) $\dfrac{l_1 T_2 + l_2 T_1}{T_2 + T_1}$.

9. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break
   (a) when the mass is at the highest point
   (b) when the mass is at the lowest point
   (c) when the wire is horizontal
   (d) at an angle of $\cos^{-1}(1/3)$ from the upward vertical.

10. When a metal wire elongates by hanging a load on it the gravitational potential energy is decreased.
    (a) This energy completely appears as the increased kinetic energy of the block.
    (b) This energy completely appears as the increased elastic potential energy of the wire.
    (c) This energy completely appears as heat.
    (d) None of these.

11. By a surface of a liquid we mean
    (a) a geometrical plane like $x = 0$
    (b) all molecules exposed to the atmosphere
    (c) a layer of thickness of the order of $10^{-8}$ m
    (d) a layer of thickness of the order of $10^{-4}$ m.

12 An ice cube is suspended in vacuum in a gravity-free hall. As the ice melts it
    (a) will retain its cubical shape
    (b) will change its shape to spherical
    (c) will fall down on the floor of the hall
    (d) will fly up.

13. When water droplets merge to form a bigger drop
    (a) energy is liberated  (b) energy is absorbed
    (c) energy is neither liberated nor absobred
    (d) energy may either be liberated or absorbed depending on the nature of the liquid.

14. The dimension $ML^{-1}T^{-2}$ can correspond to
    (a) moment of a force  (b) surface tension
    (c) modulus of elasticity  (d) coefficient of viscosity.

15. Air is pushed into a soap bubble of radius $r$ to double its radius. If the surface tension of the soap solution is $S$, the work done in the process is
    (a) $8\pi r^2 S$  (b) $12\pi r^2 S$  (c) $16\pi r^2 S$  (d) $24\pi r^2 S$.

16. If more air is pushed in a soap bubble, the pressure in it
    (a) decreases  (b) increases
    (c) remains same  (d) becomes zero.

17. If two soap bubbles of different radii are connected by a tube,
    (a) air flows from bigger bubble to the smaller bubble till the sizes become equal
    (b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
    (c) air flows from the smaller bubble to the bigger
    (d) there is no flow of air.

18. Figure (14-Q1) shows a capillary tube of radius $r$ dipped into water. If the atmospheric pressure is $P_0$, the pressure at point $A$ is

(a) $P_0$    (b) $P_0 + \dfrac{2S}{r}$    (c) $P_0 - \dfrac{2S}{r}$    (d) $P_0 - \dfrac{4S}{r}$.
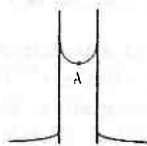


Figure 14-Q1

19. The excess pressure inside a soap bubble is twice the excess pressure inside a second soap bubble. The volume of the first bubble is $n$ times the volume of the second where $n$ is

(a) 4    (b) 2    (c) 1    (d) 0·125.

20. Which of the following graphs may represent the relation between the capillary rise $h$ and the radius $r$ of the capillary ?
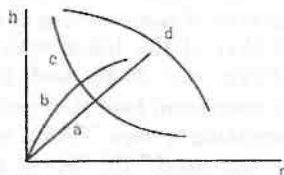


Figure 14-Q2

21. Water rises in a vertical capillary tube upto a length of 10 cm. If the tube is inclined at 45°, the length of water risen in the tube will be

(a) 10 cm          (b) $10\sqrt{2}$ cm
(c) $10/\sqrt{2}$ cm     (d) none of these.

22. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be

(a) 8 cm    (b) 6 cm    (c) 10 cm    (d) 20 cm.

23. Viscosity is a property of

(a) liquids only        (b) solids only
(c) solids and liquids only    (d) liquids and gases only.

24. The force of viscosity is

(a) electromagnetic (b) gravitational (c) nuclear (d) weak.

25. The viscous force acting between two layers of a liquid is given by $\dfrac{F}{A} = -\eta\,\dfrac{dv}{dz}$. This $F/A$ may be called

(a) pressure            (b) longitudinal stress
(c) tangential stress      (d) volume stress.

26. A raindrop falls near the surface of the earth with almost uniform velocity because

(a) its weight is negligible
(b) the force of surface tension balances its weight
(c) the force of viscosity of air balances its weight
(d) the drops are charged and atmospheric electric field balances its weight.

27. A piece of wood is taken deep inside a long column of water and released. It will move up

(a) with a constant upward acceleration
(b) with a decreasing upward acceleration
(c) with a deceleration
(d) with a uniform velocity.

28. A solid sphere falls with a terminal velocity of 20 m/s in air. If it is allowed to fall in vacuum,

(a) terminal velocity will be 20 m/s
(b) terminal velocity will be less than 20 m/s
(c) terminal velocity will be more than 20 m/s
(d) there will be no terminal velocity.

29. A spherical ball is dropped in a long column of a viscous liquid. The speed of the ball as a function of time may be best represented by the graph

(a) $A$    (b) $B$    (c) $C$    (d) $D$.
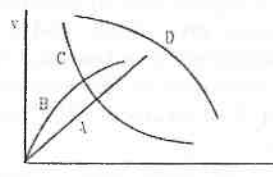


Figure 14-Q3

## OBJECTIVE II

1. A student plots a graph from his readings on the determination of Young's modulus of a metal wire but forgets to put the labels (figure 14-Q4). The quantities on $X$ and $Y$-axes may be respectively

(a) weight hung and length increased
(b) stress applied and length increased
(c) stress applied and strain developed
(d) length increased and the weight hung.
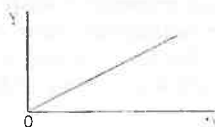


Figure 14-Q4

2. The properties of a surface are different from those of the bulk liquid because the surface molecules

(a) are smaller than other molelcules
(b) acquire charge due to collision from air molecules
(c) find different type of molecules in their range of influence
(d) feel a net force in one direction.

3. The rise of a liquid in a capillary tube depends on

(a) the material      (b) the length
(c) the outer radius    (d) the inner radius of the tube.

4. The contact angle between a solid and a liquid is a property of

(a) the material of the solid
(b) the material of the liquid

(c) the shape of the solid
(d) the mass of the solid.

5. A liquid is contained in a vertical tube of semicircular cross-section (figure 14-Q5). The contact angle is zero. The forces of surface tension on the curved part and on the flat part are in ratio
(a) 1 : 1     (b) 1 : 2     (c) $\pi$ : 2     (d) 2 : $\pi$.



Figure 14-Q5

6. When a capillary tube is dipped into a liquid, the liquid neither rises nor falls in the capillary.
(a) The surface tension of the liquid must be zero.
(b) The contact angle must be 90°.
(c) The surface tension may be zero.
(d) The contact angle may be 90°.

7. A solid sphere moves at a terminal velocity of 20 m/s in air at a place where $g = 9.8$ m/s$^2$. The sphere is taken in a gravity free hall having air at the same pressure and pushed down at a speed of 20 m/s.
(a) Its initial acceleration will be 9.8 m/s$^2$ downward.
(b) Its initial acceleration will be 9.8 m/s$^2$ upward.
(c) The magnitude of acceleration will decrease as the time passes.
(d) It will eventually stop.

# EXERCISES

1. A load of 10 kg is suspended by a metal wire 3 m long and having a cross-sectional area 4 mm$^2$. Find (a) the stress (b) the strain and (c) the elongation. Young's modulus of the metal is $2.0 \times 10^{11}$ N/m$^2$.

2. A vertical metal cylinder of radius 2 cm and length 2 m is fixed at the lower end and a load of 100 kg is put on it. Find (a) the stress (b) the strain and (c) the compression of the cylinder. Young's modulus of the metal = $2 \times 10^{11}$ N/m$^2$.

3. The elastic limit of steel is $8 \times 10^8$ N/m$^2$ and its Young's modulus $2 \times 10^{11}$ N/m$^2$. Find the maximum elongation of a half-meter steel wire that can be given without exceeding the elastic limit.

4. A steel wire and a copper wire of equal length and equal cross-sectional area are joined end to end and the combination is subjected to a tension. Find the ratio of (a) the stresses developed in the two wires and (b) the strains developed. $Y$ of steel = $2 \times 10^{11}$ N/m$^2$. $Y$ of copper = $1.3 \times 10^{11}$ N/m$^2$.

5. In figure (14-E1) the upper wire is made of steel and the lower of copper. The wires have equal cross-section. Find the ratio of the longitudinal strains developed in the two wires.
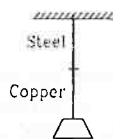


Figure 14-E1

6. The two wires shown in figure (14-E2) are made of the
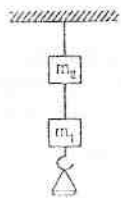


Figure 14-E2

same material which has a breaking stress of $8 \times 10^8$ N/m$^2$. The area of cross-section of the upper wire is 0.006 cm$^2$ and that of the lower wire is 0.003 cm$^2$. The mass $m_1 = 10$ kg, $m_2 = 20$ kg and the hanger is light. (a) Find the maximum load that can be put on the hanger without breaking a wire. Which wire will break first if the load is increased ? (b) Repeat the above part if $m_1 = 10$ kg and $m_2 = 36$ kg.

7. Two persons pull a rope towards themselves. Each person exerts a force of 100 N on the rope. Find the Young's modulus of the material of the rope if it extends in length by 1 cm. Original length of the rope = 2 m and the area of cross-section = 2 cm$^2$.

8. A steel rod of cross-sectional area 4 cm$^2$ and length 2 m shrinks by 0.1 cm as the temperature decreases in night. If the rod is clamped at both ends during the day hours, find the tension developed in it during night hours. Young's modulus of steel = $1.9 \times 10^{11}$ N/m$^2$.

9. Consider the situation shown in figure (14-E3). The force $F$ is equal to the $m_2 g/2$. If the area of cross-section of the string is $A$ and its Young's modulus $Y$, find the strain developed in it. The string is light and there is no friction anywhere.
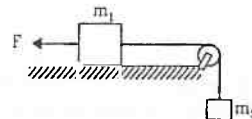


Figure 14-E3

10. A sphere of mass 20 kg is suspended by a metal wire of unstretched length 4 m and diameter 1 mm. When in equilibrium, there is a clear gap of 2 mm between the sphere and the floor. The sphere is gently pushed aside so that the wire makes an angle $\theta$ with the vertical and is released. Find the maximum value of $\theta$ so that the sphere does not rub the floor. Young's modulus of the metal of the wire is $2.0 \times 10^{11}$ N/m$^2$. Make appropriate approximations.

11. A steel wire of original length 1 m and cross-sectional area $4.00$ mm$^2$ is clamped at the two ends so that it lies horizontally and without tension. If a load of $2.16$ kg is suspended from the middle point of the wire, what would be its vertical depression ?

$Y$ of the steel = $2.0 \times 10^{11}$ N/m$^2$. Take $g$ = 10 m/s$^2$.

12. A copper wire of cross-sectional area $0.01$ cm$^2$ is under a tension of 20 N. Find the decrease in the cross-sectional area. Young's modulus of copper = $1.1 \times 10^{11}$ N/m$^2$ and Poisson's ratio = $0.32$.

[Hint : $\frac{\Delta A}{A} = 2 \frac{\Delta r}{r}$]

13. Find the increase in pressure required to decrease the volume of a water sample by $0.01\%$. Bulk modulus of water = $2.1 \times 10^9$ N/m$^2$.

14. Estimate the change in the density of water in ocean at a depth of 400 m below the surface. The density of water at the surface = 1030 kg/m$^3$ and the bulk modulus of water = $2 \times 10^9$ N/m$^2$.

15. A steel plate of face-area 4 cm$^2$ and thickness $0.5$ cm is fixed rigidly at the lower surface. A tangential force of 10 N is applied on the upper surface. Find the lateral displacement of the upper surface with respect to the lower surface. Rigidity modulus of steel = $8.4 \times 10^{10}$ N/m$^2$.

16. A $5.0$ cm long straight piece of thread is kept on the surface of water. Find the force with which the surface on one side of the thread pulls it. Surface tension of water = $0.076$ N/m.

17. Find the excess pressure inside (a) a drop of mercury of radius 2 mm (b) a soap bubble of radius 4 mm and (c) an air bubble of radius 4 mm formed inside a tank of water. Surface tension of mercury, soap solution and water are $0.465$ N/m, $0.03$ N/m and $0.076$ N/m respectively.

18. Consider a small surface area of 1 mm$^2$ at the top of a mercury drop of radius $4.0$ mm. Find the force exerted on this area (a) by the air above it (b) by the mercury below it and (c) by the mercury surface in contact with it. Atmospheric pressure = $1.0 \times 10^5$ Pa and surface tension of mercury = $0.465$ N/m. Neglect the effect of gravity. Assume all numbers to be exact.

19. The capillaries shown in figure (14-E4) have inner radii $0.5$ mm, $1.0$ mm and $1.5$ mm respectively. The liquid in the beaker is water. Find the heights of water level in the capillaries. The surface tension of water is $7.5 \times 10^{-2}$ N/m.
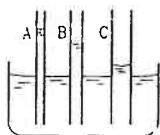


Figure 14-E4

20. The lower end of a capillary tube is immersed in mercury. The level of mercury in the tube is found to be 2 cm below the outer level. If the same tube is immersed in water, upto what height will the water rise in the capillary ?

21. A barometer is constructed with its tube having radius $1.0$ mm. Assume that the surface of mercury in the tube is spherical in shape. If the atmospheric pressure is equal to 76 cm of mercury, what will be the height raised in the barometer tube. The contact angle of mercury with glass = $135°$ and surface tension of mercury = $0.465$ N/m. Density of mercury = 13600 kg/m$^3$.

22. A capillary tube of radius $0.50$ mm is dipped vertically in a pot of water. Find the difference between the pressure of the water in the tube $5.0$ cm below the surface and the atmospheric pressure. Surface tension of water = $0.075$ N/m.

23. Find the surface energy of water kept in a cylindrical vessel of radius $6.0$ cm. Surface tension of water = $0.075$ J/m$^2$.

24. A drop of mercury of radius 2 mm is split into 8 identical droplets. Find the increase in surface energy. Surface tension of mercury = $0.465$ J/m$^2$.

25. A capillary tube of radius 1 mm is kept vertical with the lower end in water. (a) Find the height of water raised in the capillary. (b) If the length of the capillary tube is half the answer of part (a), find the angle $\theta$ made by the water surface in the capillary with the wall.

26. The lower end of a capillary tube of radius 1 mm is dipped vertically into mercury. (a) Find the depression of mercury column in the capillary. (b) If the length dipped inside is half the answer of part (a), find the angle made by the mercury surface at the end of the capillary with the vertical. Surface tension of mercury = $0.465$ N/m and the contact angle of mercury with glass = $135°$.

27. Two large glass plates are placed vertically and parallel to each other inside a tank of water with separation between the plates equal to 1 mm. Find the rise of water in the space between the plates. Surface tension of water = $0.075$ N/m.

28. Consider an ice cube of edge $1.0$ cm kept in a gravity free hall. Find the surface area of the water when the ice melts. Neglect the difference in densities of ice and water.

29. A wire forming a loop is dipped into soap solution and taken out so that a film of soap solution is formed. A loop of $6.28$ cm long thread is gently put on the film and the film is pricked with a needle inside the loop. The thread loop takes the shape of a circle. Find the tension in the thread. Surface tension of soap solution = $0.030$ N/m.

30. A metal sphere of radius 1 mm and mass 50 mg falls vertically in glycerine. Find (a) the viscous force exerted by the glycerine on the sphere when the speed of the sphere is 1 cm/s, (b) the hydrostatic force exerted by the glycerine on the sphere and (c) the terminal velocity with which the sphere will move down without acceleration. Density of glycerine = 1260 kg/m$^3$ and its coefficient of viscosity at room temperature = $8.0$ poise.

31. Estimate the speed of vertically falling raindrops from the following data. Radius of the drops = $0.02$ cm, viscosity of air = $1.8 \times 10^{-4}$ poise, $g$ = $9.9$ m/s$^2$ and density of water = 1000 kg/m$^3$.

**32.** Water flows at a speed of 6 cm/s through a tube of radius 1 cm. Coefficient of viscosity of water at room termperature is 0·01 poise. Calculate the Reynolds number. Is it a steady flow ?

□

# ANSWERS

## OBJECTIVE I

| 1. (d) | 2. (a) | 3. (b) | 4. (a) | 5. (b) | 6. (a) |
|--------|--------|--------|--------|--------|--------|
| 7. (a) | 8. (c) | 9. (b) | 10. (d) | 11. (c) | 12. (b) |
| 13. (a) | 14. (c) | 15. (d) | 16. (a) | 17. (c) | 18. (c) |
| 19. (d) | 20. (c) | 21. (b) | 22. (d) | 23. (d) | 24. (a) |
| 25. (c) | 26. (c) | 27. (b) | 28. (d) | 29. (b) | |

## OBJECTIVE II

| 1. all | 2. (c), (d) | 3. (a), (b), (d) |
|--------|-------------|------------------|
| 4. (a), (b) | 5. (c) | 6. (c), (d) |
| 7. (b), (c), (d) | | |

## EXERCISES

1. (a) $2·5 \times 10^7 \, \text{N/m}^2$   (b) $1·25 \times 10^{-4}$   (c) $3·75 \times 10^{-4} \, \text{m}$

2. (a) $7·96 \times 10^5 \, \text{N/m}^2$   (b) $4 \times 10^{-6}$   (c) $8 \times 10^{-6} \, \text{m}$

3. 2 mm

4. (a) 1   (b) $\dfrac{\text{strain in copper wire}}{\text{strain in steel wire}} = \dfrac{20}{13}$

5. $\dfrac{\text{strain in steel wire}}{\text{strain in copper wire}} = 1·54$

6. (a) 14 kg, lower   (b) 2 kg, upper

7. $1 \times 10^8 \, \text{N/m}^2$

8. $3·8 \times 10^4 \, \text{N}$

9. $\dfrac{m_2 g(2\,m_1 + m_2)}{2\,AY(m_1 + m_2)}$

10. $36·4°$

11. 1·5 cm

12. $1·164 \times 10^{-6} \, \text{cm}$

13. $2·1 \times 10^6 \, \text{N/m}^2$

14. $2 \, \text{kg/m}^3$

15. $1·5 \times 10^{-9} \, \text{m}$

16. $3·8 \times 10^{-3} \, \text{N}$

17. (a) $465 \, \text{N/m}^2$   (b) $30 \, \text{N/m}^2$
    (c) $38 \, \text{N/m}^2$

18. (a) 0·1 N   (b) 0·10023 N
    (c) 0·00023 N

19. 3 cm in $A$, 1·5 cm in $B$, 1 cm in $C$

20. 5·73 cm

21. 75·5 cm

22. $190 \, \text{N/m}^2$

23. $8·5 \times 10^{-4} \, \text{J}$

24. 23·4 μJ

25. (a) 1·5 cm   (b) 60°

26. (a) 5·34 mm   (b) 112°

27. 1·5 cm

28. $(36 \pi)^{1/3} \, \text{cm}^2$

29. $3 \times 10^{-4} \, \text{N}$

30. (a) $1·5 \times 10^{-1} \, \text{N}$   (b) $5·2 \times 10^{-5} \, \text{N}$   (c) 2·9 cm/s

31. 5 m/s

32. 120, yes.

□

9. *A sonometer wire has a total length of 1 m between the fixed ends. Where should the two bridges be placed below the wire so that the three segments of the wire have their fundamental frequencies in the ratio 1 : 2 : 3 ?*

**Solution :** Suppose the lengths of the three segments are $L_1$, $L_2$ and $L_3$ respectively. The fundamental frequencies are

$$v_1 = \frac{1}{2 L_1} \sqrt{F/\mu}$$

$$v_2 = \frac{1}{2 L_2} \sqrt{F/\mu}$$

$$v_3 = \frac{1}{2 L_3} \sqrt{F/\mu}$$

so that $v_1 L_1 = v_2 L_2 = v_3 L_3$. ... (i)

As $v_1 : v_2 : v_3 = 1 : 2 : 3$, we have

$v_2 = 2 v_1$ and $v_3 = 3 v_1$ so that by (i)

$$L_2 = \frac{v_1}{v_2} L_1 = \frac{L_1}{2}$$

and $L_3 = \frac{v_1}{v_3} L_1 = \frac{L_1}{3}$ .

As $L_1 + L_2 + L_3 = 1$ m,

we get $L_1\left(1 + \frac{1}{2} + \frac{1}{3}\right) = 1$ m

or, $L_1 = \frac{6}{11}$ m .

Thus, $L_2 = \frac{L_1}{2} = \frac{3}{11}$ m

and $L_3 = \frac{L_1}{3} = \frac{2}{11}$ m.

One bridge should be placed at $\frac{6}{11}$ m from one end and the other should be placed at $\frac{2}{11}$ m from the other end.

10. *A wire having a linear mass density $5.0 \times 10^{-3}$ kg/m is stretched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.*

**Solution :** Suppose the wire vibrates at 420 Hz in its $n$th harmonic and at 490 Hz in its $(n + 1)$th harmonic.

$$420 \text{ s}^{-1} = \frac{n}{2 L} \sqrt{F/\mu} \qquad \text{... (i)}$$

and $490 \text{ s}^{-1} = \frac{(n + 1)}{2 L} \sqrt{F/\mu}$. ... (ii)

This gives $\frac{490}{420} = \frac{n + 1}{n}$

or, $n = 6$.

Putting the value in (i),

$$420 \text{ s}^{-1} = \frac{6}{2 L} \sqrt{\frac{450 \text{ N}}{5.0 \times 10^{-3} \text{ kg/m}}} = \frac{900}{L} \text{ m/s}$$

or, $L = \frac{900}{420}$ m $= 2.1$ m.

□

## QUESTIONS FOR SHORT ANSWER

1. You are walking along a seashore and a mild wind is blowing. Is the motion of air a wave motion ?

2. The radio and TV programmes, telecast at the studio, reach our antenna by wave motion. Is it a mechanical wave or nonmechanical ?

3. A wave is represented by an equation $y = c_1 \sin (c_2 x + c_3 t)$. In which direction is the wave going ? Assume that $c_1$, $c_2$ and $c_3$ are all positive.

4. Show that the particle speed can never be equal to the wave speed in a sine wave if the amplitude is less than wavelength divided by $2\pi$.

5. Two wave pulses identical in shape but inverted with respect to each other are produced at the two ends of a stretched string. At an instant when the pulses reach the middle, the string becomes completely straight. What happens to the energy of the two pulses ?

6. Show that for a wave travelling on a string

$$\frac{y_{max}}{v_{max}} = \frac{v_{max}}{a_{max}},$$

where the symbols have usual meanings. Can we use componendo and dividendo taught in algebra to write

$$\frac{y_{max} + v_{max}}{y_{max} - v_{max}} = \frac{v_{max} + a_{max}}{v_{max} - a_{max}} \text{ ?}$$

7. What is the smallest positive phase constant which is equivalent to $7.5 \pi$ ?

8. A string clamped at both ends vibrates in its fundamental mode. Is there any position (except the ends) on the string which can be touched without disturbing the motion ? What if the string vibrates in its first overtone ?

## OBJECTIVE I

1. A sine wave is travelling in a medium. The minimum distance between the two particles, always having same speed, is
   (a) $\lambda/4$     (b) $\lambda/3$     (c) $\lambda/2$     (d) $\lambda$.

2. A sine wave is travelling in a medium. A particular particle has zero displacement at a certain instant. The particle closest to it having zero displacement is at a distance
   (a) $\lambda/4$     (b) $\lambda/3$     (c) $\lambda/2$     (d) $\lambda$.

3. Which of the following equations represents a wave travelling along Y-axis ?
   (a) $x = A \sin (ky - \omega t)$     (b) $y = A \sin (kx - \omega t)$
   (c) $y = A \sin ky \cos \omega t$     (d) $y = A \cos ky \sin \omega t$.

4. The equation $y = A \sin^2(kx - \omega t)$ represents a wave motion with
   (a) amplitude $A$, frequency $\omega/2\pi$
   (b) amplitude $A/2$, frequency $\omega/\pi$
   (c) amplitude $2A$, frequency $\omega/4\pi$
   (d) does not represent a wave motion.

5. Which of the following is a mechanical wave ?
   (a) Radio waves.          (b) $X$-rays.
   (c) Light waves.          (d) Sound waves.

6. A cork floating in a calm pond executes simple harmonic motion of frequency $\nu$ when a wave generated by a boat passes by it. The frequency of the wave is
   (a) $\nu$     (b) $\nu/2$     (c) $2\nu$     (d) $\sqrt{2}\nu$.

7. Two strings $A$ and $B$, made of same material, are stretched by same tension. The radius of string $A$ is double of the radius of $B$. A transverse wave travels on $A$ with speed $v_A$ and on $B$ with speed $v_B$. The ratio $v_A/v_B$ is
   (a) $1/2$     (b) $2$     (c) $1/4$     (d) $4$.

8. Both the strings, shown in figure (15-Q1), are made of same material and have same cross-section. The pulleys are light. The wave speed of a transverse wave in the string $AB$ is $v_1$ and in $CD$ it is $v_2$. Then $v_1/v_2$ is
   (a) $1$     (b) $2$     (c) $\sqrt{2}$     (d) $1/\sqrt{2}$.



Figure 15-Q1

9. Velocity of sound in air is 332 m/s. Its velocity in vacuum will be
   (a) > 332 m/s                    (b) = 332 m/s
   (c) < 332 m/s                    (d) meaningless.

10. A wave pulse, travelling on a two-piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has wavelength $\lambda$ and the transmitted wave $\lambda'$,

(a) $\lambda' > \lambda$          (b) $\lambda' = \lambda$          (c) $\lambda' < \lambda$
(d) nothing can be said about the relation of $\lambda$ and $\lambda'$.

11. Two waves represented by $y = a \sin(\omega t - kx)$ and $y = a \cos(\omega t - kx)$ are superposed. The resultant wave will have an amplitude
   (a) $a$     (b) $\sqrt{2}a$     (c) $2a$     (d) $0$.

12. Two wires $A$ and $B$, having identical geometrical construction, are stretched from their natural length by small but equal amount. The Young's modulus of the wires are $Y_A$ and $Y_B$ whereas the densities are $\rho_A$ and $\rho_B$. It is given that $Y_A > Y_B$ and $\rho_A > \rho_B$. A transverse signal started at one end takes a time $t_1$ to reach the other end for $A$ and $t_2$ for $B$.
   (a) $t_1 < t_2$.          (b) $t_1 = t_2$.          (c) $t_1 > t_2$.
   (d) The information is insufficient to find the relation between $t_1$ and $t_2$.

13. Consider two waves passing through the same string. Principle of superposition for displacement says that the net displacement of a particle on the string is sum of the displacements produced by the two waves individually. Suppose we state similar principles for the net velocity of the particle and the net kinetic energy of the particle. Such a principle will be valid for
   (a) both the velocity and the kinetic energy
   (b) the velocity but not for the kinetic energy
   (c) the kinetic energy but not for the velocity
   (d) neither the velocity nor the kinetic energy.

14. Two wave pulses travel in opposite directions on a string and approach each other. The shape of one pulse is inverted with respect to the other.
   (a) The pulses will collide with each other and vanish after collision.
   (b) The pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.
   (c) The pulses will pass through each other but their shapes will be modified.
   (d) The pulses will pass through each other without any change in their shapes.

15. Two periodic waves of amplitudes $A_1$ and $A_2$ pass through a region. If $A_1 > A_2$, the difference in the maximum and minimum resultant amplitude possible is
   (a) $2A_1$     (b) $2A_2$     (c) $A_1 + A_2$     (d) $A_1 - A_2$.

16. Two waves of equal amplitude $A$, and equal frequency travel in the same direction in a medium. The amplitude of the resultant wave is
   (a) $0$     (b) $A$     (c) $2A$     (d) between 0 and $2A$.

17. Two sine waves travel in the same direction in a medium. The amplitude of each wave is $A$ and the phase difference between the two waves is $120°$. The resultant amplitude will be
   (a) $A$     (b) $2A$     (c) $4A$     (d) $\sqrt{2}A$.

18. The fundamental frequency of a string is proportional to
   (a) inverse of its length     (b) the diameter
   (c) the tension                (d) the density.

19. A tuning fork of frequency 480 Hz is used to vibrate a sonometer wire having natural frequency 240 Hz. The wire will vibrate with a frequency of
(a) 240 Hz            (b) 480 Hz
(c) 720 Hz            (d) will not vibrate.

20. A tuning fork of frequency 480 Hz is used to vibrate a sonometer wire having natural frequency 410 Hz. The wire will vibrate with a frequency
(a) 410 Hz      (b) 480 Hz      (c) 820 Hz      (d) 960 Hz.

21. A sonometer wire of length $l$ vibrates in fundamental mode when excited by a tuning fork of frequency 416 Hz. If the length is doubled keeping other things same, the string will
(a) vibrate with a frequency of 416 Hz
(b) vibrate with a frequency of 208 Hz
(c) vibrate with a frequency of 832 Hz
(d) stop vibrating.

22. A sonometer wire supports a 4 kg load and vibrates in fundamental mode with a tuning fork of frequency 416 Hz. The length of the wire between the bridges is now doubled. In order to maintain fundamental mode, the load should be changed to
(a) 1 kg      (b) 2 kg      (c) 8 kg      (d) 16 kg.

## OBJECTIVE II

1. A mechanical wave propagates in a medium along the X-axis. The particles of the medium
(a) must move on the X-axis
(b) must move on the Y-axis
(c) may move on the X-axis
(d) may move on the Y-axis.

2. A transverse wave travels along the Z-axis. The particles of the medium must move
(a) along the Z-axis      (b) along the X-axis
(c) along the Y-axis      (d) in the X-Y plane.

3. Longitudinal waves cannot
(a) have a unique wavelength      (b) transmit energy
(c) have a unique wave velocity      (d) be polarized.

4. A wave going in a solid
(a) must be longitudinal      (b) may be longitudinal
(c) must be transverse      (d) may be transverse.

5. A wave moving in a gas
(a) must be longitudinal      (b) may be longitudinal
(c) must be transverse      (d) may be transverse.

6. Two particles A and B have a phase difference of $\pi$ when a sine wave passes through the region.
(a) A oscillates at half the frequency of B.
(b) A and B move in opposite directions.
(c) A and B must be separated by half of the wavelength.
(d) The displacements at A and B have equal magnitudes.

7. A wave is represented by the equation
$$y = (0.001 \text{ mm}) \sin[(50 \text{ s}^{-1})t + (2.0 \text{ m}^{-1})x].$$
(a) The wave velocity = 100 m/s.
(b) The wavelength = 2.0 m.
(c) The frequency = $25/\pi$ Hz.
(d) The amplitude = 0.001 mm.

8. A standing wave is produced on a string clamped at one end and free at the other. The length of the string
(a) must be an integral multiple of $\lambda/4$
(b) must be an integral multiple of $\lambda/2$
(c) must be an integral multiple of $\lambda$
(d) may be an integral multiple of $\lambda/2$.

9. Mark out the correct options.
(a) The energy of any small part of a string remains constant in a travelling wave.
(b) The energy of any small part of a string remains constant in a standing wave.
(c) The energies of all the small parts of equal length are equal in a travelling wave.
(d) The energies of all the small parts of equal length are equal in a standing wave.

10. In a stationary wave,
(a) all the particles of the medium vibrate in phase
(b) all the antinodes vibrate in phase
(c) the alternate antinodes vibrate in phase
(d) all the particles between consecutive nodes vibrate in phase.

## EXERCISES

1. A wave pulse passing on a string with a speed of 40 cm/s in the negative x-direction has its maximum at $x = 0$ at $t = 0$. Where will this maximum be located at $t = 5$ s ?

2. The equation of a wave travelling on a string stretched along the X-axis is given by
$$y = A\, e^{-\left(\frac{x}{a} + \frac{t}{T}\right)^2}.$$
(a) Write the dimensions of $A$, $a$ and $T$. (b) Find the wave speed. (c) In which direction is the wave travelling ? (d) Where is the maximum of the pulse located at $t = T$ ? At $t = 2T$ ?

3. Figure (15-E1) shows a wave pulse at $t = 0$. The pulse moves to the right with a speed of 10 cm/s. Sketch the shape of the string at $t = 1$ s, 2 s and 3 s.
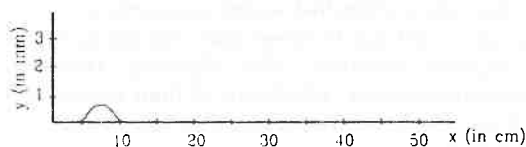


Figure 15-E1

4. A pulse travelling on a string is represented by the function

$$y = \frac{a^3}{(x - vt)^2 + a^2},$$

where $a = 5$ mm and $v = 20$ cm/s. Sketch the shape of the string at $t = 0$, 1 s and 2 s. Take $x = 0$ in the middle of the string.

5. The displacement of the particle at $x = 0$ of a stretched string carrying a wave in the positive $x$-direction is given by $f(t) = A \sin(t/T)$. The wave speed is $v$. Write the wave equation.

6. A wave pulse is travelling on a string with a speed $v$ towards the positive $X$-axis. The shape of the string at $t = 0$ is given by $g(x) = A \sin(x/a)$, where $A$ and $a$ are constants.
(a) What are the dimensions of $A$ and $a$ ? (b) Write the equation of the wave for a general time $t$, if the wave speed is $v$.

7. A wave propagates on a string in the positive $x$-direction at a velocity $v$. The shape of the string at $t = t_0$ is given by $g(x, t_0) = A \sin(x/a)$. Write the wave equation for a general time $t$.

8. The equation of a wave travelling on a string is

$$y = (0.10 \text{ mm}) \sin[(31.4 \text{ m}^{-1})x + (314 \text{ s}^{-1})t].$$

(a) In which direction does the wave travel ? (b) Find the wave speed, the wavelength and the frequency of the wave. (c) What is the maximum displacement and the maximum speed of a portion of the string ?

9. A wave travels along the positive $x$-direction with a speed of 20 m/s. The amplitude of the wave is 0.20 cm and the wavelength 2.0 cm. (a) Write a suitable wave equation which describes this wave. (b) What is the displacement and velocity of the particle at $x = 2.0$ cm at time $t = 0$ according to the wave equation written ? Can you get different values of this quantity if the wave equation is written in a different fashion ?

10. A wave is described by the equation

$$y = (1.0 \text{ mm}) \sin \pi \left( \frac{x}{2.0 \text{ cm}} - \frac{t}{0.01 \text{ s}} \right).$$

(a) Find the time period and the wavelength . (b) Write the equation for the velocity of the particles. Find the speed of the particle at $x = 1.0$ cm at time $t = 0.01$ s. (c) What are the speeds of the particles at $x = 3.0$ cm, 5.0 cm and 7.0 cm at $t = 0.01$ s ? (d) What are the speeds of the particles at $x = 1.0$ cm at $t = 0.011$, 0.012, and 0.013 s ?

11. A particle on a stretched string supporting a travelling wave, takes 5.0 ms to move from its mean position to the extreme position. The distance between two consecutive particles, which are at their mean positions, is 2.0 cm. Find the frequency, the wavelength and the wave speed.

12. Figure (15-E2) shows a plot of the transverse displacements of the particles of a string at $t = 0$ through which a travelling wave is passing in the positive $x$-direction. The wave speed is 20 cm/s. Find (a) the amplitude, (b) the wavelength, (c) the wave number and (d) the frequency of the wave.
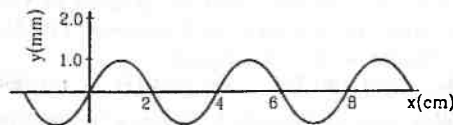


Figure 15-E2

13. A wave travelling on a string at a speed of 10 m/s causes each particle of the string to oscillate with a time period of 20 ms. (a) What is the wavelength of the wave? (b) If the displacement of a particle is 1.5 mm at a certain instant, what will be the displacement of a particle 10 cm away from it at the same instant ?

14. A steel wire of length 64 cm weighs 5 g. If it is stretched by a force of 8 N, what would be the speed of a transverse wave passing on it ?

15. A string of length 20 cm and linear mass density 0.40 g/cm is fixed at both ends and is kept under a tension of 16 N. A wave pulse is produced at $t = 0$ near an end as shown in figure (15-E3), which travels towards the other end. (a) When will the string have the shape shown in the figure again ? (b) Sketch the shape of the string at a time half of that found in part (a).
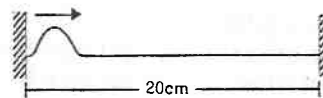


Figure 15-E3

16. A string of linear mass density 0.5 g/cm and a total length 30 cm is tied to a fixed wall at one end and to a frictionless ring at the other end (figure 15-E4). The ring can move on a vertical rod. A wave pulse is produced on the string which moves towards the ring at a speed of 20 cm/s. The pulse is symmetric about its maximum which is located at a distance of 20 cm from the end joined to the ring. (a) Assuming that the wave is reflected from the ends without loss of energy, find the time taken by the string to regain its shape. (b) The shape of the string changes periodically with time. Find this time period. (c) What is the tension in the string?



Figure 15-E4

17. Two wires of different densities but same area of cross-section are soldered together at one end and are stretched to a tension $T$. The velocity of a transverse

wave in the first wire is double of that in the second wire. Find the ratio of the density of the first wire to that of the second wire.

18. A transverse wave described by

$$y = (0.02 \text{ m}) \sin[(1.0 \text{ m}^{-1}) x + (30 \text{ s}^{-1})t]$$

propagates on a stretched string having a linear mass density of $1.2 \times 10^{-4}$ kg/m. Find the tension in the string.

19. A travelling wave is produced on a long horizontal string by vibrating an end up and down sinusoidally. The amplitude of vibration is 1.0 cm and the displacement becomes zero 200 times per second. The linear mass density of the string is 0.10 kg/m and it is kept under a tension of 90 N. (a) Find the speed and the wavelength of the wave. (b) Assume that the wave moves in the positive x-direction and at $t = 0$, the end $x = 0$ is at its positive extreme position. Write the wave equation. (c) Find the velocity and acceleration of the particle at $x = 50$ cm at time $t = 10$ ms.

20. A string of length 40 cm and weighing 10 g is attached to a spring at one end and to a fixed wall at the other end. The spring has a spring constant of 160 N/m and is stretched by 1.0 cm. If a wave pulse is produced on the string near the wall, how much time will it take to reach the spring?

21. Two blocks each having a mass of 3.2 kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB (figure 15-E5). The linear mass density of the wire AB is 10 g/m and that of CD is 8 g/m. Find the speed of a transverse wave pulse produced in AB and in CD.



Figure 15-E5

22. In the arrangement shown in figure (15-E6), the string has a mass of 4.5 g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley? Take $g = 10$ m/s$^2$.
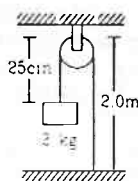


Figure 15-E6

23. A 4.0 kg block is suspended from the ceiling of an elevator through a string having a linear mass density of $19.2 \times 10^{-3}$ kg/m. Find the speed (with respect to the string) with which a wave pulse can proceed on the string if the elevator accelerates up at the rate of 2.0 m/s$^2$. Take $g = 10$ m/s$^2$.
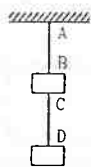
24. A heavy ball is suspended from the ceiling of a motor car through a light string. A transverse pulse travels at a speed of 60 cm/s on the string when the car is at rest and 62 cm/s when the car accelerates on a horizontal road. Find the acceleration of the car. Take $g = 10$ m/s$^2$.

25. A circular loop of string rotates about its axis on a frictionless horizontal plane at a uniform rate so that the tangential speed of any particle of the string is $v$. If a small transverse disturbance is produced at a point of the loop, with what speed (relative to the string) will this disturbance travel on the string?

26. A heavy but uniform rope of length $L$ is suspended from a ceiling. (a) Write the velocity of a transverse wave travelling on the string as a function of the distance from the lower end. (b) If the rope is given a sudden sideways jerk at the bottom, how long will it take for the pulse to reach the ceiling? (c) A particle is dropped from the ceiling at the instant the bottom end is given the jerk. Where will the particle meet the pulse?

27. Two long strings A and B, each having linear mass density $1.2 \times 10^{-2}$ kg/m, are stretched by different tensions 4.8 N and 7.5 N respectively and are kept parallel to each other with their left ends at $x = 0$. Wave pulses are produced on the strings at the left ends at $t = 0$ on string A and at $t = 20$ ms on string B. When and where will the pulse on B overtake that on A?

28. A transverse wave of amplitude 0.50 mm and frequency 100 Hz is produced on a wire stretched to a tension of 100 N. If the wave speed is 100 m/s, what average power is the source transmitting to the wire?

29. A 200 Hz wave with amplitude 1 mm travels on a long string of linear mass density 6 g/m kept under a tension of 60 N. (a) Find the average power transmitted across a given point on the string. (b) Find the total energy associated with the wave in a 2.0 m long portion of the string.

30. A tuning fork of frequency 440 Hz is attached to a long string of linear mass density 0.01 kg/m kept under a tension of 49 N. The fork produces transverse waves of amplitude 0.50 mm on the string. (a) Find the wave speed and the wavelength of the waves. (b) Find the maximum speed and acceleration of a particle of the string. (c) At what average rate is the tuning fork transmitting energy to the string?

31. Two waves, travelling in the same direction through the same region, have equal frequencies, wavelengths and amplitudes. If the amplitude of each wave is 4 mm and the phase difference between the waves is 90°, what is the resultant amplitude?

32. Figure (15-E7) shows two wave pulses at $t = 0$ travelling on a string in opposite directions with the same



Figure 15-E7

wave speed 50 cm/s. Sketch the shape of the string at $t = 4$ ms, 6 ms, 8 ms, and 12 ms.
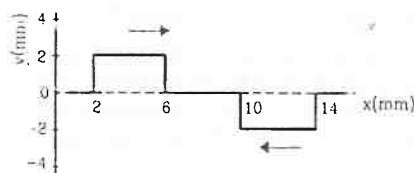
33. Two waves, each having a frequency of 100 Hz and a wavelength of 2·0 cm, are travelling in the same direction on a string. What is the phase difference between the waves (a) if the second wave was produced 0·015 s later than the first one at the same place, (b) if the two waves were produced at the same instant but the first one was produced a distance 4·0 cm behind the second one?(c) If each of the waves has an amplitude of 2·0 mm, what would be the amplitudes of the resultant waves in part (a) and (b)?

34. If the speed of a transverse wave on a stretched string of length 1 m is 60 m/s, what is the fundamental frequency of vibration?

35. A wire of length 2·00 m is stretched to a tension of 160 N. If the fundamental frequency of vibration is 100 Hz, find its linear mass density.

36. A steel wire of mass 4·0 g and length 80 cm is fixed at the two ends. The tension in the wire is 50 N. Find the frequency and wavelength of the fourth harmonic of the fundamental.

37. A piano wire weighing 6·00 g and having a length of 90·0 cm emits a fundamental frequency corresponding to the "Middle C" ($v = 261·63$ Hz). Find the tension in the wire.

38. A sonometer wire having a length of 1·50 m between the bridges vibrates in its second harmonic in resonance with a tuning fork of frequency 256 Hz. What is the speed of the transverse wave on the wire?

39. The length of the wire shown in figure (15-E8) between the pulleys is 1·5 m and its mass is 12·0 g. Find the frequency of vibration with which the wire vibrates in two loops leaving the middle point of the wire between the pulleys at rest.
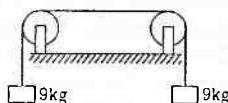


Figure 15-E8

40. A one metre long stretched string having a mass of 40 g is attached to a tuning fork. The fork vibrates at 128 Hz in a direction perpendicular to the string. What should be the tension in the string if it is to vibrate in four loops?

41. A wire, fixed at both ends is seen to vibrate at a resonant frequency of 240 Hz and also at 320 Hz. (a) What could be the maximum value of the fundamental frequency? (b) If transverse waves can travel on this string at a speed of 40 m/s what is its length?

42. A string, fixed at both ends, vibrates in a resonant mode with a separation of 2·0 cm between the consecutive nodes. For the next higher resonant frequency, this separation is reduced to 1·6 cm. Find the length of the string.

43. A 660 Hz tuning fork sets up vibration in a string clamped at both ends. The wave speed for a transverse

wave on this string is 220 m/s and the string vibrates in three loops. (a) Find the length of the string. (b) If the maximum amplitude of a particle is 0·5 cm, write a suitable equation describing the motion.

44. A particular guitar wire is 30·0 cm long and vibrates at a frequency of 196 Hz when no finger is placed on it. The next higher notes on the scale are 220 Hz, 247 Hz, 262 Hz and 294 Hz. How far from the end of the string must the finger be placed to play these notes?

45. A steel wire fixed at both ends has a fundamental frequency of 200 Hz. A person can hear sound of maximum frequency 14 kHz. What is the highest harmonic that can be played on this string which is audible to the person?

46. Three resonant frequencies of a string are 90, 150 and 210 Hz. (a) Find the highest possible fundamental frequency of vibration of this string. (b) Which harmonics of the fundamental are the given frequencies? (c) Which overtones are these frequencies. (d) If the length of the string is 80 cm, what would be the speed of a transverse wave on this string?

47. Two wires are kept tight between the same pair of supports. The tensions in the wires are in the ratio 2 : 1, the radii are in the ratio 3 : 1 and the densities are in the ratio 1 : 2. Find the ratio of their fundamental frequencies.

48. A uniform horizontal rod of length 40 cm and mass 1·2 kg is supported by two identical wires as shown in figure (15-E9). Where should a mass of 4·8 kg be placed on the rod so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone? Take $g = 10$ m/s².
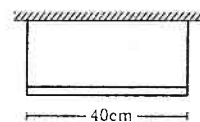


Figure 15-E9

49. Figure (15-E10) shows an aluminium wire of length 60 cm joined to a steel wire of length 80 cm and stretched between two fixed supports. The tension produced is 40 N. The cross-sectional area of the steel wire is 1·0 mm² and that of the aluminium wire is 3·0 mm². What could be the minimum frequency of a tuning fork which can produce standing waves in the system with the joint as a node? The density of aluminium is 2·6 g/cm³ and that of steel is 7·8 g/cm³.
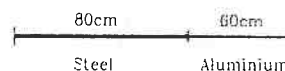


Figure 15-E10

50. A string of length $L$ fixed at both ends vibrates in its fundamental mode at a frequency $v$ and a maximum amplitude $A$. (a) Find the wavelength and the wave number $k$. (b) Take the origin at one end of the string and the X-axis along the string. Take the Y-axis along

the direction of the displacement. Take $t = 0$ at the instant when the middle point of the string passes through its mean position and is going towards the positive y-direction. Write the equation describing the standing wave.

51. A 2 m long string fixed at both ends is set into vibrations in its first overtone. The wave speed on the string is 200 m/s and the amplitude is 0.5 cm. (a) Find the wavelength and the frequency. (b) Write the equation giving the displacement of different points as a function of time. Choose the X-axis along the string with the origin at one end and $t = 0$ at the instant when the point $x = 50$ cm has reached its maximum displacement.

52. The equation for the vibration of a string, fixed at both ends vibrating in its third harmonic, is given by

$$y = (0.4 \text{ cm}) \sin[(0.314 \text{ cm}^{-1}) x] \cos[(600\pi \text{ s}^{-1})t].$$

(a) What is the frequency of vibration ? (b) What are the positions of the nodes ? (c) What is the length of the string ? (d) What is the wavelength and the speed of two travelling waves that can interfere to give this vibration ?

53. The equation of a standing wave, produced on a string fixed at both ends, is

$$y = (0.4 \text{ cm}) \sin[(0.314 \text{ cm}^{-1}) x] \cos[(600\pi \text{ s}^{-1})t].$$

What could be the smallest length of the string ?

54. A 40 cm wire having a mass of 3.2 g is stretched between two fixed supports 40.05 cm apart. In its fundamental mode, the wire vibrates at 220 Hz. If the area of cross-section of the wire is 1.0 mm$^2$, find its Young's modulus.

55. Figure (15-E11) shows a string stretched by a block going over a pulley . The string vibrates in its tenth harmonic in unison with a particular tuning fork. When a beaker containing water is brought under the block so that the block is completely dipped into the beaker, the string vibrates in its eleventh harmonic. Find the density of the material of the block.
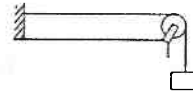


Figure 15-E11

56. A 2.00 m long rope, having a mass of 80 g, is fixed at one end and is tied to a light string at the other end. The tension in the string is 256 N. (a) Find the frequencies of the fundamental and the first two overtones. (b) Find the wavelength in the fundamental and the first two overtones.

57. A heavy string is tied at one end to a movable support and to a light thread at the other end as shown in figure (15-E12). The thread goes over a fixed pulley and supports a weight to produce a tension. The lowest frequency with which the heavy string resonates is 120 Hz. If the movable support is pushed to the right by 10 cm so that the joint is placed on the pulley, what will be the minimum frequency at which the heavy string can resonate ?
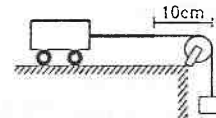


Figure 15-E12

□

# ANSWERS

## OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (c) | 3. (a) | 4. (b) | 5. (d) | 6. (a) |
| 7. (a) | 8. (d) | 9. (d) | 10. (c) | 11. (b) | 12. (d) |
| 13. (b) | 14. (d) | 15. (b) | 16. (d) | 17. (a) | 18. (a) |
| 19. (b) | 20. (b) | 21. (a) | 22. (d) | | |

## OBJECTIVE II

| | | |
|---|---|---|
| 1. (c), (d) | 2. (d) | 3. (d) |
| 4. (b), (d) | 5. (a) | 6. (b), (d) |
| 7. (c), (d) | 8. (a) | 9. (b) |
| 10. (c), (d) | | |

## EXERCISES

1. At $x = -2$ m

2. (a) $L, L, T$      (b) $a/T$
   (c) negative x-direction      (d) $x = -a$ and $x = -2a$

5. $f(x, t) = A \sin\left(\dfrac{t}{T} - \dfrac{x}{vT}\right)$

6. (a) $L, L$      (b) $f(x, t) = A \sin\dfrac{x - vt}{a}$

7. $f(x, t) = A \sin\dfrac{x - v(t - t_0)}{a}$

8. (a) negative x-direction      (b) 10 m/s, 20 cm, 50 Hz
   (c) 0.10 mm, 3.14 cm/s

9. (a) $y = (0.20 \text{ cm}) \sin[(\pi \text{ cm}^{-1}) x - (2\pi \times 10^3 \text{ s}^{-1})t]$
   (b) zero, $4\pi$ m/s

10. (a) 20 ms, 4·0 cm     (b) zero
    (c) zero     (d) 9·7 cm/s, 18 cm/s, 25 cm/s

11. 50 Hz, 4·0 cm, 2·0 m/s

12. (a) 1·0 mm     (b) 4 cm     (c) 1·6 cm$^{-1}$     (d) 5 Hz

13. (a) 20 cm     (b) $-1·5$ mm

14. 32 m/s

15. (a) 0·02 s

16. (a) 2 s     (b) 3 s     (c) $2 \times 10^{-3}$ N

17. 0·25

18. 0·108 N

19. (a) 30 m/s, 30 cm

    (b) $y = (1·0 \text{ cm}) \cos 2\pi \left[ \dfrac{x}{30 \text{ cm}} - \dfrac{t}{0·01 \text{ s}} \right]$

    (c) $-5·4$ m/s, 2·0 km/s$^2$

20. 0·05 s

21. 79 m/s and 63 m/s

22. 0·02 s

23. 50 m/s

24. 3·7 m/s$^2$

25. $v$

26. (a) $\sqrt{gx}$     (b) $\sqrt{4 L/g}$

    (c) at a distance $\dfrac{L}{3}$ from the bottom

27. at $t = 100$ ms at $x = 2·0$ m

28. 49 mW

29. (a) 0·47 W     (b) 9·4 mJ

30. (a) 70 m/s, 16 cm    (b) 1·4 m/s, 3·8 km/s    (c) 0·67 W

31. $4\sqrt{2}$ mm

33. (a) $3\pi$     (b) $4\pi$     (c) zero, 4·0 mm

34. 30 Hz

35. 1·00 g/m

36. 250 Hz, 40 cm

37. 1480 N

38. 384 m/s

39. 70 Hz

40. 164 N

41. (a) 80 Hz     (b) 25 cm

42. 8·0 cm

43. (a) 50 cm

    (b) $(0·5 \text{ cm}) \sin[(0·06\pi \text{ cm}^{-1})x] \times \cos[(1320\pi \text{ s}^{-1})t]$

44. 26·7 cm, 23·8 cm, 22·4 cm and 20·0 cm

45. 70

46. (a) 30 Hz     (b) 3rd, 5th and 7th
    (c) 2nd, 4th and 6th     (d) 48 m/s

47. 2 : 3

48. 5 cm from the left end

49. 180 Hz

50. (a) $2L$, $\pi/L$     (b) $y = A \sin(\pi x/L) \sin(2\pi v t)$

51. (a) 2 m, 100 Hz

    (b) $(0·5 \text{ cm}) \sin[(\pi \text{ m}^{-1}) x] \cos[(200\pi \text{ s}^{-1})t]$

52. (a) 300 Hz     (b) 0, 10 cm, 20 cm, 30 cm
    (c) 30 cm     (d) 20 cm, 60 m/s

53. 10 cm

54. $1·98 \times 10^{11}$ N/m$^2$

55. $5·8 \times 10^3$ kg/m$^3$

56. (a) 10 Hz, 30 Hz, 50 Hz    (b) 8·00 m, 2·67 m, 1·60 m

57. 240 Hz

□

The observer $A$ is at rest with respect to the air and the source is travelling at a velocity of 120 km/h i.e., $\frac{100}{3}$ m/s. As is clear from the figure, the person receives the sound of the whistle in a direction $BA$ making an angle $\theta$ with the track where $\cos\theta = 300/500 = 3/5$. The component of the velocity of the source (i.e., of the train)

along this direction $AB$ is $\frac{100}{3} \times \frac{3}{5}$ m/s = 20 m/s. As the source is approaching the person with this component, the frequency heard by the observer is

$$v' = \frac{v}{v - u\cos\theta} \cdot v = \frac{340}{340 - 20} \times 640\,\text{Hz} = 680\,\text{Hz}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. If you are walking on the moon, can you hear the sound of stones cracking behind you ? Can you hear the sound of your own footsteps ?

2. Can you hear your own words if you are standing in a perfect vacuum ? Can you hear your friend in the same conditions ?

3. A vertical rod is hit at one end. What kind of wave propagates in the rod if (a) the hit is made vertically (b) the hit is made horizontally ?

4. Two loudspeakers are arranged facing each other at some distance. Will a person standing behind one of the loudspeakers clearly hear the sound of the other loudspeaker or the clarity will be seriously damaged because of the 'collision' of the two sounds in between ?

5. The voice of a person, who has inhaled helium, has a remarkably high pitch. Explain on the basis of resonant vibration of vocal cord filled with air and with helium.

6. Draw a diagram to show the standing pressure wave and standing displacement wave for the 3rd overtone mode of vibration of an open organ pipe.

7. Two tuning forks vibrate with the same amplitude but the frequency of the first is double the frequency of the second. Which fork produces more intense sound in air ?

8. In discussing Doppler effect, we use the word "apparent frequency". Does it mean that the frequency of the sound is still that of the source and it is some physiological phenomenon in the listener's ear that gives rise to Doppler effect ? Think for the observer approaching the source and for the source approaching the observer.

## OBJECTIVE I

1. Consider the following statements about sound passing through a gas.
   (A) The pressure of the gas at a point oscillates in time.
   (B) The position of a small layer of the gas oscillates in time.
   (a) Both $A$ and $B$ are correct.
   (b) $A$ is correct but $B$ is wrong.
   (c) $B$ is correct but $A$ is wrong.
   (d) Both $A$ and $B$ are wrong.

2. When we clap our hands, the sound produced is best described by
   (a) $p = p_0 \sin(kx - \omega t)$
   (b) $p = p_0 \sin kx \cos \omega t$
   (c) $p = p_0 \cos kx \sin \omega t$
   (d) $p = \Sigma\, p_{0n} \sin(k_n x - \omega_n t)$.
   Here $p$ denotes the change in pressure from the equilibrium value.

3. The bulk modulus and the density of water are greater than those of air  With this much of information, we can say that velocity of sound in air
   (a) is larger than its value in water
   (b) is smaller than its value in water
   (c) is equal to its value in water
   (d) cannot be compared with its value in water.

4. A tuning fork sends sound waves in air. If the temperature of the air increases, which of the following parameters will change ?
   (a) Displacement amplitude.   (b) Frequency.
   (c) Wavelength.               (d) Time period.

5. When sound wave is refracted from air to water, which of the following will remain unchanged ?
   (a) Wave number.             (b) Wavelength.
   (c) Wave velocity.           (d) Frequency.

6. The speed of sound in a medium depends on
   (a) the elastic property but not on the inertia property
   (b) the inertia property but not on the elastic property
   (c) the elastic property as well as the inertia property
   (d) neither the elastic property nor the inertia property.

7. Two sound waves move in the same direction in the same medium. The pressure amplitudes of the waves are equal but the wavelength of the first wave is double the second. Let the average power transmitted across a cross-section by the first wave be $P_1$ and that by the second wave be $P_2$. Then
   (a) $P_1 = P_2$              (b) $P_1 = 4P_2$
   (c) $P_2 = 2P_1$             (d) $P_2 = 4P_1$.

8. When two waves with same frequency and constant phase difference interfere,
   (a) there is a gain of energy
   (b) there is a loss of energy

(c) the energy is redistributed and the distribution changes with time

(d) the energy is redistributed and the distribution remains constant in time.

9. An open organ pipe of length $L$ vibrates in its fundamental mode. The pressure variation is maximum
   (a) at the two ends
   (b) at the middle of the pipe
   (c) at distances $L/4$ inside the ends
   (d) at distances $L/8$ inside the ends.

10. An organ pipe, open at both ends, contains
    (a) longitudinal stationary waves
    (b) longitudinal travelling waves
    (c) transverse stationary waves
    (d) transverse travelling waves.

11. A cylindrical tube, open at both ends, has a fundamental frequency $v$. The tube is dipped vertically in water so that half of its length is inside the water. The new fundamental frequency is
    (a) $v/4$      (b) $v/2$      (c) $v$      (d) $2v$.

12. The phenomenon of beats can take place
    (a) for longitudinal waves only
    (b) for transverse waves only
    (c) for both longitudinal and transverse waves
    (d) for sound waves only.

13. A tuning fork of frequency 512 Hz is vibrated with a sonometer wire and 6 beats per second are heard. The beat frequency reduces if the tension in the string is slightly increased. The original frequency of vibration of the string is
    (a) 506 Hz    (b) 512 Hz    (c) 518 Hz    (d) 524 Hz.

14. The engine of a train sounds a whistle at frequency $v$. The frequency heard by a passenger is
    (a) $> v$      (b) $< v$      (c) $= \dfrac{1}{v}$      (d) $= v$.

15. The change in frequency due to Doppler effect does not depend on
    (a) the speed of the source
    (b) the speed of the observer
    (c) the frequency of the source
    (d) separation between the source and the observer.

16. A small source of sound moves on a circle as shown in figure (16-Q1) and an observer is sitting at $O$. Let $v_1$, $v_2$, $v_3$ be the frequencies heard when the source is at $A$, $B$ and $C$ respectively.
    (a) $v_1 > v_2 > v_3$        (b) $v_1 = v_2 > v_3$
    (c) $v_2 > v_3 > v_1$        (d) $v_1 > v_3 > v_2$.



Figure 16-Q1

## OBJECTIVE II

1. When you speak to your friend, which of the following parameters have a unique value in the sound produced ?
   (a) Frequency.        (b) Wavelength.
   (c) Amplitude.        (d) Wave velocity.

2. An electrically maintained tuning fork vibrates with constant frequency and constant amplitude. If the temperature of the surrounding air increases but pressure remains constant, the sound produced will have
   (a) larger wavelength      (b) larger frequency
   (c) larger velocity      (d) larger time period.

3. The fundamental frequency of a vibrating organ pipe is 200 Hz.
   (a) The first overtone is 400 Hz.
   (b) The first overtone may be 400 Hz

   (c) The first overtone may be 600 Hz.
   (d) 600 Hz is an overtone.

4. A source of sound moves towards an observer.
   (a) The frequency of the source is increased.
   (b) The velocity of sound in the medium is increased.
   (c) The wavelength of sound in the medium towards the observer is decreased.
   (d) The amplitude of vibration of the particles is increased.

5. A listener is at rest with respect to the source of sound. A wind starts blowing along the line joining the source and the observer. Which of the following quantities do not change ?
   (a) Frequency.        (b) Velocity of sound.
   (c) Wavelength.        (d) Time period.

## EXERCISES

1. A steel tube of length 1·00 m is struck at one end. A person with his ear close to the other end hears the sound of the blow twice, one travelling through the body of the tube and the other through the air in the tube. Find the time gap between the two hearings. Use the table in the text for speeds of sound in various substances.

2. At a prayer meeting, the disciples sing *JAI-RAM JAI-RAM*. The sound amplified by a loudspeaker comes back after reflection from a building at a distance of 80 m from the meeting. What maximum time interval can be kept between one *JAI-RAM* and the next *JAI-RAM* so that the echo does not disturb a listener sitting in the meeting. Speed of sound in air is 320 m/s.

3. A man stands before a large wall at a distance of 50·0 m and claps his hands at regular intervals. Initially, the interval is large. He gradually reduces the interval and fixes it at a value when the echo of a clap merges with the next clap. If he has to clap 10 times during every 3 seconds, find the velocity of sound in air.

4. A person can hear sound waves in the frequency range 20 Hz to 20 kHz. Find the minimum and the maximum wavelengths of sound that is audible to the person. The speed of sound is 360 m/s.

5. Find the minimum and maximum wavelengths of sound in water that is in the audible range (20–20000 Hz) for an average human ear. Speed of sound in water = 1450 m/s.

6. Sound waves from a loudspeaker spread nearly uniformly in all directions if the wavelength of the sound is much larger than the diameter of the loudspeaker. (a) Calculate the frequency for which the wavelength of sound in air is ten times the diameter of the speaker if the diameter is 20 cm. (b) Sound is essentially transmitted in the forward direction if the wavelength is much shorter than the diameter of the speaker. Calculate the frequency at which the wavelength of the sound is one tenth of the diameter of the speaker described above. Take the speed of sound to be 340 m/s.

7. Ultrasonic waves of frequency 4·5 MHz are used to detect tumour in soft tissues. The speed of sound in tissue is 1·5 km/s and that in air is 340 m/s. Find the wavelength of this ultrasonic wave in air and in tissue.

8. The equation of a travelling sound wave is $y = 6·0 \sin(600\, t - 1·8\, x)$ where $y$ is measured in $10^{-5}$ m, $t$ in second and $x$ in metre. (a) Find the ratio of the displacement amplitude of the particles to the wavelength of the wave. (b) Find the ratio of the velocity amplitude of the particles to the wave speed.

9. A sound wave of frequency 100 Hz is travelling in air. The speed of sound in air is 350 m/s. (a) By how much is the phase changed at a given point in 2·5 ms ? (b) What is the phase difference at a given instant between two points separated by a distance of 10·0 cm along the direction of propagation ?

10. Two point sources of sound are kept at a separation of 10 cm. They vibrate in phase to produce waves of wavelength 5·0 cm. What would be the phase difference between the two waves arriving at a point 20 cm from one source (a) on the line joining the sources and (b) on the perpendicular bisector of the line joining the sources ?

11. Calculate the speed of sound in oxygen from the following data. The mass of 22·4 litre of oxygen at STP ($T = 273$ K and $p = 1·0 \times 10^5$ N/m$^2$) is 32 g, the molar heat capacity of oxygen at constant volume is $C_v = 2·5\, R$ and that at constant pressure is $C_p = 3·5\, R$.

12. The speed of sound as measured by a student in the laboratory on a winter day is 340 m/s when the room temperature is 17°C. What speed will be measured by another student repeating the experiment on a day when the room temperature is 32°C ?

13. At what temperature will the speed of sound be double of its value at 0°C ?

14. The absolute temperature of air in a region linearly increases from $T_1$ to $T_2$ in a space of width $d$. Find the time taken by a sound wave to go through the region in terms of $T_1$, $T_2$, $d$ and the speed $v$ of sound at 273 K. Evaluate this time for $T_1 = 280$ K, $T_2 = 310$ K, $d = 33$ m and $v = 330$ m/s.

15. Find the change in the volume of 1·0 litre kerosene when it is subjected to an extra pressure of $2·0 \times 10^5$ N/m$^2$ from the following data. Density of kerosene = 800 kg/m$^3$ and speed of sound in kerosene = 1330 m/s.

16. Calculate the bulk modulus of air from the following data about a sound wave of wavelength 35 cm travelling in air. The pressure at a point varies between $(1·0 \times 10^5 + 14)$ Pa and the particles of the air vibrate in simple harmonic motion of amplitude $5·5 \times 10^{-6}$ m.

17. A source of sound operates at 2·0 kHz, 20 W emitting sound uniformly in all directions. The speed of sound in air is 340 m/s and the density of air is 1·2 kg/m$^3$. (a) What is the intensity at a distance of 6·0 m from the source ? (b) What will be the pressure amplitude at this point ? (c) What will be the displacement amplitude at this point ?

18. The intensity of sound from a point source is $1·0 \times 10^{-8}$ W/m$^2$ at a distance of 5·0 m from the source. What will be the intensity at a distance of 25 m from the source ?

19. The sound level at a point 5·0 m away from a point source is 40 dB. What will be the level at a point 50 m away from the source ?

20. If the intensity of sound is doubled, by how many decibels does the sound level increase ?

21. Sound with intensity larger than 120 dB appears painful to a person. A small speaker delivers 2·0 W of audio output. How close can the person get to the speaker without hurting his ears ?

22. If the sound level in a room is increased from 50 dB to 60 dB, by what factor is the pressure amplitude increased ?

23. The noise level in a class-room in absence of the teacher is 50 dB when 50 students are present. Assuming that on the average each student outputs same sound energy per second, what will be the noise level if the number of students is increased to 100 ?

24. In a Quincke's experiment the sound detected is changed from a maximum to a minimum when the sliding tube is moved through a distance of 2·50 cm. Find the frequency of sound if the speed of sound in air is 340 m/s.

25. In a Quincke's experiment, the sound intensity has a minimum value $I$ at a particular position. As the sliding tube is pulled out by a distance of 16·5 mm, the intensity

increases to a maximum of $9\,I$. Take the speed of sound in air to be 330 m/s. (a) Find the frequency of the sound source. (b) Find the ratio of the amplitudes of the two waves arriving at the detector assuming that it does not change much between the positions of minimum intensity and maximum intensity.

26. Two audio speakers are kept some distance apart and are driven by the same amplifier system. A person is sitting at a place 6·0 m from one of the speakers and 6·4 m from the other. If the sound signal is continuously varied from 500 Hz to 5000 Hz, what are the frequencies for which there is a destructive interference at the place of the listener? Speed of sound in air = 320 m/s.

27. A source of sound $S$ and a detector $D$ are placed at some distance from one another. A big cardboard is placed near the detector and perpendicular to the line $SD$ as shown in figure (16-E1). It is gradually moved away and it is found that the intensity changes from a maximum to a minimum as the board is moved through a distance of 20 cm. Find the frequency of the sound emitted. Velocity of sound in air is 336 m/s.
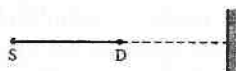


Figure 16-E1

28. A source $S$ and a detector $D$ are placed at a distance $d$ apart. A big cardboard is placed at a distance $H$ from the source and the detector as shown in figure (16-E2). The source emits a wave of wavelength $= d/2$ which is received by the detector after reflection from the cardboard. It is found to be in phase with the direct wave received from the source. By what minimum distance should the cardboard be shifted away so that the reflected wave becomes out of phase with the direct wave?
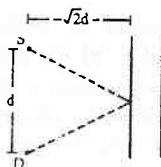


Figure 16-E2

29. Two stereo speakers are separated by a distance of 2·40 m. A person stands at a distance of 3·20 m directly in front of one of the speakers as shown in figure (16-E3). Find the frequencies in the audible range (20-2000 Hz) for which the listener will hear a minimum sound intensity. Speed of sound in air = 320 m/s.
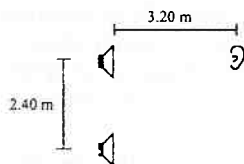


Figure 16-E3

30. Two sources of sound, $S_1$ and $S_2$, emitting waves of equal wavelength 20·0 cm, are placed with a separation of 20·0 cm between them. A detector can be moved on a line parallel to $S_1 S_2$ and at a distance of 20·0 cm from it. Initially, the detector is equidistant from the two sources. Assuming that the waves emitted by the sources are in phase, find the minimum distance through which the detector should be shifted to detect a minimum of sound.

31. Two speakers $S_1$ and $S_2$, driven by the same amplifier, are placed at $y = 1·0$ m and $y = -1·0$ m (figure 16-E4). The speakers vibrate in phase at 600 Hz. A man stands at a point on the X-axis at a very large distance from the origin and starts moving parallel to the Y-axis. The speed of sound in air is 330 m/s. (a) At what angle $\theta$ will the intensity of sound drop to a minimum for the first time? (b) At what angle will he hear a maximum of sound intensity for the first time? (c) If he continues to walk along the line, how many more maxima can he hear?
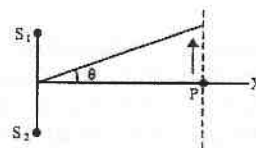


Figure 16-E4

32. Three sources of sound $S_1$, $S_2$ and $S_3$ of equal intensity are placed in a straight line with $S_1 S_2 = S_2 S_3$ (figure 16-E5). At a point $P$, far away from the sources, the wave coming from $S_2$ is 120° ahead in phase of that from $S_1$. Also, the wave coming from $S_3$ is 120° ahead of that from $S_2$. What would be the resultant intensity of sound at $P$?



Figure 16-E5

33. Two coherent narrow slits emitting sound of wavelength $\lambda$ in the same phase are placed parallel to each other at a small separation of $2\lambda$. The sound is detected by moving a detector on the screen $\Sigma$ at a distance $D(\gg \lambda)$ from the slit $S_1$ as shown in figure (16-E6). Find the distance $x$ such that the intensity at $P$ is equal to the intensity at $O$.
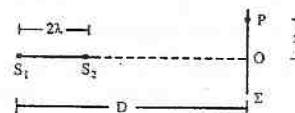


Figure 16-E6

34. Figure (16-E7) shows two coherent sources $S_1$ and $S_2$ which emit sound of wavelength $\lambda$ in phase. The separation between the sources is $3\lambda$. A circular wire of large radius is placed in such a way that $S_1 S_2$ lies in its plane and the middle point of $S_1 S_2$ is at the centre of

the wire. Find the angular positions $\theta$ on the wire for which constructive interference takes place.
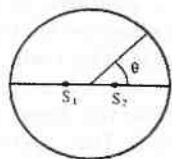


Figure 16-E7

35. Two sources of sound $S_1$ and $S_2$ vibrate at same frequency and are in phase (figure 16-E8). The intensity of sound detected at a point $P$ as shown in the figure is $I_0$. (a) If $\theta$ equals 45°, what will be the intensity of sound detected at this point if one of the sources is switched off? (b) What will be the answer of the previous part if $\theta$ = 60° ?
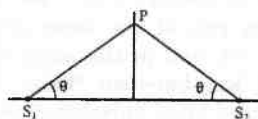


Figure 16-E8

36. Find the fundamental, first overtone and second overtone frequencies of an open organ pipe of length 20 cm. Speed of sound in air is 340 m/s.

37. A closed organ pipe can vibrate at a minimum frequency of 500 Hz. Find the length of the tube. Speed of sound in air = 340 m/s.

38. In a standing wave pattern in a vibrating air column, nodes are formed at a distance of 4·0 cm. If the speed of sound in air is 328 m/s, what is the frequency of the source ?

39. The separation between a node and the next antinode in a vibrating air column is 25 cm. If the speed of sound in air is 340 m/s, find the frequency of vibration of the air column.

40. A cylindrical metal tube has a length of 50 cm and is open at both ends. Find the frequencies between 1000 Hz and 2000 Hz at which the air column in the tube can resonate. Speed of sound in air is 340 m/s.

41. In a resonance column experiment, a tuning fork of frequency 400 Hz is used. The first resonance is observed when the air column has a length of 20·0 cm and the second resonance is observed when the air column has a length of 62·0 cm. (a) Find the speed of sound in air. (b) How much distance above the open end does the pressure node form ?

42. The first overtone frequency of a closed organ pipe $P_1$ is equal to the fundamental frequency of an open organ pipe $P_2$. If the length of the pipe $P_1$ is 30 cm, what will be the length of $P_2$ ?

43. A copper rod of length 1·0 m is clamped at its middle point. Find the frequencies between 20 Hz-20,000 Hz at which standing longitudinal waves can be set up in the rod. The speed of sound in copper is 3·8 km/s.

44. Find the greatest length of an organ pipe open at both ends that will have its fundamental frequency in the normal hearing range (20-20,000 Hz). Speed of sound in air = 340 m/s.

45. An open organ pipe has a length of 5 cm. (a) Find the fundamental frequency of vibration of this pipe. (b) What is the highest harmonic of such a tube that is in the audible range ? Speed of sound in air is 340 m/s and the audible range is 20-20,000 Hz.

46. An electronically driven loudspeaker is placed near the open end of a resonance column apparatus. The length of air column in the tube is 80 cm. The frequency of the loudspeaker can be varied between 20 Hz-2 kHz. Find the frequencies at which the column will resonate. Speed of sound in air = 320 m/s.

47. Two successive resonance frequencies in an open organ pipe are 1944 Hz and 2592 Hz. Find the length of the tube. The speed of sound in air is 324 m/s.

48. A piston is fitted in a cylindrical tube of small cross-section with the other end of the tube open. The tube resonates with a tuning fork of frequency 512 Hz. The piston is gradually pulled out of the tube and it is found that a second resonance occurs when the piston is pulled out through a distance of 32·0 cm. Calculate the speed of sound in the air of the tube.

49. A U-tube having unequal arm-lengths has water in it. A tuning fork of frequency 440 Hz can set up the air in the shorter arm in its fundamental mode of vibration and the same tuning fork can set up the air in the longer arm in its first overtone vibration. Find the length of the air columns. Neglect any end effect and assume that the speed of sound in air = 330 m/s.

50. Consider the situation shown in figure (16-E9). The wire which has a mass of 4·00 g oscillates in its second harmonic and sets the air column in the tube into vibrations in its fundamental mode. Assuming that the speed of sound in air is 340 m/s, find the tension in the wire.
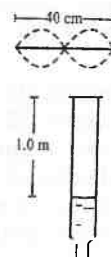


Figure 16-E9

51. A 30·0-cm-long wire having a mass of 10·0 g is fixed at the two ends and is vibrated in its fundamental mode. A 50·0-cm-long closed organ pipe, placed with its open end near the wire, is set up into resonance in its fundamental mode by the vibrating wire. Find the tension in the wire. Speed of sound in air = 340 m/s.

52. Show that if the room temperature changes by a small amount from $T$ to $T + \Delta T$, the fundamental frequency of an organ pipe changes from $v$ to $v + \Delta v$, where

$$\frac{\Delta v}{v} = \frac{1}{2} \frac{\Delta T}{T}.$$

53. The fundamental frequency of a closed pipe is 293 Hz when the air in it is at a temperature of 20°C. What will be its fundamental frequency when the temperature changes to 22°C ?

54. A Kundt's tube apparatus has a copper rod of length 1·0 m clamped at 25 cm from one of the ends. The tube contains air in which the speed of sound is 340 m/s. The powder collects in heaps separated by a distance of 5·0 cm. Find the speed of sound waves in copper.

55. A Kundt's tube apparatus has a steel rod of length 1·0 m clamped at the centre. It is vibrated in its fundamental mode at a frequency of 2600 Hz. The lycopodium powder dispersed in the tube collects into heaps separated by 6·5 cm. Calculate the speed of sound in steel and in air.

56. A source of sound with adjustable frequency produces 2 beats per second with a tuning fork when its frequency is either 476 Hz or 480 Hz. What is the frequency of the tuning fork ?

57. A tuning fork produces 4 beats per second with another tuning fork of frequency 256 Hz. The first one is now loaded with a little wax and the beat frequency is found to increase to 6 per second. What was the original frequency of the tuning fork ?

58. Calculate the frequency of beats produced in air when two sources of sound are activated, one emitting a wavelength of 32 cm and the other of 32·2 cm. The speed of sound in air is 350 m/s.

59. A tuning fork of unknown frequency makes 5 beats per second with another tuning fork which can cause a closed organ pipe of length 40 cm to vibrate in its fundamental mode. The beat frequency decreases when the first tuning fork is slightly loaded with wax. Find its original frequency. The speed of sound in air is 320 m/s.

60. A piano wire A vibrates at a fundamental frequency of 600 Hz. A second identical wire B produces 6 beats per second with it when the tension in A is slightly increased. Find the ratio of the tension in A to the tension in B.

61. A tuning fork of frequency 256 Hz produces 4 beats per second with a wire of length 25 cm vibrating in its fundamental mode. The beat frequency decreases when the length is slightly shortened. What could be the minimum length by which the wire be shortened so that it produces no beats with the tuning fork ?

62. A traffic policeman standing on a road sounds a whistle emitting the main frequency of 2·00 kHz. What could be the appparent frequency heard by a scooter-driver approaching the policeman at a speed of 36·0 km/h ? Speed of sound in air = 340 m/s.

63. The horn of a car emits sound with a dominant frequency of 2400 Hz. What will be the apparent dominant frequency heard by a person standing on the road in front of the car if the car is going at 18·0 km/h ? Speed of sound in air = 340 m/s.

64. A person riding a car moving at 72 km/h sounds a whistle emitting a wave of frequency 1250 Hz. What frequency will be heard by another person standing on the road (a) in front of the car (b) behind the car ? Speed of sound in air = 340 m/s.

65. A train approaching a platform at a speed of 54 km/h sounds a whistle. An observer on the platform finds its frequency to be 1620 Hz. The train passes the platform keeping the whistle on and without slowing down. What frequency will the observer hear after the train has crossed the platfrom ? The speed of sound in air = 332 m/s.

66. A bat emitting an ultrasonic wave of frequency 4·5 × 10⁴ Hz flies at a speed of 6 m/s between two parallel walls. Find the two frequencies heard by the bat and the beat frequency between the two. The speed of sound is 330 m/s.

67. A bullet passes past a person at a speed of 220 m/s. Find the fractional change in the frequency of the whistling sound heard by the person as the bullet crosses the person. Speed of sound in air = 330 m/s.

68. Two electric trains run at the same speed of 72 km/h along the same track and in the same direction with a separation of 2·4 km between them. The two trains simultaneously sound brief whistles. A person is situated at a perpendicular distance of 500 m from the track and is equidistant from the two trains at the instant of the whistling. If both the whistles were at 500 Hz and the speed of sound in air is 340 m/s, find the frequencies heard by the person.

69. A violin player riding on a slow train plays a 440 Hz note. Another violin player standing near the track plays the same note. When the two are close by and the train approaches the person on the ground, he hears 4·0 beats per second. The speed of sound in air = 340 m/s. (a) Calculate the speed of the train. (b) What beat frequency is heard by the player in the train ?

70. Two identical tuning forks vibrating at the same frequency 256 Hz are kept fixed at some distance apart. A listener runs between the forks at a speed of 3·0 m/s so that he approaches one tuning fork and recedes from the other (figure 16-E10). Find the beat frequency observed by the listener. Speed of sound in air = 332 m/s.



Figure 16-E10

71. Figure (16-E11) shows a person standing somewhere in between two identical tuning forks, each vibrating at
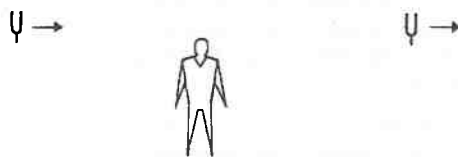


Figure 16-E11

512 Hz. If both the tuning forks move towards right at a speed of 5·5 m/s, find the number of beats heard by the listener. Speed of sound in air = 330 m/s.

72. A small source of sound vibrating at frequency 500 Hz is rotated in a circle of radius 100/π cm at a constant angular speed of 5·0 revolutions per second. A listener situates himself in the plane of the circle. Find the minimum and the maximum frequency of the sound observed. Speed of sound in air = 332 m/s.

73. Two trains are travelling towards each other both at a speed of 90 km/h. If one of the trains sounds a whistle at 500 Hz, what will be the apparent frequency heard in the other train ? Speed of sound in air = 350 m/s.

74. A traffic policeman sounds a whistle to stop a car-driver approaching towards him. The car-driver does not stop and takes the plea in court that because of the Doppler shift, the frequency of the whistle reaching him might have gone beyond the audible limit of 20 kHz and he did not hear it. Experiments showed that the whistle emits a sound with frequency close to 16 kHz. Assuming that the claim of the driver is true, how fast was he driving the car ? Take the speed of sound in air to be 330 m/s. Is this speed practical with today's technology ?

75. A car moving at 108 km/h finds another car in front of it going in the same direction at 72 km/h. The first car sounds a horn that has a dominant frequency of 800 Hz. What will be the apparent frequency heard by the driver in the front car ? Speed of sound in air = 330 m/s.

76. Two submarines are approaching each other in a calm sea. The first submarine travels at a speed of 36 km/h and the other at 54 km/h relative to the water. The first submarine sends a sound signal (sound waves in water are also called *sonar*) at a frequency of 2000 Hz. (a) At what frequency is this signal received by the second submarine ? (b) The signal is reflected from the second submarine. At what frequency is this signal received by the first submarine. Take the speed of the sound wave in water to be 1500 m/s.

77. A small source of sound oscillates in simple harmonic motion with an amplitude of 17 cm. A detector is placed along the line of motion of the source. The source emits a sound of frequency 800 Hz which travels at a speed of 340 m/s. If the width of the frequency band detected by the detector is 8 Hz, find the time period of the source.

78. A boy riding on his bike is going towards east at a speed of 4 √2 m/s. At a certain point he produces a sound pulse of frequency 1650 Hz that travels in air at a speed of 334 m/s. A second boy stands on the ground 45° south of east from him. Find the frequency of the pulse as received by the second boy.

79. A sound source, fixed at the origin, is continuously emitting sound at a frequency of 660 Hz. The sound travels in air at a speed of 330 m/s. A listener is moving along the line x = 336 m at a constant speed of 26 m/s. Find the frequency of the sound as observed by the listener when he is (a) at y = −140 m, (b) at y = 0 and (c) at y = 140 m.

80. A train running at 108 km/h towards east whistles at a dominant frequency of 500 Hz. Speed of sound in air is 340 m/s. What frequency will a passenger sitting near the open window hear ? (b) What frequency will a person standing near the track hear whom the train has just passed ? (c) A wind starts blowing towards east at a speed of 36 km/h. Calculate the frequencies heard by the passenger in the train and by the person standing near the track.

81. A boy riding on a bicycle going at 12 km/h towards a vertical wall whistles at his dog on the ground. If the frequency of the whistle is 1600 Hz and the speed of sound in air is 330 m/s, find (a) the frequency of the whistle as received by the wall (b) the frequency of the reflected whistle as received by the boy.

82. A person standing on a road sends a sound signal to the driver of a car going away from him at a speed of 72 km/h. The signal travelling at 330 m/s in air and having a frequency of 1600 Hz gets reflected from the body of the car and returns. Find the frequency of the reflected signal as heard by the person.

83. A car moves with a speed of 54 km/h towards a cliff. The horn of the car emits sound of frequency 400 Hz at a speed of 335 m/s. (a) Find the wavelength of the sound emitted by the horn in front of the car. (b) Find the wavelength of the wave reflected from the cliff. (c) What frequency does a person sitting in the car hear for the reflected sound wave ? (d) How many beats does he hear in 10 seconds between the sound coming directly from the horn and that coming after the reflection ?

84. An operator sitting in his base camp sends a sound signal of frequency 400 Hz. The signal is reflected back from a car moving towards him. The frequency of the reflected sound is found to be 410 Hz. Find the speed of the car. Speed of sound in air = 324 m/s.

85. Figure (16-E12) shows a source of sound moving along the X-axis at a speed of 22 m/s continuously emitting a sound of frequency 2·0 kHz which travels in air at a speed of 330 m/s. A listener Q stands on the Y-axis at a distance of 330 m from the origin. At t = 0, the source crosses the origin P. (a) When does the sound emitted from the source at P reach the listener Q ? (b) What will be the frequency heard by the listener at this instant ? (c) Where will the source be at this instant ?



Figure 16-E12

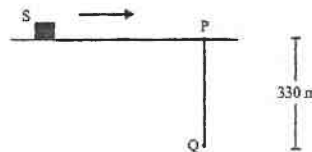86. A source emitting sound at frequency 4000 Hz, is moving along the Y-axis with a speed of 22 m/s. A listener is situated on the ground at the position (660 m, 0). Find the frequency of the sound received by the listener at the instant the source crosses the origin. Speed of sound in air = 330 m/s.

87. A source of sound emitting a 1200 Hz note travels along a straight line at a speed of 170 m/s. A detector is placed

at a distance of 200 m from the line of motion of the source. (a) Find the frequency of sound received by the detector at the instant when the source gets closest to it. (b) Find the distance between the source and the detector at the instant it detects the frequency 1200 Hz. Velocity of sound in air = 340 m/s.

**88.** A small source of sound $S$ of frequency 500 Hz is attached to the end of a light string and is whirled in a vertical circle of radius 1·6 m. The string just remains tight when the source is at the highest point. (a) An observer is located in the same vertical plane at a large distance and at the same height as the centre of the circle (figure 16-E13). The speed of sound in air = 330 m/s and $g = 10$ m/s². Find the maximum frequency heard by the observer. (b) An observer is situated at a large distance vertically above the centre of the circle. Find the frequencies heard by the observer corresponding to

the sound emitted by the source when it is at the same height as the centre.
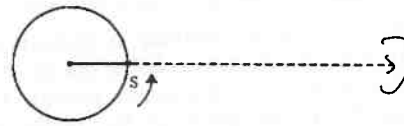


Figure 16-E13

**89.** A source emitting a sound of frequency ν is placed at a large distance from an observer. The source starts moving towards the observer with a uniform acceleration $a$. Find the frequency heard by the observer corresponding to the wave emitted just after the source starts. The speed of sound in the medium is $v$.

□

# ANSWERS

## OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (a) | 2. (d) | 3. (d) | 4. (c) | 5. (d) | 6. (c) |
| 7. (a) | 8. (d) | 9. (b) | 10. (a) | 11. (c) | 12. (c) |
| 13. (a) | 14. (d) | 15. (d) | 16. (c) | | |

## OBJECTIVE II

1. (d)            2. (a), (c)            3. (b), (c), (d)
4. (c)            5. (a), (d)

## EXERCISES

1. 2·75 ms
2. 0·5 s
3. 333 m/s
4. 18 mm, 18 m
5. 7·25 cm, 72·5 m
6. (a) 170 Hz        (b) 17 kHz
7. $7·6 \times 10^{-5}$ m, $3·3 \times 10^{-4}$ m
8. (a) $1·7 \times 10^{-5}$        (b) $1·1 \times 10^{-4}$
9. (a) $\pi/2$        (b) $2\pi/35$
10. (a) zero        (b) zero
11. 310 m/s
12. 349 m/s
13. 819°C
14. $\dfrac{2d}{v} \cdot \dfrac{\sqrt{273}}{\sqrt{T_1} + \sqrt{T_2}}$, 96 ms
15. 0·14 cm³
16. $1·4 \times 10^9$ N/m²
17. (a) 44 mW/m²        (b) 6·0 Pa        (c) $1·2 \times 10^{-6}$ m

18. $4·0 \times 10^{-10}$ W/m²
19. 20 dB
20. 3 dB
21. 40 cm
22. $\sqrt{10}$
23. 53 dB
24. 3·4 kHz
25. (a) 5·0 kHz        (b) 2
26. 1200 Hz, 2000 Hz, 2800 Hz, 3600 Hz and 4400 Hz
27. 420 Hz
28. 0·13 $d$
29. 200 $(2n+1)$ Hz where $n = 0, 1, 2, \ldots 49$
30. 12·6 cm
31. (a) 7·9°        (b) 16°        (c) two
32. zero
33. $\sqrt{3}\, D$
34. 0°, 48·2°, 70·5°, 90° and similar points in other quadrants
35. (a) $I_0/4$        (b) $I_0/4$
36. 850 Hz, 1700 Hz and 2550 Hz
37. 17 cm
38. 4·1 kHz
39. 340 Hz
40. 1020 Hz, 1360 Hz and 1700 Hz
41. (a) 336 m/s        (b) 1 cm
42. 20 cm
43. 1·9 $n$ kHz where $n = 1, 2, 3, \ldots, 10$
44. 8·5 m
45. (a) 3·4 kHz        (b) 5
46. 100$(2n+1)$ Hz where $n = 0, 1, 2, 3, \ldots, 9$
47. 25 cm

48. 328 m/s

49. 18·8 cm, 56·3 cm

50. 11·6 N

51. 347 N

53. 294 Hz

54. 3400 m/s

55. 5200 m/s, 338 m/s

56. 478 Hz

57. 252 Hz

58. 7 per second

59. 205 Hz

60. 1·02

61. 0·39 cm

62. 2·06 kHz

63. 2436 Hz

64. (a) 1328 Hz        (b) 1181 Hz

65. 1480 Hz

66. $4·67 \times 10^{4}$ Hz, $4·34 \times 10^{4}$ Hz, 3270 Hz

67. 0·8

68. 529 Hz, 474 Hz

69. (a) 11 km/h        (b) a little less than 4 beats/s

70. 4·6 Hz

71. 17·5 Hz, may not be able to distinguish

72. 485 Hz and 515 Hz

73. 577 Hz

74. 300 km/h

75. 827 Hz

76. (a) 2034 Hz        (b) 2068 Hz

77. 0·63 s

78. 1670 Hz

79. (a) 680 Hz        (b) 660 Hz        (c) 640 Hz

80. (a) 500 Hz        (b) 459 Hz
    (c) 500 Hz by the passenger and 458 by the person near the track

81. (a) 1616 Hz        (b) 1632 Hz

82. 1417 Hz

83. (a) 80 cm        (b) 80 cm        (c) 437 Hz
    (d) No beat may be heard

84. 12 m/s

85. (a) $t = 1$ second        (b) 2·0 kHz        (c) at $x = 22$ m

86. 4018 Hz

87. (a) 1600 Hz        (b) 224 m

88. (a) 506 Hz        (b) 490 Hz and 511 Hz

89. $\dfrac{2\upsilon v^{2}}{2\upsilon v - a}$

□

# QUESTIONS FOR SHORT ANSWER

1. Is the colour of 620 nm light and 780 nm light same ? Is the colour of 620 nm light and 621 nm light same ? How many colours are there in white light ?

2. The wavelength of light in a medium is $\lambda = \lambda_0/\mu$, where $\lambda$ is the wavelength in vacuum. A beam of red light ($\lambda_0 = 720$ nm) enters into water. The wavelength in water is $\lambda = \lambda_0/\mu = 540$ nm. To a person under water does this light appear green ?

3. Whether the diffraction effects from a slit will be more clearly visible or less clearly, if the slit-width is increased ?

4. If we put a cardboard (say 20 cm × 20 cm) between a light source and our eyes, we can't see the light. But when we put the same cardboard between a sound source and our ear, we hear the sound almost clearly. Explain.

5. TV signals broadcast by Delhi studio cannot be directly received at Patna which is about 1000 km away. But the same signal goes some 36000 km away to a satellite, gets reflected and is then received at Patna. Explain.

6. Can we perform Young's double slit experiment with sound waves ? To get a reasonable "fringe pattern", what should be the order of separation between the slits ? How can the bright fringes and the dark fringes be detected in this case ?

7. Is it necessary to have two waves of equal intensity to study interference pattern ? Will there be an effect on clarity if the waves have unequal intenstity ?

8. Can we conclude from the interference phenomenon whether light is a transverse wave or a longitudinal wave ?

9. Why don't we have interference when two candles are placed close to each other and the intensity is seen at a distant screen ? What happens if the candles are replaced by laser sources ?

10. If the separation between the slits in a Young's double slit experiment is increased, what happens to the fringe-width ? If the separation is increased too much, will the fringe pattern remain detectable ?

11. Suppose white light falls on a double slit but one slit is covered by a violet filter (allowing $\lambda = 400$ nm). Describe the nature of the fringe pattern observed.

# OBJECTIVE I

1. Light is
   (a) wave phenomenon      (b) particle phenomenon
   (c) both particle and wave phenomenon.

2. The speed of light depends
   (a) on elasticity of the medium only
   (b) on inertia of the medium only
   (c) on elasticity as well as inertia
   (d) neither on elasticity nor on inertia.

3. The equation of a light wave is written as $y = A \sin(kx - \omega t)$. Here, $y$ represents
   (a) displacement of ether particles
   (b) pressure in the medium
   (c) density of the medium
   (d) electric field.

4. Which of the following properties show that light is a transverse wave ?
   (a) Reflection.          (b) Interference.
   (c) Diffraction.         (d) Polarization.

5. When light is refracted into a medium,
   (a) its wavelength and frequency both increase
   (b) its wavelength increases but frequency remains unchanged
   (c) its wavelength decreases but frequency remains unchanged
   (d) its wavelength and frequency both decrease.

6. When light is refracted, which of the following does not change ?
   (a) Wavelength.          (b) Frequency.
   (c) Velocity.            (d) Amplitude.

7. The amplitude modulated (AM) radio wave bends appreciably round the corners of a 1 m × 1 m board but the frequency modulated (FM) wave only negligibly bends. If the average wavelengths of AM and FM waves are $\lambda_a$ and $\lambda_f$,
   (a) $\lambda_a > \lambda_f$          (b) $\lambda_a = \lambda_f$          (c) $\lambda_a < \lambda_f$
   (d) we don't have sufficient information to decide about the relation of $\lambda_a$ and $\lambda_f$

8. Which of the following sources gives best monochromatic light ?
   (a) A candle.(b) A bulb.(c) A mercury tube. (d) A laser .

9. The wavefronts of a light wave travelling in vacuum are given by $x + y + z = c$. The angle made by the direction of propagation of light with the $X$-axis is
   (a) 0°          (b) 45°          (c) 90°          (d) $\cos^{-1}(1/\sqrt{3})$.

10. The wavefronts of light coming from a distant source of unknown shape are nearly
    (a) plane    (b) elliptical    (c) cylindrical    (d) spherical.

11. The inverse square law of intensity (i.e., the intensity $\propto \dfrac{1}{r^2}$) is valid for a
    (a) point source          (b) line source
    (c) plane source          (d) cylindrical source.

12. Two sources are called coherent if they produce waves
    (a) of equal wavelength          (b) of equal velocity
    (c) having same shape of wavefront
    (d) having a constant phase difference.

13. When a drop of oil is spread on a water surface, it displays beautiful colours in daylight because of

(a) disperson of light      (b) reflection of light
(c) polarization of light     (d) interference of light.

14. Two coherent sources of different intensities send waves which interfere. The ratio of maximum intensity to the minimum intensity is 25. The intensities of the sources are in the ratio
(a) 25 : 1     (b) 5 : 1     (c) 9 : 4     (d) 625 : 1.

15. The slits in a Young's double slit experiment have equal width and the source is placed symmetrically with respect to the slits. The intensity at the central fringe is $I_0$. If one of the slits is closed, the intensity at

this point will be
(a) $I_0$     (b) $I_0/4$     (c) $I_0/2$     (d) $4I_0$.

16. A thin transparent sheet is placed in front of a Young's double slit. The fringe-width will
(a) increase           (b) decrease
(c) remain same       (d) become nonuniform.

17. If Young's double slit experiment is performed in water,
(a) the fringe width will decrease
(b) the fringe width will increase
(c) the fringe width will remain unchanged
(d) there will be no fringe.

## OBJECTIVE II

1. A light wave can travel
(a) in vacuum          (b) in vacuum only
(c) in a material medium   (d) in a material medium only.

2. Which of the following properties of light conclusively support wave theory of light ?
(a) Light obeys laws of reflection.
(b) Speed of light in water is smaller than the speed in vacuum.
(c) Light shows interference.
(d) Light shows photoelectric effect.

3. When light propagates in vacuum there is an electric field and a magnetic field. These fields
(a) are constant in time
(b) have zero average value
(c) are perpendicular to the direction of propagation of light.
(d) are mutually perpendicular.

4. Huygens' principle of secondary wavelets may be used to
(a) find the velocity of light in vacuum
(b) explain the particle behaviour of light
(c) find the new position of a wavefront
(d) explain Snell's law.

5. Three observers $A$, $B$ and $C$ measure the speed of light coming from a source to be $v_A$, $v_B$ and $v_C$. The observer $A$ moves towards the source and $C$ moves away from the source at the same speed. The observer $B$ stays stationary. The surrounding space is vacuum everywhere.

(a) $v_A > v_B > v_C$.          (b) $v_A < v_B < v_C$.
(c) $v_A = v_B = v_C$.          (d) $v_B = \frac{1}{2}(v_A + v_C)$.

6. Suppose the medium in the previous question is water. Select the correct option(s) from the list given in that question.

7. Light waves travel in vacuum along the $X$-axis. Which of the following may represent the wavefronts ?
(a) $x = c$.   (b) $y = c$.   (c) $z = c$.   (d) $x + y + z = c$.

8. If the source of light used in a Young's double slit experiment is changed from red to violet,
(a) the fringes will become brighter
(b) consecutive fringes will come closer
(c) the intensity of minima will increase
(d) the central bright fringe will become a dark fringe.

9. A Young's double slit experiment is performed with white light.
(a) The central fringe will be white.
(b) There will not be a completely dark fringe.
(c) The fringe next to the central will be red.
(d) The fringe next to the central will be violet.

10. Four light waves are represented by
(i) $y = a_1 \sin \omega t$.         (ii) $y = a_2 \sin(\omega t + \varepsilon)$.
(iii) $y = a_1 \sin 2\omega t$.       (iv) $y = a_2 \sin 2(\omega t + \varepsilon)$.
Interference fringes may be observed due to superposition of
(a) (i) and (ii)           (b) (i) and (iii)
(c) (ii) and (iv)         (d) (iii) and (iv).

## EXERCISES

1. Find the range of frequency of light that is visible to an average human being (400 nm < $\lambda$ < 700 nm)

2. The wavelength of sodium light in air is 589 nm. (a) Find its frequency in air. (b) Find its wavelength in water (refractive index = 1·33). (c) Find its frequency in water. (d) Find its speed in water.

3. The index of refraction of fused quartz is 1·472 for light of wavelength 400 nm and is 1·452 for light of

wavelength 760 nm. Find the speeds of light of these wavelengths in fused quartz.

4. The speed of the yellow light in a certain liquid is $2·4 \times 10^8$ m/s. Find the refractive index of the liquid.

5. Two narrow slits emitting light in phase are separated by a distance of 1·0 cm. The wavelength of the light is $5·0 \times 10^{-7}$ m. The interference pattern is observed on a screen placed at a distance of 1·0 m. (a) Find the separation between the consecutive maxima. Can you

expect to distinguish between these maxima ? (b) Find the separation between the sources which will give a separation of 1·0 mm between the consecutive maxima.

6. The separation between the consecutive dark fringes in a Young's double slit experiment is 1·0 mm. The screen is placed at a distance of 2·5 m from the slits and the separation between the slits is 1·0 mm. Calculate the wavelength of light used for the experiment.

7. In a double slit interference experiment, the separation between the slits is 1·0 mm, the wavelength of light used is $5·0 \times 10^{-7}$ m and the distance of the screen from the slits is 1·0 m. (a) Find the distance of the centre of the first minimum from the centre of the central maximum. (b) How many bright fringes are formed in one centimeter width on the screen ?

8. In a Young's double slit experiment, two narrow vertical slits placed 0·800 mm apart are illuminated by the same source of yellow light of wavelength 589 nm. How far are the adjacent bright bands in the interference pattern observed on a screen 2·00 m away ?

9. Find the angular separation between the consecutive bright fringes in a Young's double slit experiment with blue-green light of wavelength 500 nm. The separation between the slits is $2·0 \times 10^{-3}$ m.

10. A source emitting light of wavelengths 480 nm and 600 nm is used in a double slit interference experiment. The separation between the slits is 0·25 mm and the interference is observed on a screen placed at 150 cm from the slits. Find the linear separation between the first maximum (next to the central maximum) corresponding to the two wavelengths.

11. White light is used in a Young's double slit experiment. Find the minimum order of the violet fringe ($\lambda$ = 400 nm) which overlaps with a red fringe ($\lambda$ = 700 nm).

12. Find the thickness of a plate which will produce a change in optical path equal to half the wavelength $\lambda$ of the light passing through it normally. The refractive index of the plate is $\mu$.

13. A plate of thickness $t$ made of a material of refractive index $\mu$ is placed in front of one of the slits in a double slit experiment. (a) Find the change in the optical path due to introduction of the plate. (b) What should be the minimum thickness $t$ which will make the intensity at the centre of the fringe pattern zero ? Wavelength of the light used is $\lambda$. Neglect any absorption of light in the plate.

14. A transparent paper (refractive index = 1·45) of thickness 0·02 mm is pasted on one of the slits of a Young's double slit experiment which uses monochromatic light of wavelength 620 nm. How many fringes will cross through the centre if the paper is removed ?

15. In a Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1·6 and thickness 1·964 micron (1 micron = $10^{-6}$ m) is introduced in the path of one of the interfering waves. The mica sheet is then removed and

the distance between the screen and the slits is doubled. It is found that the distance between the successive maxima now is the same as the observed fringe-shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

16. A mica strip and a polysterene strip are fitted on the two slits of a double slit apparatus. The thickness of the strips is 0·50 mm and the separation between the slits is 0·12 cm. The refractive index of mica and polysterene are 1·58 and 1·55 respectively for the light of wavelength 590 nm which is used in the experiment. The interference is observed on a screen a distance one meter away. (a) What would be the fringe-width ? (b) At what distance from the centre will the first maximum be located ?

17. Two transparent slabs having equal thickness but different refractive indices $\mu_1$ and $\mu_2$ are pasted side by side to form a composite slab. This slab is placed just after the double slit in a Young's experiment so that the light from one slit goes through one material and the light from the other slit goes through the other material. What should be the minimum thickness of the slab so that there is a minimum at the point $P_0$ which is equidistant from the slits ?

18. A thin paper of thickness 0·02 mm having a refractive index 1·45 is pasted across one of the slits in a Young's double slit experiment. The paper transmits 4/9 of the light energy falling on it. (a) Find the ratio of the maximum intensity to the minimum intensity in the fringe pattern. (b) How many fringes will cross through the centre if an identical paper piece is pasted on the other slit also ? The wavelength of the light used is 600 nm.

19. A Young's double slit apparatus has slits separated by 0·28 mm and a screen 48 cm away from the slits. The whole apparatus is immersed in water and the slits are illuminated by the red light ($\lambda$ = 700 nm in vacuum). Find the fringe-width of the pattern formed on the screen.

20. A parallel beam of monochromatic light is used in a Young's double slit experiment. The slits are separated by a distance $d$ and the screen is placed parallel to the plane of the slits. Show that if the incident beam makes an angle $\theta = \sin^{-1}\left(\dfrac{\lambda}{2d}\right)$ with the normal to the plane of the slits, there will be a dark fringe at the centre $P_0$ of the pattern.

21. A narrow slit $S$ transmitting light of wavelength $\lambda$ is placed a distance $d$ above a large plane mirror as shown in figure (17-E1). The light coming directly from the slit
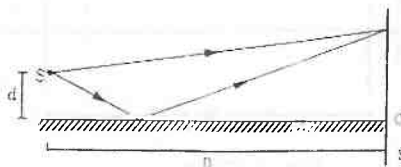


Figure 17-E1

and that coming after the reflection interfere at a screen $\Sigma$ placed at a distance $D$ from the slit. (a) What will be the intensity at a point just above the mirror, i.e., just above $O$ ? (b) At what distance from $O$ does the first maximum occur ?

22. A long narrow horizontal slit is placed 1 mm above a horizontal plane mirror. The interference between the light coming directly from the slit and that after reflection is seen on a screen 1·0 m away from the slit. Find the fringe-width if the light used has a wavelength of 700 nm.

23. Consider the situation of the previous problem. If the mirror reflects only 64% of the light energy falling on it, what will be the ratio of the maximum to the minimum intensity in the interference pattern observed on the screen ?

24. A double slit $S_1 - S_2$ is illuminated by a coherent light of wavelength $\lambda$. The slits are separated by a distance $d$. A plane mirror is placed in front of the double slit at a distance $D_1$ from it and a screen $\Sigma$ is placed behind the double slit at a distance $D_2$ from it (figure 17-E2). The screen $\Sigma$ receives only the light reflected by the mirror. Find the fringe-width of the interference pattern on the screen.
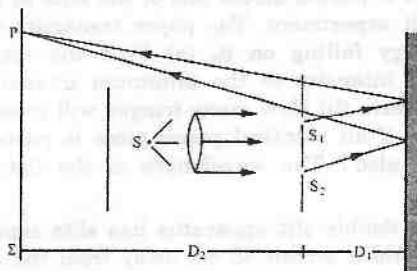


Figure 17-E2

25. White coherent light (400 nm–700 nm) is sent through the slits of a Young's double slit experiment (figure 17-E3). The separation between the slits is 0·5 mm and the screen is 50 cm away from the slits. There is a hole in the screen at a point 1 0 mm away (along the width of the fringes) from the central line. (a) Which wavelength(s) will be absent in the light coming from the hole ? (b) which wavelength(s) will have a strong intensity ?
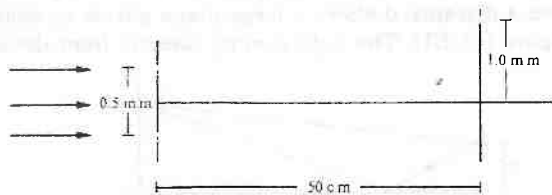


Figure 17-E3

26. Consider the arrangement shown in figure (17-E4). The distance $D$ is large compared to the separation $d$ between the slits. (a) Find the minimum value of $d$ so that there is a dark fringe at $O$. (b) Suppose $d$ has this value. Find the distance $x$ at which the next bright fringe is formed. (c) Find the fringe-width.
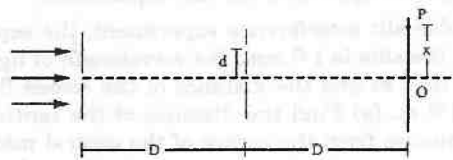


Figure 17-E4

27. Two coherent point sources $S_1$ and $S_2$ vibrating in phase emit light of wavelength $\lambda$. The separation between the sources is $2\lambda$. Consider a line passing through $S_2$ and perpendicular to the line $S_1 S_2$. What is the smallest distance from $S_2$ where a minimum of intensity occurs ?

28. Figure (17-E5) shows three equidistant slits being illuminated by a monochromatic parallel beam of light. Let $BP_0 - AP_0 = \lambda/3$ and $D >> \lambda$. (a) Show that in this case $d = \sqrt{2\lambda D/3}$. (b) Show that the intensity at $P_0$ is three times the intensity due to any of the three slits individually.
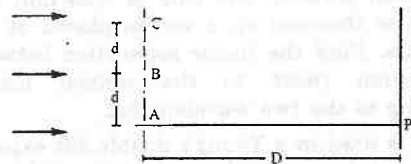


Figure 17-E5

29. In a Young's double slit experiment, the separation between the slits = 2·0 mm, the wavelength of the light = 600 nm and the distance of the screen from the slits = 2·0 m. If the intensity at the centre of the central maximum is 0·20 W/m², what will be the intensity at a point 0·5 cm away from this centre along the width of the fringes ?

30. In a Young's double slit interference experiment the fringe pattern is observed on a screen placed at a distance $D$ from the slits. The slits are separated by a distance $d$ and are illuminated by monochromatic light of wavelength $\lambda$. Find the distance from the central point where the intensity falls to (a) half the maximum, (b) one fourth of the maximum.

31. In a Young's double slit experiment $\lambda = 500$ nm. $d = 1·0$ mm and $D = 1·0$ m. Find the minimum distance from the central maximum for which the intensity is half of the maximum intensity.

32. The linewidth of a bright fringe is sometimes defined as the separation between the points on the two sides of the central line where the intensity falls to half the maximum. Find the linewidth of a bright fringe in a Young's double slit experiment in terms of $\lambda$, $d$ and $D$ where the symbols have their usual meanings.

33. Consider the situation shown in figure (17-E6). The two slits $S_1$ and $S_2$ placed symmetrically around the central line are illuminated by a monochromatic light of wavelength $\lambda$. The separation between the slits is $d$. The light transmitted by the slits falls on a screen $\Sigma_1$ placed at a distance $D$ from the slits. The slit $S_3$ is at the central line and the slit $S_4$ is at a distance $z$ from $S_3$. Another screen $\Sigma_2$ is placed a further distance $D$ away from $\Sigma_1$. Find the ratio of the maximum to minimum intensity observed on $\Sigma_2$ if $z$ is equal to

(a) $z = \dfrac{\lambda D}{2d}$, (b) $\dfrac{\lambda D}{d}$, (c) $\dfrac{\lambda D}{4d}$.
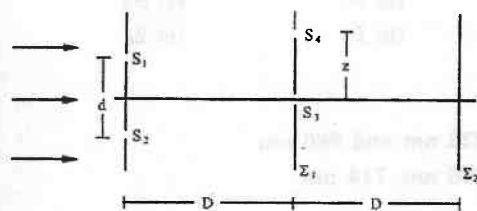


Figure 17-E6

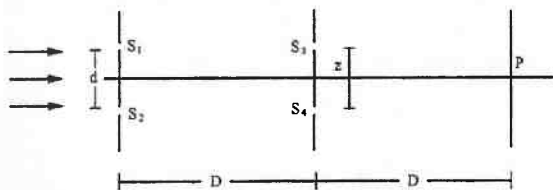34. Consider the arrangement shown in figure (17-E7). By some mechanism, the separation between the slits $S_3$ and $S_4$ can be changed. The intensity is measured at the point $P$ which is at the common perpendicular bisector of $S_1S_2$ and $S_3S_4$. When $z = \dfrac{D\lambda}{2d}$, the intensity measured at $P$ is $I$. Find this intensity when $z$ is equal to

(a) $\dfrac{D\lambda}{d}$, (b) $\dfrac{3D\lambda}{2d}$ and (c) $\dfrac{2D\lambda}{d}$.



Figure 17-E7

35. A soap film of thickness 0·0011 mm appears dark when seen by the reflected light of wavelength 580 nm. What is the index of refraction of the soap solution, if it is known to be between 1·2 and 1·5 ?

36. A parallel beam of light of wavelength 560 nm falls on a thin film of oil (refractive index = 1·4). What should be the minimum thickness of the film so that it strongly reflects the light ?

37. A parallel beam of white light is incident normally on a water film $1.0 \times 10^{-4}$ cm thick. Find the wavelength in the visible range (400 nm-700 nm) which are strongly transmitted by the film. Refractive index of water = 1·33.

38. A glass surface is coated by an oil film of uniform thickness $1.00 \times 10^{-4}$ cm. The index of refraction of the oil is 1·25 and that of the glass is 1·50. Find the wavelengths of light in the visible region (400 nm-750 nm) which are completely transmitted by the oil film under normal incidence.

39. Plane microwaves are incident on a long slit having a width of 5·0 cm. Calculate the wavelength of the microwaves if the first diffraction minimum is formed at $\theta = 30°$.

40. Light of wavelength 560 nm goes through a pinhole of diameter 0·20 mm and falls on a wall at a distance of 2·00 m. What will be the radius of the central bright spot formed on the wall ?

41. A convex lens of diameter 8·0 cm is used to focus a parallel beam of light of wavelength 620 nm. If the light be focused at a distance of 20 cm from the lens, what would be the radius of the central bright spot formed ?

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (d) | 3. (d) | 4. (d) | 5. (c) | 6. (b) |
| 7. (a) | 8. (d) | 9. (d) | 10. (a) | 11. (a) | 12. (d) |
| 13. (d) | 14. (c) | 15. (b) | 16. (c) | 17. (a) | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (c) | 2. (b), (c) | 3. (b), (c), (d) |
| 4. (c), (d) | 5. (c), (d) | 6. (a), (d) |
| 7. (a) | 8. (b) | 9. (a), (b), (d) |
| 10. (a), (d) | | |

### EXERCISES

1. $774.3 \times 10^{14}$ Hz-$7.5 \times 10^{14}$ Hz

2. (a) $5.09 \times 10^{14}$ Hz (b) 443 nm
   (c) $5.09 \times 10^{14}$ Hz (d) $2.25 \times 10^8$ m/s

3. $2.04 \times 10^8$ m/s, $2.07 \times 10^8$ m/s

4. 1·25

5. (a) 0·05 mm (b) 0·50 mm

6. 400 nm

7. (a) 0·25 mm (b) 20

8. 1·47 mm

9. 0·014 degree

10. 0·72 mm

11. 7

12. $\dfrac{\lambda}{2(\mu-1)}$

13. (a) $(\mu-1)t$,                         (b) $\dfrac{\lambda}{2(\mu-1)}$

14. 14 5

15. 590 nm

16. (a) $4.9 \times 10^{-7}$ m

    (b) 0·021 cm on one side and 0·028 cm on the other
       side

17. $\dfrac{\lambda}{2}\left|\dfrac{1}{\mu_1-\mu_2}\right|$

18. (a) 25                              (b) 15

19. 0·90 mm

21. (a) zero                           (b) $\dfrac{D\lambda}{4d}$

22. 0·35 mm

23. 81 : 1

24. $\lambda(2D_1+D_2)/d$

25. (a) 400 nm, 667 nm,        (b) 500 nm

26. (a) $\sqrt{\dfrac{\lambda D}{2}}$        (b) $d$        (c) $2d$

27. $7\lambda/12$

29. 0·05 W/m²

30. (a) $\dfrac{D\lambda}{4d}$                (b) $\dfrac{D\lambda}{3d}$

31. $1.25 \times 10^{-7}$ m

32. $\dfrac{D\lambda}{2d}$

33. (a) 1                (b) $\infty$          (c) 34

34. (a) zero            (b) $I$              (c) $2I$

35. 1·32

36. 100 nm

37. 443 nm, 532 nm and 666 nm

38. 455 nm, 556 nm, 714 nm

39. 2·5 cm

40. 1·37 cm

41. $3.8 \times 10^{-6}$ m
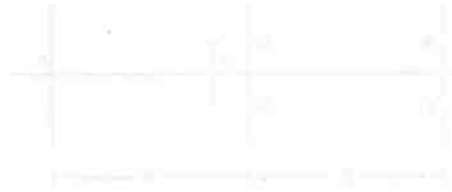
$$= \frac{1}{20 \text{ cm}} + \frac{1}{20 \text{ cm}} - \frac{60 \text{ cm}}{(20 \text{ cm})^2}$$

or,     $F = -20$ cm.

It is a divergent lens. It should be kept at a distance

$$D = \frac{dF}{f_1} \text{ behind the second lens.}$$

Here,     $D = \frac{(60 \text{ cm})(-20 \text{ cm})}{20 \text{ cm}} = -60 \text{ cm}.$

Thus, the equivalent divergent lens should be placed at a distance of 60 cm to the right of the second lens. The final image is formed at the focus of this divergent lens i.e., 20 cm to the left of it. It is, therefore, 40 cm to the right of the second lens.

□

## QUESTIONS FOR SHORT ANSWER

1. Is the formula "Real depth/Apparent depth = $\mu$" valid if viewed from a position quite away from the normal ?

2. Can you ever have a situation in which a light ray goes undeviated through a prism ?

3. Why does a diamond shine more than a glass piece cut to the same shape ?

4. A narrow beam of light passes through a slab obliquely and is then received by an eye (figure 18-Q1). The index of refraction of the material in the slab fluctuates slowly with time. How will it appear to the eye? The twinkling of stars has a similar explanation.
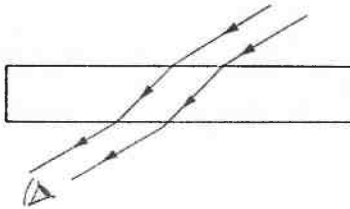


Figure 18-Q1

5. Can a plane mirror ever form a real image ?

6. If a piece of paper is placed at the position of a virtual image of a strong light source, will the paper burn after sufficient time ? What happens if the image is real? What happens if the image is real but the source is virtual ?

7. Can a virtual image be photographed by a camera ?

8. In motor vehicles, a convex mirror is attached near the driver's seat to give him the view of the traffic behind. What is the special function of this convex mirror which a plane mirror can not do ?

9. If an object far away from a convex mirror moves towards the mirror, the image also moves. Does it move faster, slower or at the same speed as compared to the object ?

10. Suppose you are inside the water in a swimming pool near an edge. A friend is standing on the edge. Do you find your friend taller or shorter than his usual height ?

11. The equation of refraction at a spherical surface is
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}.$$
Taking $R = \infty$, show that this equation leads to the equation
$$\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{\mu_1}{\mu_2}$$
for refraction at a plane surface.

12. A thin converging lens is formed with one surface convex and the other plane. Does the position of image depend on whether the convex surface or the plane surface faces the object ?

13. A single lens is mounted in a tube. A parallel beam enters the tube and emerges out of the tube as a divergent beam. Can you say with certainty that there is a diverging lens in the tube ?

14. An air bubble is formed inside water. Does it act as a converging lens or a diverging lens ?

15. Two converging lenses of unequal focal lengths can be used to reduce the aperture of a parallel beam of light without losing the energy of the light. This increases the intensity. Describe how the converging lenses should be placed to do this.

16. If a spherical mirror is dipped in water, does its focal length change ?

17. If a thin lens is dipped in water, does its focal length change ?

18. Can mirrors give rise to chromatic aberration ?

19. A laser light is focused by a converging lens. Will there be a significant chromatic aberration ?

## OBJECTIVE I

1. A point source of light is placed in front of a plane mirror.
   (a) All the reflected rays meet at a point when produced backward.

(b) Only the reflected rays close to the normal meet at a point when produced backward.
(c) Only the reflected rays making a small angle with

the mirror, meet at a point when produced backward.
(d) Light of different colours make different images.

2. Total internal reflection can take place only if
(a) light goes from optically rarer medium (smaller refractive index) to optically denser medium
(b) light goes from optically denser medium to rarer medium
(c) the refractive indices of the two media are close to each other
(d) the refractive indices of the two media are widely different.

3. In image formation from spherical mirrors, only paraxial rays are considered because they
(a) are easy to handle geometrically
(b) contain most of the intensity of the incident light
(c) form nearly a point image of a point source
(d) show minimum dispersion effect.

4. A point object is placed at a distance of 30 cm from a convex mirror of focal length 30 cm. The image will form at
(a) infinity
(b) pole
(c) focus
(d) 15 cm behind the mirror.

5. Figure (18-Q2) shows two rays $A$ and $B$ being reflected by a mirror and going as $A'$ and $B'$. The mirror
(a) is plane
(b) is convex
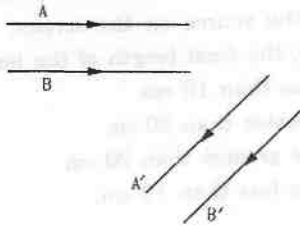(c) is concave
(d) may be any spherical mirror.



Figure 18-Q2

6. The image formed by a concave mirror
(a) is always real
(b) is always virtual
(c) is certainly real if the object is virtual
(d) is certainly virtual if the object is real.

7. Figure (18-Q3) shows three transparent media of refractive indices $\mu_1$, $\mu_2$ and $\mu_3$. A point object $O$ is placed in the medium $\mu_2$. If the entire medium on the right of the spherical surface has refractive index $\mu_1$, the image forms at $O'$. If this entire medium has refractive index $\mu_3$, the image forms at $O''$. In the situation shown,
(a) the image forms between $O'$ and $O''$
(b) the image forms to the left of $O'$
(c) the image forms to the right of $O''$
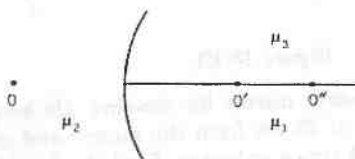(d) two images form, one at $O'$ and the other at $O''$.



Figure 18-Q3

8. Four modifications are suggested in the lens formula to include the effect of the thickness $t$ of the lens. Which one is likely to be correct?
(a) $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{t}{uf}$
(b) $\dfrac{t}{v} - \dfrac{1}{u} = \dfrac{1}{f}$
(c) $\dfrac{1}{v-t} - \dfrac{1}{u+t} = \dfrac{1}{f}$
(d) $\dfrac{1}{v} - \dfrac{1}{u} + \dfrac{t}{uv} = \dfrac{t}{f}$.

9. A double convex lens has two surfaces of equal radii $R$ and refractive index $\mu = 1.5$. We have,
(a) $f = R/2$
(b) $f = R$
(c) $f = -R$
(d) $f = 2R$.

10. A point source of light is placed at a distance of $2f$ from a converging lens of focal length $f$. The intensity on the other side of the lens is maximum at a distance
(a) $f$
(b) between $f$ and $2f$
(c) $2f$
(d) more than $2f$.

11. A parallel beam of light is incident on a converging lens parallel to its principal axis. As one moves away from the lens on the other side on its principal axis, the intensity of light
(a) remains constant
(b) continuously increases
(c) continuously decreases
(d) first increases then decreases.

12. A symmetric double convex lens is cut in two equal parts by a plane perpendicular to the principal axis. If the power of the original lens was 4 D, the power of a cut-lens will be
(a) 2 D
(b) 3 D
(c) 4 D
(d) 5 D.

13. A symmetric double convex lens is cut in two equal parts by a plane containing the principal axis. If the power of the original lens was 4 D, the power of a divided lens will be
(a) 2 D
(b) 3 D
(c) 4 D
(d) 5 D.

14. Two concave lenses $L_1$ and $L_2$ are kept in contact with each other. If the space between the two lenses is filled with a material of refractive index $\mu \sim 1$, the magnitude of the focal length of the combination
(a) becomes undefined
(b) remains unchanged
(c) increases
(d) decreases.

15. A thin lens is made with a material having refractive index $\mu = 1.5$. Both the sides are convex. It is dipped in water ($\mu = 1.33$). It will behave like
(a) a convergent lens
(b) a divergent lens
(c) a rectangular slab
(d) a prism.

16. A convex lens is made of a material having refractive index $1.2$. Both the surfaces of the lens are convex. If it is dipped into water ($\mu = 1.33$), it will behave like
(a) a convergent lens
(b) a divergent lens
(c) a rectangular slab
(d) a prism.

17. A point object $O$ is placed on the principal axis of a convex lens of focal length $f = 20$ cm at a distance of 40 cm to the left of it. The diameter of the lens is 10 cm. An eye is placed 60 cm to right of the lens and a distance $h$ below the principal axis. The maximum value of $h$ to see the image is
(a) 0
(b) 2.5 cm
(c) 5 cm
(d) 10 cm.

18. The rays of different colours fail to converge at a point after going through a converging lens. This defect is called
(a) spherical aberration
(b) distortion
(c) coma
(d) chromatic aberration.

## OBJECTIVE II

1. If the light moving in a straight line bends by a small but fixed angle, it may be a case of
   (a) reflection               (b) refraction
   (c) diffraction             (d) dispersion.

2. Mark the correct options.
   (a) If the incident rays are converging, we have a real object.
   (b) If the final rays are converging, we have a real image.
   (c) The image of a virtual object is called a virtual image.
   (d) If the image is virtual, the corresponding object is called a virtual object.

3. Which of the following (referred to a spherical mirror) do (does) not depend on whether the rays are paraxial or not ?
   (a) Pole.                 (b) Focus.
   (c) Radius of curvature.     (d) Principal axis.

4. The image of an extended object, placed perpendicular to the principal axis of a mirror, will be erect if
   (a) the object and the image are both real
   (b) the object and the image are both virtual
   (c) the object is real but the image is virtual
   (d) the object is virtual but the image is real.

5. A convex lens forms a real image of a point object placed on its principal axis. If the upper half of the lens is painted black,
   (a) the image will be shifted downward
   (b) the image will be shifted upward
   (c) the image will not be shifted
   (d) the intensity of the image will decrease.

6. Consider three converging lenses $L_1$, $L_2$ and $L_3$ having identical geometrical construction. The index of refraction of $L_1$ and $L_2$ are $\mu_1$ and $\mu_2$ respectively. The upper half of the lens $L_3$ has a refractive index $\mu_1$ and the lower half has $\mu_2$ (figure 18-Q4). A point object $O$ is imaged at $O_1$ by the lens $L_1$ and at $O_2$ by the lens $L_2$ placed in same position. If $L_3$ is placed at the same place,
   (a) there will be an image at $O_1$
   (b) there will be an image at $O_2$.
   (c) the only image will form somewhere between $O_1$ and $O_2$
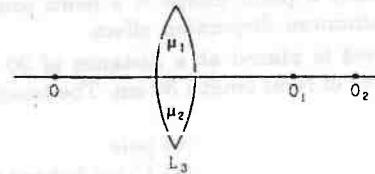   (d) the only image will form away from $O_2$.



Figure 18-Q4

7. A screen is placed a distance 40 cm away from an illuminated object. A converging lens is placed between the source and the screen and it is attempted to form the image of the source on the screen. If no position could be found, the focal length of the lens
   (a) must be less than 10 cm
   (b) must be greater than 20 cm
   (c) must not be greater than 20 cm
   (d) must not be less than 10 cm.

## EXERCISES

1. A concave mirror having a radius of curvature 40 cm is placed in front of an illuminated point source at a distance of 30 cm from it. Find the location of the image.

2. A concave mirror forms an image of 20 cm high object on a screen placed 5·0 m away from the mirror. The height of the image is 50 cm. Find the focal length of the mirror and the distance between the mirror and the object.

3. A concave mirror has a focal length of 20 cm. Find the position or positions of an object for which the image-size is double of the object-size.

4. A 1 cm object is placed perpendicular to the principal axis of a convex mirror of focal length 7·5 cm. Find its distance from the mirror if the image formed is 0·6 cm in size.

5. A candle flame 1·6 cm high is imaged in a ball bearing of diameter 0·4 cm. If the ball bearing is 20 cm away from the flame, find the location and the height of the image.

6. A 3 cm tall object is placed at a distance of 7·5 cm from a convex mirror of focal length 6 cm. Find the location, size and nature of the image.

7. A U-shaped wire is placed before a concave mirror having radius of curvature 20 cm as shown in figure (18-E1). Find the total length of the image.



Figure 18-E1

8. A man uses a concave mirror for shaving. He keeps his face at a distance of 25 cm from the mirror and gets an image which is 1·4 times enlarged. Find the focal length of the mirror.

9. Find the diameter of the image of the moon formed by a spherical concave mirror of focal length 7·6 m. The diameter of the moon is 3450 km and the distance between the earth and the moon is $3·8 \times 10^{5}$ km.

10. A particle goes in a circle of radius 2·0 cm. A concave mirror of focal length 20 cm is placed with its principal axis passing through the centre of the circle and perpendicular to its plane. The distance between the pole of the mirror and the centre of the circle is 30 cm. Calculate the radius of the circle formed by the image.

11. A concave mirror of radius $R$ is kept on a horizontal table (figure 18-E2). Water (refractive index = $\mu$) is poured into it upto a height $h$. Where should an object be placed so that its image is formed on itself ?
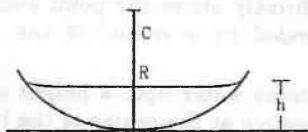


Figure 18-E2

12. A point source $S$ is placed midway between two converging mirrors having equal focal length $f$ as shown in figure (18-E3). Find the values of $d$ for which only one image is formed.
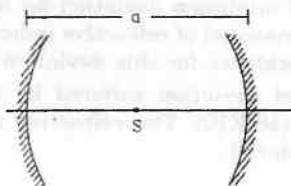


Figure 18-E3

13. A converging mirror $M_1$, a point source $S$ and a diverging mirror $M_2$ are arranged as shown in figure (18-E4). The source is placed at a distance of 30 cm from $M_1$. The focal length of each of the mirrors is 20 cm. Consider only the images formed by a maximum of two reflections. It is found that one image is formed on the source itself. (a) Find the distance between the two mirrors. (b) Find the location of the image formed by the single reflection from $M_2$.
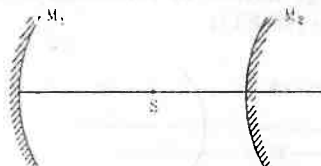


Figure 18-E4

14. A light ray falling at an angle of 45° with the surface of a clean slab of ice of thickness 1·00 m is refracted into it at an angle of 30°. Calculate the time taken by the light rays to cross the slab. Speed of light in vacuum = $3 \times 10^{8}$ m/s.

15. A pole of length 1·00 m stands half dipped in a swimming pool with water level 50·0 cm higher than the bed. The refractive index of water is 1·33 and sunlight is coming at an angle of 45° with the vertical. Find the length of the shadow of the pole on the bed.

16. A small piece of wood is floating on the surface of a 2·5 m deep lake. Where does the shadow form on the bottom when the sun is just setting? Refractive index of water = 4/3.

17. An object $P$ is focused by a microscope $M$. A glass slab of thickness 2·1 cm is introduced between $P$ and $M$. If the refractive index of the slab is 1·5, by what distance should the microscope be shifted to focus the object again ?

18. A vessel contains water upto a height of 20 cm and above it an oil upto another 20 cm. The refractive indices of the water and the oil are 1·33 and 1·30 respectively. Find the apparent depth of the vessel when viewed from above.

19. Locate the image of the point $P$ as seen by the eye in the figure (18-E5).
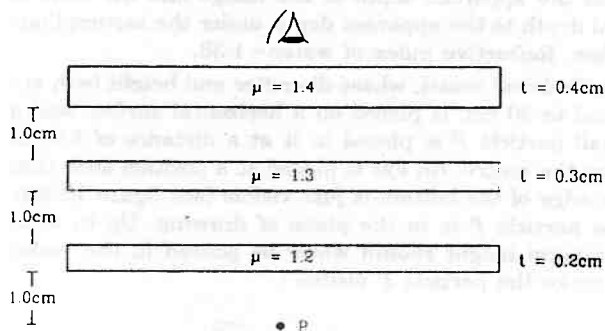


Figure 18-E5

20. $k$ transparent slabs are arranged one over another. The refractive indices of the slabs are $\mu_1, \mu_2, \mu_3, \ldots \mu_k$ and the thicknesses are $t_1, t_2, t_3, \ldots t_k$. An object is seen through this combination with nearly perpendicular light. Find the equivalent refractive index of the system which will allow the image to be formed at the same place.

21. A cylindrical vessel of diameter 12 cm contains $800\pi$ cm$^3$ of water. A cylindrical glass piece of diameter 8·0 cm and height 8·0 cm is placed in the vessel. If the bottom of the vessel under the glass piece is seen by the paraxial rays (see figure 18-E6), locate its image. The index of refraction of glass is 1·50 and that of water is 1·33.
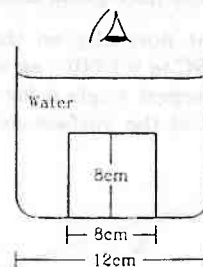


Figure 18-E6

22. Consider the situation in figure (18-E7). The bottom of the pot is a reflecting plane mirror, $S$ is a small fish and $T$ is a human eye. Refractive index of water is $\mu$. (a) At what distance(s) from itself will the fish see the image(s) of the eye ? (b) At what distance(s) from itself will the eye see the image(s) of the fish.
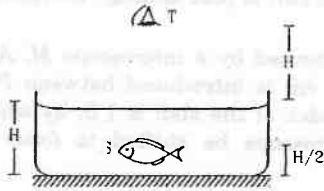


Figure 18-E7

23. A small object is placed at the centre of the bottom of a cylindrical vessel of radius 3 cm and height 4 cm filled completely with water. Consider the ray leaving the vessel through a corner. Suppose this ray and the ray along the axis of the vessel are used to trace the image. Find the apparent depth of the image and the ratio of real depth to the apparent depth under the assumptions taken. Refractive index of water = 1·33.

24. A cylindrical vessel, whose diameter and height both are equal to 30 cm, is placed on a horizontal surface and a small particle $P$ is placed in it at a distance of 5·0 cm from the centre. An eye is placed at a position such that the edge of the bottom is just visible (see figure 18-E8). The particle $P$ is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle $P$ visible ?
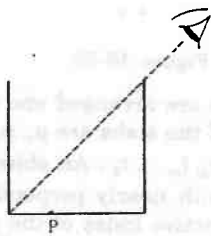


Figure 18-E8

25. A light ray is incident at an angle of 45° with the normal to a $\sqrt{2}$ cm thick plate ($\mu$ = 2·0). Find the shift in the path of the light as it emerges out from the plate.

26. An optical fibre ($\mu$ = 1·72) is surrounded by a glass coating ($\mu$ = 1·50). Find the critical angle for total internal reflection at the fibre-glass interface.

27. A light ray is incident normally on the face $AB$ of a right-angled prism $ABC$ ($\mu$ = 1·50) as shown in figure (18-E9). What is the largest angle $\phi$ for which the light ray is totally reflected at the surface $AC$ ?
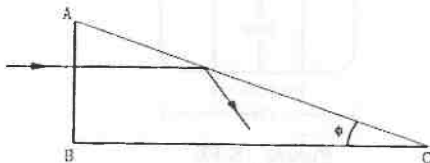


Figure 18-E9

28. Find the maximum angle of refraction when a light ray is refracted from glass ($\mu$ = 1·50) to air.

29. Light is incident from glass ($\mu$ = 1·5) to air. Sketch the variation of the angle of deviation $\delta$ with the angle of incident $i$ for $0 < i < 90°$.

30. Light is incident from glass ($\mu$ = 1·50) to water ($\mu$ = 1·33). Find the range of the angle of deviation for which there are two angles of incidence.

31. Light falls from glass ($\mu$ = 1·5) to air. Find the angle of incidence for which the angle of deviation is 90°.

32. A point source is placed at a depth $h$ below the surface of water (refractive index = $\mu$). (a) Show that light escapes through a circular area on the water surface with its centre directly above the point source. (b) Find the angle subtended by a radius of the area on the source.

33. A container contains water upto a height of 20 cm and there is a point source at the centre of the bottom of the container. A rubber ring of radius $r$ floats centrally on the water. The ceiling of the room is 2·0 m above the water surface. (a) Find the radius of the shadow of the ring formed on the ceiling if $r$ = 15 cm. (b) Find the maximum value of $r$ for which the shadow of the ring is formed on the ceiling. Refractive index of water = 4/3.

34. Find the angle of minimum deviation for an equilateral prism made of a material of refractive index 1·732. What is the angle of incidence for this deviation ?

35. Find the angle of deviation suffered by the light ray shown in figure (18-E10). The refractive index $\mu$ = 1·5 for the prism material.
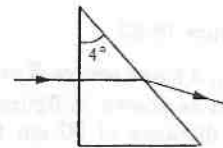


Figure 18-E10

36. A light ray, going through a prism with the angle of prism 60°, is found to deviate by 30°. What limit on the refractive index can be put from these data ?

37. Locate the image formed by refraction in the situation shown in figure (18-E11).
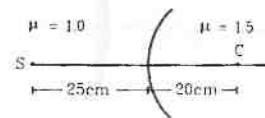


Figure 18-E11

38. A spherical surface of radius 30 cm separates two transparent media $A$ and $B$ with refractive indices 1·33 and 1·48 respectively. The medium $A$ is on the convex side of the surface. Where should a point object be placed in medium $A$ so that the paraxial rays become parallel after refraction at the surface ?

39. Figure (18-E12) shows a transparent hemisphere of radius 3.0 cm made of a material of refractive index 2.0. (a) A narrow beam of parallel rays is incident on the hemisphere as shown in the figure. Are the rays totally reflected at the plane surface ? (b) Find the image formed by the refraction at the first surface. (c) Find the image formed by the reflection or by the refraction at the plane surface. (d) Trace qualitatively the final rays as they come out of the hemisphere.
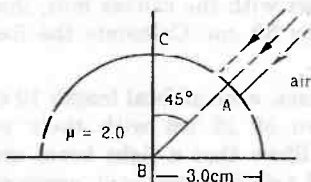


Figure 18-E12

40. A small object is embedded in a glass sphere ($\mu = 1.5$) of radius 5.0 cm at a distance 1.5 cm left to the centre. Locate the image of the object as seen by an observer standing (a) to the left of the sphere and (b) to the right of the sphere.

41. A biconvex thick lens is constructed with glass ($\mu = 1.50$). Each of the surfaces has a radius of 10 cm and the thickness at the middle is 5 cm. Locate the image of an object placed far away from the lens.

42. A narrow pencil of parallel light is incident normally on a solid transparent sphere of radius $r$. What should be the refractive index if the pencil is to be focused (a) at the surface of the sphere, (b) at the centre of the sphere.

43. One end of a cylindrical glass rod ($\mu = 1.5$) of radius 1.0 cm is rounded in the shape of a hemisphere. The rod is immersed in water ($\mu = 4/3$) and an object is placed in the water along the axis of the rod at a distance of 8.0 cm from the rounded edge. Locate the image of the object.

44. A paperweight in the form of a hemisphere of radius 3.0 cm is used to hold down a printed page. An observer looks at the page vertically through the paperweight. At what height above the page will the printed letters near the centre appear to the observer ?

45. Solve the previous problem if the paperweight is inverted at its place so that the spherical surface touches the paper.

46. A hemispherical portion of the surface of a solid glass sphere ($\mu = 1.5$) of radius $r$ is silvered to make the inner side reflecting. An object is placed on the axis of the hemisphere at a distance $3r$ from the centre of the sphere. The light from the object is refracted at the unsilvered part, then reflected from the silvered part and again refracted at the unsilvered part. Locate the final image formed.

47. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm.

The convex side is silvered and placed on a horizontal surface as shown in figure (18-E13). (a) Where should a pin be placed on the axis so that its image is formed at the same place ? (b) If the concave part is filled with water ($\mu = 4/3$), find the distance through which the pin should be moved so that the image of the pin again coincides with the pin.
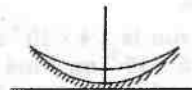


Figure 18-E13

48. A double convex lens has focal length 25 cm. The radius of curvature of one of the surfaces is double of the other. Find the radii, if the refractive index of the material of the lens is 1.5.

49. The radii of curvature of a lens are + 20 cm and + 30 cm. The material of the lens has a refracting index 1.6. Find the focal length of the lens (a) if it is placed in air, and (b) if it is placed in water ($\mu = 1.33$).

50. Lenses are constructed by a material of refractive index 1.50. The magnitude of the radii of curvature are 20 cm and 30 cm. Find the focal lengths of the possible lenses with the above specifications.

51. A thin lens made of a material of refractive index $\mu_2$ has a medium of refractive index $\mu_1$ on one side and a medium of refractive index $\mu_3$ on the other side. The lens is biconvex and the two radii of curvature have equal magnitude $R$. A beam of light travelling parallel to the principal axis is incident on the lens. Where will the image be formed if the beam is incident from (a) the medium $\mu_1$ and (b) from the medium $\mu_3$ ?

52. A convex lens has a focal length of 10 cm. Find the location and nature of the image if a point object is placed on the principal axis at a distance of (a) 9.8 cm, (b) 10.2 cm from the lens.

53. A slide projector has to project a 35 mm slide (35 mm × 23 mm) on a 2 m × 2 m screen at a distance of 10 m from the lens. What should be the focal length of the lens in the projector ?

54. A particle executes a simple harmonic motion of amplitude 1.0 cm along the principal axis of a convex lens of focal length 12 cm. The mean position of oscillation is at 20 cm from the lens. Find the amplitude of oscillation of the image of the particle.

55. An extended object is placed at a distance of 5.0 cm from a convex lens of focal length 8.0 cm. (a) Draw the ray diagram (to the scale) to locate the image and from this, measure the distance of the image from the lens. (b) Find the position of the image from the lens formula and see how close the drawing is to the correct result.

56. A pin of length 2.00 cm is placed perpendicular to the principal axis of a converging lens. An inverted image of size 1.00 cm is formed at a distance of 40.0 cm from the pin. Find the focal length of the lens and its distance from the pin.

**57.** A convex lens produces a double size real image when an object is placed at a distance of 18 cm from it. Where should the object be placed to produce a triple size real image ?

**58.** A pin of length 2.0 cm lies along the principal axis of a converging lens, the centre being at a distance of 11 cm from the lens. The focal length of the lens is 6 cm. Find the size of the image.

**59.** The diameter of the sun is $1.4 \times 10^9$ m and its distance from the earth is $1.5 \times 10^{11}$ m. Find the radius of the image of the sun formed by a lens of focal length 20 cm.

**60.** A 5.0 diopter lens forms a virtual image which is 4 times the object placed perpendicularly on the principal axis of the lens. Find the distance of the object from the lens.

**61.** A diverging lens of focal length 20 cm and a converging mirror of focal length 10 cm are placed coaxially at a separation of 5 cm. Where should an object be placed so that a real image is formed at the object itself ?

**62.** A converging lens of focal length 12 cm and a diverging mirror of focal length 7.5 cm are placed 5.0 cm apart with their principal axes coinciding. Where should an object be placed so that its image falls on itself ?

**63.** A converging lens and a diverging mirror are placed at a separation of 15 cm. The focal length of the lens is 25 cm and that of the mirror is 40 cm. Where should a point source be placed between the lens and the mirror so that the light, after getting reflected by the mirror and then getting transmitted by the lens, comes out parallel to the principal axis ?

**64.** A converging lens of focal length 15 cm and a converging mirror of focal length 10 cm are placed 50 cm apart with common principal axis. A point source is placed in between the lens and the mirror at a distance of 40 cm from the lens. Find the locations of the two images formed.

**65.** Consider the situation described in the previous problem. Where should a point source be placed on the principal axis so that the two images form at the same place ?

**66.** A converging lens of focal length 15 cm and a converging mirror of focal length 10 cm are placed 50 cm apart. If a pin of length 2.0 cm is placed 30 cm from the lens farther away from the mirror, where will the final image form and what will be the size of the final image ?

**67.** A point object is placed on the principal axis of a convex lens ($f = 15$ cm) at a distance of 30 cm from it. A glass plate ($u = 1.50$) of thickness 1 cm is placed on the other side of the lens perpendicular to the axis. Locate the image of the point object.

**68.** A convex lens of focal length 20 cm and a concave lens of focal length 10 cm are placed 10 cm apart with their principal axes coinciding. A beam of light travelling parallel to the principal axis and having a beam diameter 5.0 mm, is incident on the combination. Show that the emergent beam is parallel to the incident one. Find the beam diameter of the emergent beam.

**69.** A diverging lens of focal length 20 cm and a converging lens of focal length 30 cm are placed 15 cm apart with their principal axes coinciding. Where should an object

be placed on the principal axis so that its image is formed at infinity ?

**70.** A 5 mm high pin is placed at a distance of 15 cm from a convex lens of focal length 10 cm. A second lens of focal length 5 cm is placed 40 cm from the first lens and 55 cm from the pin. Find (a) the position of the final image, (b) its nature and (c) its size.

**71.** A point object is placed at a distance of 15 cm from a convex lens. The image is formed on the other side at a distance of 30 cm from the lens. When a concave lens is placed in contact with the convex lens, the image shifts away further by 30 cm. Calculate the focal lengths of the two lenses.

**72.** Two convex lenses, each of focal length 10 cm, are placed at a separation of 15 cm with their principal axes coinciding. (a) Show that a light beam coming parallel to the principal axis diverges as it comes out of the lens system. (b) Find the location of the virtual image formed by the lens system of an object placed far away. (c) Find the focal length of the equivalent lens. (Note that the sign of the focal length is positive although the lens system actually diverges a parallel beam incident on it).

**73.** A ball is kept at a height $h$ above the surface of a heavy transparent sphere made of a material of refractive index $\mu$. The radius of the sphere is $R$. At $t = 0$, the ball is dropped to fall normally on the sphere. Find the speed of the image formed as a function of time for $t < \sqrt{\dfrac{2h}{g}}$. Consider only the image by a single refraction.

**74.** A particle is moving at a constant speed $V$ from a large distance towards a concave mirror of radius $R$ along its principal axis. Find the speed of the image formed by the mirror as a function of the distance $x$ of the particle from the mirror.

**75.** A small block of mass $m$ and a concave mirror of radius $R$ fitted with a stand lie on a smooth horizontal table with a separation $d$ between them. The mirror together with its stand has a mass $m$. The block is pushed at $t = 0$ towards the mirror so that it starts moving towards the mirror at a constant speed $V$ and collides with it. The collision is perfectly elastic. Find the velocity of the image (a) at a time $t < d/V$, (b) at a time $t > d/V$.

**76.** A gun of mass $M$ fires a bullet of mass $m$ with a horizontal speed $V$. The gun is fitted with a concave mirror of focal length $f$ facing towards the receding bullet. Find the speed of separation of the bullet and the image just after the gun was fired.

**77.** A mass $m = 50$ g is dropped on a vertical spring of spring constant 500 N/m from a height $h = 10$ cm as shown in figure (18-E14). The mass sticks to the spring and
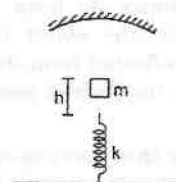


Figure 18-E14

executes simple harmonic oscillations after that. A concave mirror of focal length 12 cm facing the mass is fixed with its principal axis coinciding with the line of motion of the mass, its pole being at a distance of 30 cm from the free end of the spring. Find the length in which the image of the mass oscillates.

78. Two concave mirrors of equal radii of curvature $R$ are fixed on a stand facing opposite directions. The whole system has a mass $m$ and is kept on a frictionless horizontal table (figure 18-E15).
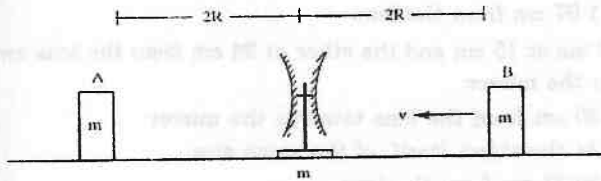


Figure 18-E15

Two blocks $A$ and $B$, each of mass $m$, are placed on the two sides of the stand. At $t = 0$, the separation between $A$ and the mirrors is $2R$ and also the separation between $B$ and the mirrors is $2R$. The block $B$ moves towards the mirror at a speed $v$. All collisions which take place are elastic. Taking the original position of the

mirrors-stand system to be $x = 0$ and $X$-axis along $AB$. find the position of the images of $A$ and $B$ at $t =$

(a) $\dfrac{R}{v}$     (b) $\dfrac{3R}{v}$     (c) $\dfrac{5R}{v}$ .

79. Consider the situation shown in figure (18-E16). The elevator is going up with an acceleration of $2.00$ m/s² and the focal length of the mirror is $12.0$ cm. All the surfaces are smooth and the pulley is light. The mass-pulley system is released from rest (with respect to the elevator) at $t = 0$ when the distance of $B$ from the mirror is $42.0$ cm. Find the distance between the image of the block $B$ and the mirror at $t = 0.200$ s. Take $g = 10$ m/s².
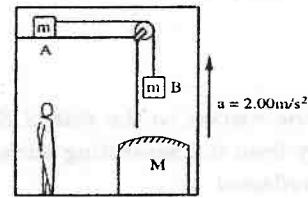


Figure 18-E16

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (a) | 2. (b) | 3. (c) | 4. (d) | 5. (a) | 6. (c) |
| 7. (d) | 8. (c) | 9. (b) | 10. (c) | 11. (d) | 12. (a) |
| 13. (c) | 14. (c) | 15. (a) | 16. (b) | 17. (b) | 18. (d) |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (b) | 2. (b) | 3. (a). (c), (d) |
| 4. (c), (d) | 5. (c), (d) | 6. (a), (b) |
| 7. (b) | | |

### EXERCISES

1. 60 cm from the mirror on the side of the object
2. 1·43 m, 2·0 m
3. 10 cm or 30 cm from the mirror
4. 5 cm
5. 1·0 mm inside the ball bearing, 0·08 mm
6. $\dfrac{10}{3}$ cm from the mirror on the side opposite to the object, 1·33 cm, virtual and erect
7. 10 cm
8. 87·5 cm
9. 6·9 cm

10. 4·0 cm

11. $\dfrac{(R-h)}{\mu}$ above the water surface

12. $2f$, $4f$

13. (a) 50 cm (b) 10 cm from the diverging mirror farther from the converging mirror

14. 5·44 ns

15. 81·5 cm

16. 2·83 m shifted from the position directly below the piece of the wood.

17. 0·70 cm

18. 30·4 cm

19. 0·2 cm above $P$

20. $\dfrac{\sum\limits_{i=1}^{k} t_i}{\sum\limits_{i=1}^{k} (t_i/\mu_i)}$

21. 7·1 cm above the bottom

22. (a) $H\left(\mu + \dfrac{1}{2}\right)$ above itself, $H\left(\mu + \dfrac{3}{2}\right)$ below itself

(b) $H\left(1 + \dfrac{1}{2\mu}\right)$ below itself and $H\left(1 + \dfrac{3}{2\mu}\right)$ below itself

23. 2·25 cm, 1·78
24. 26·7 cm
25. 0·62 cm
26. $\sin^{-1}\dfrac{75}{86}$
27. $\cos^{-1}(2/3)$
28. 90°
30. 0 to $\cos^{-1}(8/9)$
31. 45°
32. (b) $\sin^{-1}(1/\mu)$
33. (a) 2·8 m  (b) 22·6 cm
34. 60°, 60°
35. 2°
36. $\mu < \sqrt{2}$
37. 100 cm from the surface on the side of $S$
38. 266·0 cm away from the separating surface
39. (a) They are reflected

(b) If the sphere is completed, the image forms at the point diametrically opposite to $A$

(c) At the mirror image of $A$ in $BC$

40. (a) 2 cm left to the centre
    (b) 2·65 cm left to the centre
41. 9·1 cm from the farther surface on the other side of the lens
42. (a) 2,

(b) not possible, it will focus close to the centre if the refractive index is large.

43. At infinity
44. No shift is observed
45. 1 cm
46. At the reflecting surface of the sphere
47. (a) 15 cm from the lens on the axis
    (b) 1·14 cm towards the lens
48. 18·75 cm, 37·5 cm
49. (a) 100 cm  (b) 300 cm
50. ± 24 cm, + 120 cm
51. (a) $\dfrac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$ (b) $\dfrac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$
52. (a) 490 cm on the side of the object, virtual
    (b) 510 cm on the other side, real

53. 17·2 cm
54. 2·3 cm
56. 8·89 cm, 26·7 cm
57. 16 cm
58. 3 cm
59. 0·93 mm
60. 15 cm
61. 60 cm from the lens further away from the mirror
62. 30 cm from the lens further away from the mirror
63. 1·67 cm from the lens
64. One at 15 cm and the other at 24 cm from the lens away from the mirror
65. 30 cm from the lens towards the mirror
66. At the object itself, of the same size
67. 30·33 cm from the lens
68. 1·0 cm if the light is incident from the side of concave lens and 2·5 mm if it is incident from the side of the convex lens
69. 60 cm from the diverging lens or 210 cm from the converging lens
70. (a) 10 cm from the second lens further away,
    (b) erect and real, (c) 10 mm
71. 10 cm for convex lens and 60 cm for concave lens
72. (b) 5 cm from the first lens towards the second lens
    (c) 20 cm
73. $\dfrac{\mu R\, gt}{\left[(\mu - 1)\left(h - \frac{1}{2}gt^2\right) - R\right]^2}$
74. $\dfrac{R^2 V}{(2\,x - R)^2}$
75. (a) $-\dfrac{R^2 V}{[2(d - Vt) - R]^2}$

(b) $V\left[1 - \dfrac{R^2}{[2(Vt - d) - R]^2}\right]$

76. $2(1 + m/M)V$
77. 1·2 cm
78. (a) $x = -\dfrac{2R}{3}, R$ (b) $x = -2R, 0$ (c) $x = -3R, -\dfrac{4R}{3}$
79. 8·57 cm

**Solution** : If an object is placed at 25 cm from the correcting lens, it should produce the virtual image at 40 cm. Thus, $u = -25$ cm, $v = -40$ cm.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= -\frac{1}{40 \text{ cm}} + \frac{1}{25 \text{ cm}}$$

or,        $f = \frac{200}{3}$ cm $= +\frac{2}{3}$ m

or,        $P = \frac{1}{f} = +1.5$ D.

The unaided eye can see a maximum distance of 250 cm. Suppose the maximum distance for clear vision is $d$ when the lens is used. Then the object at a distance $d$ is imaged by the lens at 250 cm. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

or,        $-\frac{1}{250 \text{ cm}} - \frac{1}{d} = \frac{3}{200 \text{ cm}}$

or,        $d = -53$ cm.

Thus, the person will be able to see upto a maximum distance of 53 cm.

□

10. *A young boy can adjust the power of his eye-lens between 50 D and 60 D. His far point is infinity. (a) What is the distance of his retina from the eye-lens ? (b) What is his near point ?*

**Solution** :

(a) When the eye is fully relaxed, its focal length is largest and the power of the eye-lens is minimum. This power is 50 D according to the given data. The focal length is $\frac{1}{50}$ m = 2 cm. As the far point is at infinity, the parallel rays coming from infinity are focused on the retina in the fully relaxed condition. Hence, the distance of the retina from the lens equals the focal length which is 2 cm.

(b) When the eye is focused at the near point, the power is maximum which is 60 D. The focal length in this case is $f = \frac{1}{60}$ m $= \frac{5}{3}$ cm. The image is formed on the retina and thus $v = 2$ cm. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or,        $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{2 \text{ cm}} - \frac{3}{5 \text{ cm}}$

or,        $u = -10$ cm.

The near point is at 10 cm.

# QUESTIONS FOR SHORT ANSWER

1. Can virtual image be formed on the retina in a seeing process ?

2. Can the image formed by a simple microscope be projected on a screen without using any additional lens or mirror ?

3. The angular magnification of a system is less than one. Does it mean that the image formed is inverted ?

4. A simple microscope using a single lens often shows coloured image of a white source. Why ?

5. A magnifying glass is a converging lens placed close to the eye. A farsighted person uses spectacles having converging lenses. Compare the functions of a converging lens used as a magnifying glass and as spectacles.

6. A person is viewing an extended object. If a converging lens is placed in front of his eyes, will he feel that the size has increased ?

7. The magnifying power of a converging lens used as a simple microscope is $\left(1 + \frac{D}{f}\right)$. A compound microscope is a combination of two such converging lenses. Why don't we have magnifying power $\left(1 + \frac{D}{f_o}\right)\left(1 + \frac{D}{f_e}\right)$ ? In other words, why can the objective not be treated as a simple microscope but the eyepiece can ?

8. By mistake, an eye surgeon puts a concave lens in place of the lens in the eye after a cataract operation. Will the patient be able to see clearly any object placed at any distance ?

9. The magnifying power of a simple microscope is given by $1 + \frac{D}{f}$, where $D$ is the least distance for clear vision. For farsighted persons, $D$ is greater than the usual. Does it mean that the magnifying power of a simple microscope is greater for a farsighted person as compared to a normal person ? Does it mean that a farsighted person can see an insect more clearly under a microscope than a normal person ?

10. Why are the magnification properties of microscopes and telescopes defined in terms of the ratio of angles and not in terms of the ratio of sizes of objects and images ?

11. An object is placed at a distance of 30 cm from a converging lens of focal length 15 cm. A normal eye (near point 25 cm, far point infinity) is placed close to the lens on the other side. (a) Can the eye see the object clearly ? (b) What should be the minimum separation between the lens and the eye so that the eye can clearly see the

object ? (c) Can a diverging lens, placed in contact with the converging lens, help in seeing the object clearly when the eye is close to the lens ?

12. A compound microscope forms an inverted image of an object. In which of the following cases it is likely to

create difficulties ? (a) Looking at small germs. (b) Looking at circular spots. (c) Looking at a vertical tube containing some water.

## OBJECTIVE I

1. The size of an object as perceived by an eye depends primarily on
   (a) actual size of the object
   (b) distance of the object from the eye
   (c) aperture of the pupil
   (d) size of the image formed on the retina.

2. The muscles of a normal eye are least strained when the eye is focused on an object
   (a) far away from the eye
   (b) very close to the eye
   (c) at about 25 cm from the eye
   (d) at about 1 m from the eye.

3. A normal eye is not able to see objects closer than 25 cm because
   (a) the focal length of the eye is 25 cm
   (b) the distance of the retina from the eye-lens is 25 cm
   (c) the eye is not able to decrease the distance between the eye-lens and the retina beyond a limit
   (d) the eye is not able to decrease the focal length beyond a limit.

4. When objects at different distances are seen by the eye, which of the following remain constant ?
   (a) The focal length of the eye-lens.
   (b) The object-distance from the eye-lens.
   (c) The radii of curvature of the eye-lens.
   (d) The image-distance from the eye-lens.

5. A person $A$ can clearly see objects between 25 cm and 200 cm. Which of the following may represent the range of clear vision for a person $B$ having muscles stronger

than $A$, but all other parameters of eye identical to that of $A$ ?
   (a) 25 cm to 200 cm          (b) 18 cm to 200 cm
   (c) 25 cm to 300 cm          (d) 18 cm to 300 cm.

6. The focal length of a normal eye-lens is about
   (a) 1 mm       (b) 2 cm       (c) 25 cm       (d) 1 m.

7. The distance of the eye-lens from the retina is $x$. For a normal eye, the maximum focal length of the eye-lens
   (a) $= x$       (b) $< x$       (c) $> x$       (d) $= 2x$.

8. A man wearing glasses of focal length $+1$ m cannot clearly see beyond 1 m
   (a) if he is farsighted       (b) if he is nearsighted
   (c) if his vision is normal    (d) in each of these cases.

9. An object is placed at a distance $u$ from a simple microscope of focal length $f$. The angular magnification obtained depends
   (a) on $f$ but not on $u$       (b) on $u$ but not on $f$
   (c) on $f$ as well as $u$       (d) neither on $f$ nor on $u$.

10. To increase the angular magnification of a simple microscope, one should increase
    (a) the focal length of the lens
    (b) the power of the lens
    (c) the aperture of the lens
    (d) the object size.

11. A man is looking at a small object placed at his near point. Without altering the position of his eye or the object, he puts a simple microscope of magnifying power 5 X before his eyes. The angular magnification achieved is
    (a) 5       (b) 2·5       (c) 1       (d) 0·2.

## OBJECTIVE II

1. When we see an object, the image formed on the retina is
   (a) real       (b) virtual       (c) erect       (d) inverted.

2. In which of the following the final image is erect ?
   (a) Simple microscope.       (b) Compound microscope.
   (c) Astronomical telescope.   (d) Galilean telescope.

3. The maximum focal length of the eye-lens of a person is greater than its distance from the retina. The eye is
   (a) always strained in looking at an object
   (b) strained for objects at large distances only
   (c) strained for objects at short distances only
   (d) unstrained for all distances.

4. Mark the correct options.
   (a) If the far point goes ahead, the power of the

divergent lens should be reduced.
   (b) If the near point goes ahead, the power of the convergent lens should be reduced.
   (c) If the far point is 1 m away from the eye, divergent lens should be used.
   (d) If the near point is 1 m away from the eye, divergent lens should be used.

5. The focal length of the objective of a compound microscope is $f_o$ and its distance from the eyepiece is $L$. The object is placed at a distance $u$ from the objective. For proper working of the instrument,
   (a) $L < u$       (b) $L > u$       (c) $f_o < L < 2f_o$       (d) $L > 2f_o$.

# EXERCISES

1. A person looks at different trees in an open space with the following details. Arrange the trees in decreasing order of their apparent sizes.

| Tree | Height(m) | Distance from the eye(m) |
|------|-----------|--------------------------|
| A | 2·0 | 50 |
| B | 2·5 | 80 |
| C | 1·8 | 70 |
| D | 2·8 | 100 |

2. An object is to be seen through a simple microscope of focal length 12 cm. Where should the object be placed so as to produce maximum angular magnification ? The least distance for clear vision is 25 cm.

3. A simple microscope has a magnifying power of 3·0 when the image is formed at the near point (25 cm) of a normal eye. (a) What is its focal length ? (b) What will be its magnifying power if the image is formed at infinity ?

4. A child has near point at 10 cm. What is the maximum angular magnification the child can have with a convex lens of focal length 10 cm ?

5. A simple microscope is rated 5 X for a normal relaxed eye. What will be its magnifying power for a relaxed farsighted eye whose near point is 40 cm ?

6. Find the maximum magnifying power of a compound microscope having a 25 diopter lens as the objective, a 5 diopter lens as the eyepiece and the separation 30 cm between the two lenses. The least distance for clear vision is 25 cm.

7. The separation between the objective and the eyepiece of a compound microscope can be adjusted between 9·8 cm to 11·8 cm. If the focal lengths of the objective and the eyepiece are 1·0 cm and 6 cm respectively, find the range of the magnifying power if the image is always needed at 24 cm from the eye.

8. An eye can distinguish between two points of an object if they are separated by more than 0·22 mm when the object is placed at 25 cm from the eye. The object is now seen by a compound microscope having a 20 D objective and 10 D eyepiece separated by a distance of 20 cm. The final image is formed at 25 cm from the eye. What is the minimum separation between two points of the object which can now be distinguished ?

9. A compound microscope has a magnifying power of 100 when the image is formed at infinity. The objective has a focal length of 0·5 cm and the tube length is 6·5 cm. Find the focal length of the eyepiece.

10. A compound microscope consists of an objective of focal length 1 cm and an eyepiece of focal length 5 cm. An object is placed at a distance of 0·5 cm from the objective. What should be the separation between the lenses so that the microscope projects an inverted real image of the object on a screen 30 cm behind the eyepiece ?

11. An optical instrument used for angular magnification has a 25 D objective and a 20 D eyepiece. The tube length is 25 cm when the eye is least strained.

(a) Whether it is a microscope or a telescope ? (b) What is the angular magnification produced ?

12. An astronomical telescope is to be designed to have a magnifying power of 50 in normal adjustment. If the length of the tube is 102 cm, find the powers of the objective and the eyepiece.

13. The eyepiece of an astronomical telescope has a focal length of 10 cm. The telescope is focused for normal vision of distant objects when the tube length is 1·0 m. Find the focal length of the objective and the magnifying power of the telescope.

14. A Galilean telescope is 27 cm long when focused to form an image at infinity. If the objective has a focal length of 30 cm, what is the focal length of the eyepiece ?

15. A farsighted person cannot see objects placed closer to 50 cm. Find the power of the lens needed to see the objects at 20 cm.

16. A nearsighted person cannot clearly see beyond 200 cm. Find the power of the lens needed to see objects at large distances.

17. A person wears glasses of power – 2·5 D. Is the person farsighted or nearsighted ? What is the far point of the person without the glasses ?

18. A professor reads a greeting card received on his 50th birthday with + 2·5 D glasses keeping the card 25 cm away. Ten years later, he reads his farewell letter with the same glasses but he has to keep the letter 50 cm away. What power of lens should he now use ?

19. A normal eye has retina 2 cm behind the eye-lens. What is the power of the eye-lens when the eye is (a) fully relaxed, (b) most strained ?

20. The near point and the far point of a child are at 10 cm and 100 cm. If the retina is 2·0 cm behind the eye-lens, what is the range of the power of the eye-lens ?

21. A nearsighted person cannot see beyond 25 cm. Assuming that the separation of the glass from the eye is 1 cm, find the power of lens needed to see distant objects.

22. A person has near point at 100 cm. What power of lens is needed to read at 20 cm if he/she uses (a) contact lens, (b) spectacles having glasses 2·0 cm separated from the eyes?

23. A lady uses + 1·5 D glasses to have normal vision from 25 cm onwards. She uses a 20 D lens as a simple microscope to see an object. Find the maximum magnifying power if she uses the microscope (a) together with her glass (b) without the glass. Do the answers suggest that an object can be more clearly seen through a microscope without using the correcting glasses ?

24. A lady cannot see objects closer than 40 cm from the left eye and closer than 100 cm from the right eye. While on a mountaineering trip, she is lost from her team. She tries to make an astronomical telescope from her reading glasses to look for her teammates. (a) Which glass should she use as the eyepiece ? (b) What magnification can she get with relaxed eye ?

# ANSWERS

### OBJECTIVE I

1. (d)  2. (a)  3. (d)  4. (d)  5. (b)  6. (b)
7. (a)  8. (d)  9. (c)  10. (b)  11. (c)

### OBJECTIVE II

1. (a), (d)  2. (a), (d)  3. (a)
4. (a), (c)  5. (b), (d)

### EXERCISES

1. A, B, D, C
2. 8·1 cm from the lens
3. (a) 12·5 cm (b) 2 0
4. 2
5. 8 X
6. 8·4

7. 20 to 30
8. 0·04 mm
9. 2 cm
10. 5 cm
11. microscope, 20
12. 1 D, 50 D
13. 90 cm, 9
14. 3 cm
15. 3 D
16. − 0·5 D
17. nearsighted, 40 cm
18. + 4·5 D
19. 50 D, 54 D
20. + 60 D to + 51 D
21. − 4·2 D
22. + 4 D, + 4·53 D
23. 6, 9
24. right lens, 2

□

# CHAPTER 20

# DISPERSION AND SPECTRA

## 20.1 DISPERSION

As mentioned earlier, the refractive index of a material depends slightly on the wavelength of light. The relation between the two may be approximately described by the equation

$$\mu = \mu_0 + \frac{A}{\lambda^2},$$

where $A$ is a small positive constant known as Cauchy's constant. The refractive index decreases as the wavelength increases. For visible light, it is maximum for the violet end and minimum for the red end. Figure (20.1) shows the variation of refractive index with wavelength for some transparent materials.
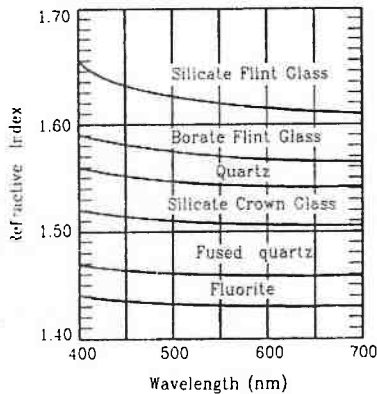


Figure 20.1

Because of the difference in refractive indices, light of different colours bend through different angles on refraction. If white light passes through a glass prism (figure 20.2), the violet rays deviate the most and the
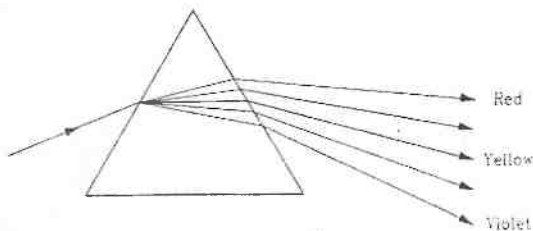


Figure 20.2

red rays deviate the least. Thus, white light is separated into its various component colours. This phenomenon of separation of different constituent colours of light while passing through a transparent medium is known as *dispersion of light*.

## 20.2 DISPERSIVE POWER

Consider a prism of a transparent material. When a beam of white light is passed through the prism, light of different wavelengths are deviated by different amounts. The overall deviation of the light beam is measured by the deviation of the yellow light as this deviation is roughly the average of all deviations. In figure (20.3), this deviation is shown by the symbol $\delta_y$. It is clear that if $\delta_r$ and $\delta_v$ are the deviations for red and violet components, the angular divergence of the transmitted beam is $\delta_v - \delta_r$. This divergence is called *angular dispersion*.
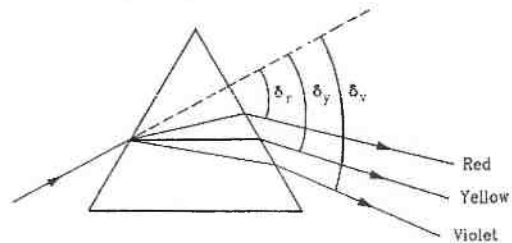


Figure 20.3

The mean deviation depends on the average refractive index $\mu$ and the angular dispersion depends on the difference $\mu_v - \mu_r$. It may be seen from figure (20.1) that if the average value of $\mu$ is small (fluorite), $\mu_v - \mu_r$ is also small and if the average value of $\mu$ is large (silicate flint glass), $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

The *dispersive power* of a material is defined as the ratio of angular dispersion to the average deviation when a light beam is transmitted through a thin prism placed in a position so that the mean ray (ray having

*Refractive indices for red, yellow and violet light are 1·514, 1·517 and 1·523 respectively for crown glass and 1·613, 1·620 and 1·632 for flint glass.*

**Solution :** The deviation produced by the crown prism is

$$\delta = (\mu - 1)A$$

and by the flint prism is

$$\delta' = (\mu' - 1)A'.$$

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'. \qquad \dots \text{(i)}$$

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'.$$

or,

$$A' = \frac{(\mu - 1)}{(\mu' - 1)} A = \frac{1·517 - 1}{1·620 - 1} \times 5°$$

$$= 4·2°.$$

(b) The angular dispersion produced by the crown prism is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is

$$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A'.$$

The net angular dispersion is,

$$\delta = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$

$$= (1·523 - 1·514) \times 5° - (1·632 - 1·613) \times 4·2°$$

$$= -0·0348°.$$

The angular dispersion has magnitude 0·0348°.

3. *The dispersive powers of crown and flint glasses are 0·03 and 0·05 respectively. The refractive indices for yellow light for these glasses are 1·517 and 1·621 respectively. It is desired to form an achromatic combination of prisms of crown and flint glasses which can produce a deviation of 1° in the yellow ray. Find the refracting angles of the two prisms needed.*

**Solution :** Suppose, the angle of the crown prism needed is $A$ and that of the flint prism is $A'$. We have

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1}$$

or,

$$\mu_v - \mu_r = (\mu - 1)\omega.$$

The angular dispersion produced by the crown prism is

$$(\mu_v - \mu_r)A = (\mu - 1)\omega A.$$

Similarly, the angular dispersion produced by the flint prism is

$$(\mu' - 1)\omega'A'.$$

For achromatic combination, the net dispersion should be zero. Thus,

$$(\mu - 1)\omega A = (\mu' - 1)\omega'A'$$

or,

$$\frac{A'}{A} = \frac{(\mu - 1)\omega}{(\mu' - 1)\omega'} = \frac{0·517 \times 0·03}{0·621 \times 0·05} = 0·50. \qquad \dots \text{(i)}$$

The deviation in the yellow ray produced by the crown prism is $\delta = (\mu - 1)A$ and by the the flint prism is $\delta' = (\mu' - 1)A'$. The net deviation produced by the combination is

$$\delta - \delta' = (\mu - 1)A - (\mu' - 1)A'$$

or,

$$1° = 0·517 A - 0·621 A'. \qquad \dots \text{(ii)}$$

Solving (i) and (ii), $A = 4·8°$ and $A' = 2·4°$. Thus, the crown prism should have its refracting angle 4·8° and that of the flint prism should be 2·4°.

□

## QUESTIONS FOR SHORT ANSWER

1. The equation $\omega = \dfrac{\mu_v - \mu_r}{\mu - 1}$ was derived for a prism having small refracting angle. Is it also valid for a prism of large refracting angle ? Is it also valid for a glass slab or a glass sphere ?

2. Can the dispersive power $\omega = \dfrac{\mu_v - \mu_r}{\mu - 1}$ be negative ? What is the sign of $\omega$ if a hollow prism is immersed into water ?

3. If three identical prisms are combined, is it possible to pass a beam that emerges undeviated ? Undispersed ?

4. "Monochromatic light should be used to produce pure spectrum". Comment on this statement.

5. Does focal length of a lens depend on the colour of the light used ? Does focal length of a mirror depend on the colour ?

6. Suggest a method to produce a rainbow in your house.

## OBJECTIVE I

1. The angular dispersion produced by a prism
   (a) increases if the average refractive index increases
   (b) increases if the average refractive index decreases
   (c) remains constant whether the average refractive index increases or decreases.
   (d) has no relation with average refractive index.

2. If a glass prism is dipped in water, its dispersive power
   (a) increases    (b) decreases    (c) does not change
   (d) may increase or decrease depending on whether the angle of the prism is less than or greater than 60°.

3. A prism can produce a minimum deviation δ in a light beam. If three such prisms are combined, the minimum deviation that can be produced in this beam is
   (a) 0    (b) δ    (c) 2δ    (d) 3δ.

4. Consider the following two statements :
   (A) Line spectra contain information about atoms.

(B) Band spectra contain information about molecules.
   (a) both A and B are wrong
   (b) A is correct but B is wrong
   (c) B is correct but A is wrong
   (d) both A and B are correct.

5. The focal length of a converging lens are $f_v$ and $f_r$ for violet and red light respectively.
   (a) $f_v > f_r$    (b) $f_v = f_r$    (c) $f_v < f_r$
   (d) Any of the three is possible depending on the value of the average refractive index μ.

## OBJECTIVE II

1. A narrow beam of white light goes through a slab having parallel faces.
   (a) The light never splits in different colours.
   (b) The emergent beam is white.
   (c) The light inside the slab is split into different colours.
   (d) The light inside the slab is white.

2. By properly combining two prisms made of different materials, it is possible to
   (a) have dispersion without average deviation
   (b) have deviation without dispersion
   (c) have both dispersion and average deviation
   (d) have neither dispersion nor average deviation.

3. In producing a pure spectrum, the incident light is passed through a narrow slit placed in the focal plane of an achromatic lens because a narrow slit

(a) produces less diffraction
   (b) increases intensity
   (c) allows only one colour at a time
   (d) allows a more parallel beam when it passes through the lens.

4. Which of the following quantities related to a lens depend on the wavelength or wavelengths of the incident light ?
   (a) Power.    (b) Focal length.
   (c) Chromatic aberration.    (d) Radii of curvature.

5. Which of the following quantities increase when wavelength is increased ? Consider only the magnitudes.
   (a) The power of a converging lens.
   (b) The focal length of a converging lens.
   (c) The power of a diverging lens.
   (d) The focal length of a diverging lens.

## EXERCISES

1. A flint glass prism and a crown glass prism are to be combined in such a way that the deviation of the mean ray is zero. The refractive index of flint and crown glasses for the mean ray are 1·620 and 1·518 respectively. If the refracting angle of the flint prism is 6 0°, what would be the refracting angle of the crown prism ?

2. A certain material has refractive indices 1·56, 1·60 and 1·68 for red, yellow and violet light respectively.
   (a) Calculate the dispersive power. (b) Find the angular dispersion produced by a thin prism of angle 6° made of this material.

3. The focal lengths of a convex lens for red, yellow and violet rays are 100 cm, 98 cm and 96 cm respectively. Find the dispersive power of the material of the lens.

4. The refractive index of a material changes by 0·014 as the colour of the light changes from red to violet. A rectangular slab of height 2·00 cm made of this material is placed on a newspaper. When viewed normally in yellow light, the letters appear 1·32 cm below the top surface of the slab. Calculate the dispersive power of the material.

5. A thin prism is made of a material having refractive indices 1·61 and 1·65 for red and violet light. The

dispersive power of the material is 0·07. It is found that a beam of yellow light passing through the prism suffers a minimum deviation of 4·0° in favourable conditions. Calculate the angle of the prism.

6. The minimum deviations suffered by red, yellow and violet beams passing through an equilateral transparent prism are 38·4°, 38·7° and 39·2° respectively. Calculate the dispersive power of the medium.

7. Two prisms of identical geometrical shape are combined with their refracting angles oppositely directed. The materials of the prisms have refractive indices 1·52 and 1·62 for violet light. A violet ray is deviated by 1·0° when passes symmetrically through this combination. What is the angle of the prisms ?

8. Three thin prisms are combined as shown in figure (20-E1). The refractive indices of the crown glass for red, yellow and violet rays are $\mu_r$, $\mu_y$ and $\mu_v$ respectively and
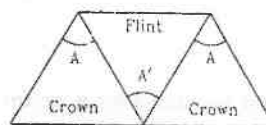


Figure 20-E1

those for the flint glass are $\mu'_r$, $\mu'_y$ and $\mu'_v$ respectively. Find the ratio $A'/A$ for which (a) there is no net angular dispersion, and (b) there is no net deviation in the yellow ray.

9. A thin prism of crown glass ($\mu_r = 1.515$, $\mu_v = 1.525$) and a thin prism of flint glass ($\mu_r = 1.612$, $\mu_v = 1.632$) are placed in contact with each other. Their refracting angles are $5.0°$ each and are similarly directed. Calculate the angular dispersion produced by the combination.

10. A thin prism of angle $6.0°$, $\omega = 0.07$ and $\mu_y = 1.50$ is combined with another thin prism having $\omega = 0.08$ and $\mu_y = 1.60$. The combination produces no deviation in the mean ray. (a) Find the angle of the second prism. (b) Find the net angular dispersion produced by the

combination when a beam of white light passes through it. (c) If the prisms are similarly directed, what will be the deviation in the mean ray? (d) Find the angular dispersion in the situation described in (c).

11. The refractive index of a material $M1$ changes by $0.014$ and that of another material $M2$ changes by $0.024$ as the colour of the light is changed from red to violet. Two thin prisms one made of $M1(A = 5.3°)$ and other made of $M2(A = 3.7°)$ are combined with their refracting angles oppositely directed. (a) Find the angular dispersion produced by the combination. (b) The prisms are now combined with their refracting angles similarly directed. Find the angular dispersion produced by the combination.

□

# ANSWERS

## OBJECTIVE I

1. (a)    2. (b)    3. (b)    4. (d)    5. (c)

## OBJECTIVE II

1. (b),(c)    2. (a), (b), (c)    3. (d)    4. (a), (b), (c)
5. (b), (d)

## EXERCISES

1. $7.2°$

2. (a) $0.2$    (b) $0.72°$

3. $0.041$

4. $0.026$

5. $7°$

6. $0.0206$

7. $10°$

8. (a) $\dfrac{2(\mu_y - 1)}{\mu'_y - 1}$    (b) $\dfrac{2(\mu_v - \mu_r)}{\mu'_v - \mu'_r}$

9. $0.15°$

10. (a) $5°$    (b) $0.03°$    (c) $6°$    (d) $0.45°$

11. (a) $0.0146°$ (b) $0.163°$

□

# CHAPTER 21

# SPEED OF LIGHT

## 21.1 HISTORICAL INTRODUCTION

The speed of light in vacuum is a fundamental constant in physics. The most interesting fact about this speed is that if an object moves with this speed in one frame, it has the same speed in any other frame. This led to a major revision of our concept of space and time and is the key fact on which the special theory of relativity is based.

In 1983, the speed of light was defined to be exactly 299, 792, 458 m/s. In fact, the length of an object is now defined to be 299, 792, 458 m/s multiplied by the time taken by the light to cross it. Thus, when one sends light from one place to another place and measures the time taken by the light to do so, one is not measuring the speed of light, rather one is measuring the distance between the two places.

Prior to 1983, the length was defined independently and one had a separate metre. The speed of light could then be measured as the length divided by the time taken by the light to cross it.

Perhaps, the great Indian talents in the Vedic age had the knowledge of the speed of light. G.V.Raghavrao in his book quotes a verse from Rigveda (I, 50-4) *Yojananam Sahastra Dwe Dwe Shate Dwe Cha Yojane Aken Nimishardhena Krammana Namostute*. In this verse, the author pays respects to the one (the reference is to the sun light) who moves 2202 *yojans* in half *nimish*. Yojan is a quite common unit in India, it means 4 kose, each kose measuring 8000 British yards and each yard measuring $0.9144$ m. The definition of the time unit nimish can be found in Shrimadbhagwat (III, 11-3 to 10) where it is mentioned that 15 nimishas make 1 *kashta*, 15 kashtas make one *laghu*, 30 laghus make 1 *muhurta* and 30 muhurtas make 1 *diva-ratri*. A diva-ratri is, of course, a day-night which is 24 hours in modern language. When you convert 2202 yojans per half nimish into SI units, it turns out to be $3.0 \times 10^8$ m/s up to two significant digits, a value quite accurate as we know it today.

In the modern era, perhaps the first attempt to measure the speed of light was made by Galileo. The design of the experiment was as follows. Two experimenters $A$ and $B$, each having a lantern and a shutter, stand on two small hills. The shutter can cover or uncover the lantern. Initially, both the lanterns are covered. One of the persons $A$ uncovers the lantern. The second person $B$ uncovers his lantern when he sees the light from the lantern of $A$. The first person $A$ covers his lantern when he sees the light from the lantern of $B$. The time elapsed between the uncovering and covering of the first lantern is measured. During this time, the light travels from the first person to the second person and then back. Knowing the distance and time, the speed of light may be calculated.

The proposed method failed because the speed of light is so large that a human being cannot respond with the required accuracy of timing. If the distance between the hills is as large as 15 km, the time taken by light in going back and forth is only one ten thousandth part of a second. The first recorded speed of light in modern era came through the astronomical observations by the Danish astronomer Olaf Roemer in 1676. The value obtained was about $2.1 \times 10^8$ m/s, somewhat smaller than the actual. In 1728, English astronomer Bradley measured the speed of light from his observations. The value was quite close to the correct one.

The first measurement of the speed of light from purely terrestrial experiments was reported by the French physicist Fizeau in 1849. The method was improved by another French physicist Foucault. Yet another method was proposed by American physicist Michelson. We now describe these three methods.

## 21.2 FIZEAU METHOD

Figure (21.1a) shows a schematic diagram of the arrangement used in this method. Light from a source $S$ passes through a convex lens $L_1$. The transmitted beam is intercepted by a semi-transparent inclined
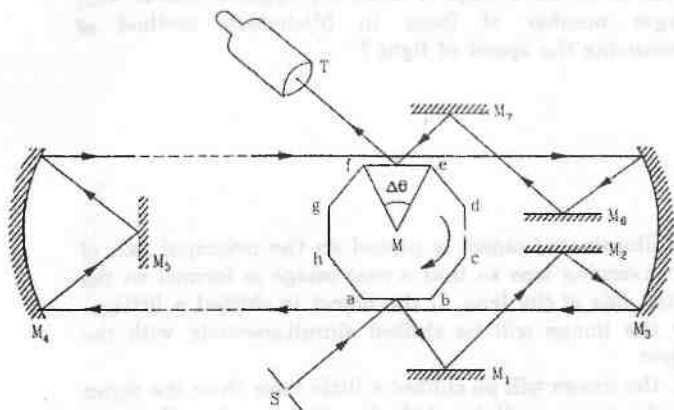
## 21.4 MICHELSON METHOD



Figure 21.3

The scheme of Michelson method to measure the speed of light is shown in figure (21.3). Light from an intense source $S$ is incident upon one face of a polygon-shaped mirror $M$. The light reflected from this surface is sent to the lower portion of a concave mirror $M_3$ after reflections from two plane mirrors $M_1$ and $M_2$. The geometry is set so that the light reflected from the concave mirror becomes parallel. This parallel beam of light is allowed to travel through a long distance (several kilometers) and falls on the lower portion of another concave mirror $M_4$. The parallel beam is converged at the focus of $M_4$ where a plane mirror $M_5$ is placed. $M_5$ reflects the beam back to the concave mirror $M_4$, this time at the upper portion. As $M_5$ is at the focus, the beam reflected by $M_4$ becomes parallel and travels back to the concave mirror $M_3$. After proper reflections from $M_3$ and the plane mirrors, it is sent to the polygonal mirror. A telescope is adjusted to receive the rays reflected by the polygonal mirror and hence, to form an image of the source.

Suppose the polygonal mirror $M$ is stationary. Light from the source falls on the face $ab$ of the mirror $M$ and after reflections from all the mirrors, finally falls on the face $ef$ of the mirror $M$. The image of $S$ is seen in the telescope. If the polygonal mirror rotates, the face $ef$ also turns a little while light travels between the two reflections from the polygonal mirror. The light thus fails to enter into the telescope and the image is not seen. If the rotational speed of the mirror is gradually increased, a stage comes when the adjacent face $fg$ takes the place of $ef$ by the time light comes there. Then, the light is again sent into the telescope.

In the experiment, one looks through the telescope and gradually increases the angular speed of the polygonal mirror. The image flickers initially and becomes steady at a particular angular speed of the mirror. This angular speed is measured.

Suppose,

$N$ = the number of faces in the polygonal mirror,

$\omega$ = the angular speed of rotation of the mirror when the image becomes steady,

$D$ = the distance travelled by the light between the reflections from the polygonal mirror.

If the speed of light is $c$, the time taken by the light to travel the distance $D$ is $\Delta t = D/c$. The angle rotated by the mirror during this time is $\Delta\theta = 2\pi/N$.

The angular speed of the mirror is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi/N}{D/c} = \frac{2\pi c}{DN}$$

or,

$$c = \frac{D\omega N}{2\pi}.$$

If $v$ be the frequency of rotation, $\omega/2\pi = v$ and

$$c = DvN. \qquad \ldots \ (21.3)$$

Michelson and his co-workers made a series of similar experiments. The first determination was made in 1879 with an octagonal rotating mirror. The latest in the series was underway at the time of the death of Michelson and was completed in 1935 by Pease and Pearson. This experiment used a rotating mirror with 32 faces.

□

## QUESTIONS FOR SHORT ANSWER

1. The speed of sound in air is 332 m/s. Is it advisable to define the length 1 m as the distance travelled by sound in 1/332 s ?

2. Consider Galileo's method of measuring the speed of light using two lanterns. To get an accuracy of about 10%, the time taken by the experimenter in closing or opening the shutter should be about one tenth of the time taken by the light in going from one experimenter to the other. Assume that it takes 1/100 second for an experimenter to close or open the shutter. How far should the two experimenters be to get a 10% accuracy ? What are the difficulties in having this separation ?

3. In Fizeau method of measuring the speed of light, the toothed wheel is placed in the focal plane of a converging

lens. How would the experiment be affected if the wheel is slightly away from the focal plane?

4. In the original Fizeau method, the light travelled 8·6 km and then returned. What could be the difficulty if this distance is taken as 8·6 m?

5. What is the advantage of using a polygonal mirror with larger number of faces in Michelson method of measuring the speed of light?

## OBJECTIVE I

1. Light passes through a closed cylindrical tube containing a gas. If the gas is gradually pumped out, the speed of light inside the tube will
   (a) increase     (b) decrease     (c) remain constant
   (d) first increase and then decrease.

2. The speeds of red light and yellow light are exactly same
   (a) in vacuum but not in air
   (b) in air but not in vacuum
   (c) in vacuum as well as in air
   (d) neither in vacuum nor in air.

3. An illuminated object is placed on the principal axis of a converging lens so that a real image is formed on the other side of the lens. If the object is shifted a little,
   (a) the image will be shifted simultaneously with the object
   (b) the image will be shifted a little later than the object
   (c) the image will be shifted a little earlier than the object
   (d) the image will not shift.

## OBJECTIVE II

1. The speed of light is 299,792,458 m/s
   (a) with respect to the earth
   (b) with respect to the sun
   (c) with respect to a train moving on the earth
   (d) with respect to a spaceship going in outer space.

2. Which of the following methods can be used to measure the speed of light in laboratory?

   (a) Roemer method.            (b) Fizeau method.
   (c) Focault method.           (d) Michelson method.

3. Which of the following methods can be used to measure the speed of light in water?
   (a) Roemer method.            (b) Fizeau method.
   (c) Focault method.           (d) Michelson method.

## EXERCISES

1. In an experiment to measure the speed of light by Fizeau's apparatus, following data are used :
      Distance between the mirrors = 12·0 km,
      Number of teeth in the wheel = 180.
   Find the minimum angular speed of the wheel for which the image is not seen.

2. In an experiment with Foucault's apparatus, the various distances used are as follows :
   Distance between the rotating and the fixed mirror = 16 m
   Distance between the lens and the rotating mirror = 6 m,

   Distance between the source and the lens = 2 m.
   When the mirror is rotated at a speed of 356 revolutions per second, the image shifts by 0·7 mm. Calculate the speed of light from these data.

3. In a Michelson experiment for measuring speed of light, the distance travelled by light between two reflections from the rotating mirror is 4·8 km. The rotating mirror has a shape of a regular octagon. At what minimum angular speed of the mirror (other than zero) the image is formed at the position where a non-rotating mirror forms it?

□

## ANSWERS

### OBJECTIVE I

1. (a)           2. (a)           3. (b)

### OBJECTIVE II

1. (a), (b), (c), (d)     2. (c)          3. (c)

### EXERCISES

1. $1·25 \times 10^4$ deg/s

2. $2·984 \times 10^8$ m/s

3. $7·8 \times 10^3$ rev/s

□

# PHOTOMETRY

We see an object when light coming from the object enters our eyes and excites the sensation of vision. The brightness sensed by the eye depends on the amount of light energy entering into it and the wavelength distribution of this energy. In this chapter, we shall study the factors responsible for the sensation of brightness.

## 22.1 TOTAL RADIANT FLUX

The total energy of radiation emitted by a source per unit time is called its *total radiant flux*. This radiation contains components of various wavelengths extending even beyond the visible range. However, not all wavelengths have equal contribution in making up the total radiation. In calculating total radiant flux of a source, the total energy emitted per unit time in the whole range of wavelengths must be calculated.

The SI unit of total radiant flux of a source is *watt*.

## 22.2 LUMINOSITY OF RADIANT FLUX

The brightness produced by radiation depends on the wavelength of the radiation besides depending on the total radiant flux. For example, consider two 10 W sources of light, one emitting yellow light and the other red light. Though both emit equal energy per unit time, yellow will look brighter than the red. The *luminosity of radiant flux* measures the capacity to produce brightness sensation in eye. A relative comparison of luminosity of radiant flux of different wavelengths can be made by the curve in figure (22.1). The figure represents relative luminosity under normal light conditions for an average person. The scale on the vertical axis is chosen arbitrarily. We see that for normal light conditions, the luminosity is maximum for wavelength around 555 nm and falls off on both sides. Radiation is "visible" if its luminosity is not zero. As the luminosity falls off gradually, there are no sharp cut-offs of visible region.
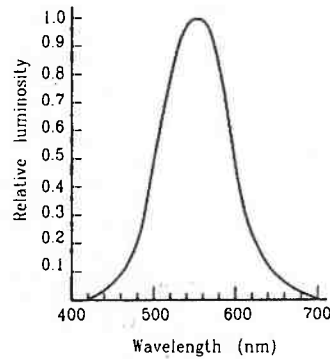


Figure 22.1

## 22.3 LUMINOUS FLUX : RELATIVE LUMINOSITY

In general, the radiation emitted by a source has components corresponding to a wide range of wavelengths. Different component wavelengths have different energies (in a given time) and different brightness producing capacities. The radiant flux is a quantity directly representing the total energy emitted per unit time. The *luminous flux* is a quantity directly representing the total brightness producing capacity of the source. Its unit is called *lumen*. The luminous flux of a source of 1/685 W emitting monochromatic light of wavelength 555 nm is called one lumen. In other words, a 1 W source emitting monochromatic light of wavelength 555 nm emits 685 lumen.

*Relative luminosity* of a wavelength refers to the fraction

$$\frac{\text{luminous flux of a source of given wavelength}}{\text{luminous flux of a 555 nm source of same power}}$$

It is often represented as a percentage. Thus, figure (22.1) represents the relative luminosity as a function of wavelength.

It should be clear that the luminous flux depends on the radiant flux as well as on the wavelength distribution.

4. *A point source emitting uniformly in all directions is placed above a table-top at a distance of 0·50 m from it. The luminous flux of the source is 1570 lumen. Find the illuminance at a small surface area of the table-top (a) directly below the source and (b) at a distance of 0·80 m from the source.*

**Solution :** Consider the situation shown in figure (22-W1). Let A be the point directly below the source S and B be the point at 0·80 m from the source.
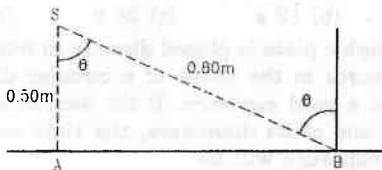


Figure 22-W1

The luminous flux of 1570 lumen is emitted uniformly in the solid angle $4\pi$. The luminous intensity of the source in any direction is

$$I = \frac{1570 \text{ lumen}}{4\pi \text{ sr}}$$

$$= 125 \text{ cd}.$$

The illuminance is

$$E = \frac{I \cos\theta}{r^2}.$$

At the point A, $r = 0·50$ m and $\theta = 0$. Thus,

$$E_A = \frac{125 \text{ cd}}{0·25 \text{ m}^2} = 500 \text{ lux}.$$

At the point B, $r = 0·80$ m and $\cos\theta = \dfrac{SA}{SB} = \dfrac{0·50}{0·80} = \dfrac{5}{8}$.

Thus, $$E_B = \frac{(125 \text{ cd}) \times \dfrac{5}{8}}{0·64 \text{ m}^2}$$

$$= 122 \text{ lux}.$$

5. *The luminous intensity of a small plane source of light along the forward normal is 160 candela. Assuming the source to be perfectly diffused, find the luminous flux emitted into a cone of solid angle 0·02 sr around a line making an angle of 60° with the forward normal.*
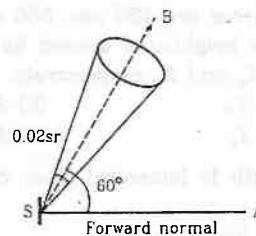


Figure 22-W2

**Solution :** The situation is shown in figure (22-W2). By Lambert's cosine law, the intensity in the direction SB is

$$I = I_0 \cos 60°,$$

where $I_0 = 160$ candela is the intensity along the forward normal.

Thus, $$I = (160 \text{ candela})\left(\frac{1}{2}\right)$$

$$= 80 \text{ candela}.$$

The luminous flux emitted in the cone shown in the figure is

$$\Delta F = I \Delta\omega$$

$$= (80 \text{ candela}) (0·02 \text{ sr})$$

$$= 1·6 \text{ lumen}.$$

□

# QUESTIONS FOR SHORT ANSWER

1. What is the luminous flux of a source emitting radio waves ?

2. The luminous flux of a 1 W sodium vapour lamp is more than that of a 10 kW source of ultraviolet radiation. Comment.

3. Light is incident normally on a small plane surface. If the surface is rotated by an angle of 30° about the incident light, does the illuminance of the surface increase, decrease or remain same ? Does your answer change if the light did not fall normally on the surface ?

4. A bulb is hanging over a table. At which portion of the table is the illuminance maximum ? If a plane mirror is placed above the bulb facing the table, will the illuminance on the table increase ?

5. The sun is less bright at morning and evening as compared to at noon although its distance from the observer is almost the same. Why ?

6. Why is the luminous efficiency small for a filament bulb as compared to a mercury vapour lamp ?

7. The yellow colour has a greater luminous efficiency as compared to the other colours. Can we increase the illuminating power of a white light source by putting a yellow plastic paper around this source ?

## OBJECTIVE I

1. The one parameter that determines the brightness of a light source sensed by an eye is
   (a) energy of light entering the eye per second
   (b) wavelength of the light
   (c) total radiant flux entering the eye
   (d) total luminous flux entering the eye.

2. Three light sources $A$, $B$ and $C$ emit equal amount of radiant energy per unit time. The wavelengths emitted by the three sources are 450 nm, 555 nm and 700 nm respectively. The brightness sensed by an eye for the sources are $X_A$, $X_B$ and $X_C$ respectively. Then,
   (a) $X_A > X_B$, $X_C > X_B$          (b) $X_A > X_B$, $X_B > X_C$
   (c) $X_B > X_A$, $X_B > X_C$          (d) $X_B > X_A$, $X_C > X_B$.

3. As the wavelength is increased from violet to red, the luminosity
   (a) continuously increases
   (b) continuously decreases
   (c) increases then decreases
   (d) decreases then increases.

4. An electric bulb is hanging over a table at a height of 1 m above it. The illuminance on the table directly below the bulb is 40 lux. The illuminance at a point on the table 1 m away from the first point will be about
   (a) 10 lux     (b) 14 lux     (c) 20 lux     (d) 28 lux.

5. Light from a point source falls on a screen. If the separation between the source and the screen is increased by 1%, the illuminance will decrease (nearly) by
   (a) 0·5%     (b) 1%     (c) 2%     (d) 4%.

6. A battery-operated torch is adjusted to send an almost parallel beam of light. It produces an illuminance of 40 lux when the light falls on a wall 2 m away. The illuminance produced when it falls on a wall 4 m away is close to
   (a) 40 lux     (b) 20 lux     (c) 10 lux     (d) 5 lux.

7. The intensity produced by a long cylindrical light source at a small distance $r$ from the source is proportional to
   (a) $\frac{1}{r^2}$     (b) $\frac{1}{r^3}$     (c) $\frac{1}{r}$     (d) none of these.

8. A photographic plate placed at a distance of 5 cm from a weak point source is exposed for 3 s. If the plate is kept at a distance of 10 cm from the source, the time needed for the same exposure is
   (a) 3 s     (b) 12 s     (c) 24 s     (d) 48 s.

9. A photographic plate is placed directly in front of a small diffused source in the shape of a circular disc. It takes 12 s to get a good exposure. If the source is rotated by 60° about one of its diameters, the time needed to get the same exposure will be
   (a) 6 s     (b) 12 s     (c) 24 s     (d) 48 s.

10. A point source of light moves in a straight line parallel to a plane table. Consider a small portion of the table directly below the line of movement of the source. The illuminance at this portion varies with its distance $r$ from the source as
    (a) $I \propto \frac{1}{r}$     (b) $I \propto \frac{1}{r^2}$     (c) $I \propto \frac{1}{r^3}$     (d) $I \propto \frac{1}{r^4}$.

11. Figure (22-Q1) shows a glowing mercury tube. The intensities at point $A$, $B$ and $C$ are related as
    (a) $B > C > A$          (b) $A > C > B$
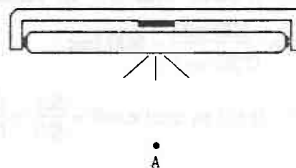    (c) $B = C > A$          (d) $B = C < A$.



Figure 22-Q1

## OBJECTIVE II

1. The brightness producing capacity of a source
   (a) does not depend on its power
   (b) does not depend on the wavelength emitted
   (c) depends on its power
   (d) depends on the wavelength emitted.

2. A room is illuminated by an extended source. The illuminance at a particular portion of a wall can be increased by
   (a) moving the source          (b) rotating the source
   (c) bringing some mirrors in proper positions
   (d) changing the colour of the source.

3. Mark the correct options.
   (a) The luminous efficiency of a monochromatic source is always greater than that of a white light source of same power.
   (b) The luminous efficiency of a monochromatic source

of wavelength 555 nm is always greater than that of a white light source of same power.
   (c) The illuminating power of a monochromatic source of wavelength 555 nm is always greater than that of a white light source of same power.
   (d) The illuminating power of a monochromatic source is always greater than that of a white light source of same power.

4. Mark the correct options.
   (a) Luminous flux and radiant flux have same dimensions.
   (b) Luminous flux and luminous intensity have same dimensions.
   (c) Radiant flux and power have same dimensions.
   (d) Relative luminosity is a dimensionless quantity.

## EXERCISES

1. A source emits 45 joules of energy in 15 s. What is the radiant flux of the source ?

2. A photographic plate records sufficiently intense lines when it is exposed for 12 s to a source of 10 W. How long should it be exposed to a 12 W source radiating the light of same colour to get equally intense lines ?

3. Using figure (22.1), find the relative luminosity of wavelength (a) 480 nm, (b) 520 nm (c) 580 nm and (d) 600 nm.

4. The relative luminosity of wavelength 600 nm is 0·6. Find the radiant flux of 600 nm needed to produce the same brightness sensation as produced by 120 W of radiant flux at 555 nm.

5. The luminous flux of a monochromatic source of 1 W is 450 lumen/watt. Find the relative luminosity at the wavelength emitted.

6. A source emits light of wavelengths 555 nm and 600 nm. The radiant flux of the 555 nm part is 40 W and of the 600 nm part is 30 W. The relative luminosity at 600 nm is 0·6. Find (a) the total radiant flux, (b) the total luminous flux, (c) the luminous efficiency.

7. A light source emits monochromatic light of wavelength 555 nm. The source consumes 100 W of electric power and emits 35 W of radiant flux. Calculate the overall luminous efficiency.

8. A source emits 31·4 W of radiant flux distributed uniformly in all directions. The luminous efficiency is 60 lumen/watt. What is the luminous intensity of the source ?

9. A point source emitting 628 lumen of luminous flux uniformly in all directions is placed at the origin. Calculate the illuminance on a small area placed at (1·0 m, 0, 0) in such a way that the normal to the area makes an angle of 37° with the X-axis.

10. The illuminance of a small area changes from 900 lumen/m$^2$ to 400 lumen/m$^2$ when it is shifted along its normal by 10 cm. Assuming that it is illuminated by a point source placed on the normal, find the distance between the source and the area in the original position.

11. A point source emitting light uniformly in all directions is placed 60 cm above a table-top. The illuminance at a point on the table-top, directly below the source, is 15 lux. Find the illuminance at a point on the table-top 80 cm away from the first point.

12. Light from a point source falls on a small area placed perpendicular to the incident light. If the area is rotated about the incident light by an angle of 60°, by what fraction will the illuminance change ?

13. A student is studying a book placed near the edge of a circular table of radius $R$. A point source of light is suspended directly above the centre of the table. What should be the height of the source above the table so as to produce maximum illuminance at the position of the book ?

14. Figure (22-E1) shows a small diffused plane source $S$ placed over a horizontal table-top at a distance of 2·4 m with its plane parallel to the table-top. The illuminance at the point $A$ directly below the source is 25 lux. Find the illuminance at a point $B$ of the table at a distance of 1·8 m from $A$.
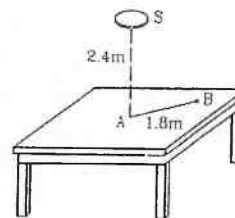


Figure 22-E1

15. An electric lamp and a candle produce equal illuminance at a photometer screen when they are placed at 80 cm and 20 cm from the screen respectively. The lamp is now covered with a thin paper which transmits 49% of the luminous flux. By what distance should the lamp be moved to balance the intensities at the screen again ?

16. Two light sources of intensities 8 cd and 12 cd are placed on the same side of a photometer screen at a distance of 40 cm from it. Where should a 80 cd source be placed to balance the illuminance ?

□

## ANSWERS

### OBJECTIVE I

1. (d)      2. (c)      3. (c)      4. (b)      5. (c)      6. (a)
7. (c)      8. (b)      9. (c)      10. (c)      11. (d)

### OBJECTIVE II

1. (c), (d)          2. (a), (b), (c), (d)      3. (b), (c)
4. (b), (c), (d)

### EXERCISES

1. 3 W
2. 10 s
3. 0·14          (b) 0·68          (c) 0·92          (d) 0·66
4. 200 W
5. 66%
6. (a) 70 W      (b) 39730 lumen      (c) 568 lumen/W

7. 240 lumen/W
8. 150 cd
9. 40 lux
10. 20 cm
11. 3·24 lux

12. it will not change
13. $R/\sqrt{2}$
14. 6·1 lux
15. 24 cm
16. 80 cm